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MARS FREE RETURNS VIA GRAVITY ASSIST FROM VENUS

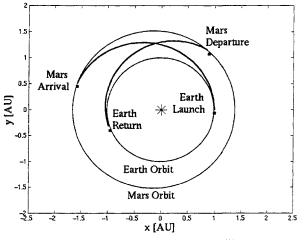
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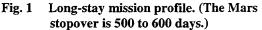
The safety of the crew is the top priority for human exploration of Mars. If an unexpected emergency occurs, a free-return trajectory brings the spacecraft back to the Earth without a deterministic maneuver. We use an automated design tool to search for Mars free-return trajectories, which satisfy NASA's Design Reference Mission (DRM) constraints. While no cases exist for Mars alone, a Mars-Venus free return does meet the DRM requirements for a launch in 2014. This trajectory is remarkably fortuitous as it does not exist for many years prior to or after the 2014 date.

Introduction

VARIOUS concepts have been proposed to achieve the first human mission to Mars.¹⁻¹³ Of chief concern in all proposals is how to minimize the exposure of the crew to the hazardous space environment, while keeping the propellant costs to an acceptable level. The problem falls into the classic one, familiar to all mission designers: minimize some combination of flight time and ΔV .

In this paper, we analyze Mars free returns via gravity assist from Venus. We briefly review free returns using Mars without Venus and then consider the advantages that a Venus gravity assist confers. We approach the problem both analytically and numerically. A recently discovered graphical technique based on Tisserand's criterion gives insight into how gravity-assist paths can be discovered. We also make use of two software tools, the Satellite Tour Design Program (STOUR)¹⁴⁻¹⁸ and the Mission Design and Analysis Software (MIDAS),¹⁹ which employ a patched-conic method and treat gravity assist as impulsive. Finally, to assess Mars-Venus flybys as an abort option for the future Mars mission NASA's DRM is used as the most realistic baseline.





Trajectory Classifications

The type of trajectory to be used is the foremost among the choices that must be made to satisfy mission objectives and constraints. In this section we briefly review some of the classic trajectories of the literature.

Long-Stay Mission

Due to the energy and phasing constraints, Mars trajectory options are characterized by the length of the Mars stopover. The long-stay mission, often referred to as conjunction class missions, keep ΔV cost low by using near-Hohmann transfers for both inbound and outbound arcs (Fig. 1). Outbound and inbound transits take about 250 days each, although a modest increase in launch ΔV reduces TOF to as few as 100 days.¹ The

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phasing requires a relatively long stay time (500 to 600 days) at Mars.

Short-Stay Mission

The short-stay mission, often referred to the opposition class mission, is as characterized by one transit leg having a large transfer angle (Fig. 2). Transfer to Mars may be either Type I (with transfer angle less than 180°) or Type II (with transfer angle greater than 180°). The short-stay mission achieves a short Mars stopover of 30 to 90 days for a total mission duration of 400 to 650 days. (In the extreme case of a 0-day stopover at Mars we have a free-return trajectory.) A spacecraft placed in a Type II leg may fly inside the orbit of Venus, raising concerns about the higher solar radiation dose. In general, the opposition class has a higher energy requirement than the conjunction class mission.

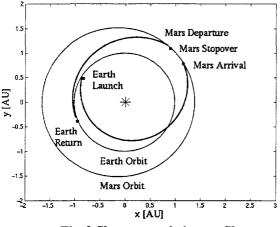


Fig. 2 Short-stay mission profile.

Free Returns

In the past, free-return trajectories have been critical in the event of an emergency (e.g. Apollo 13). Manned missions require numerous complex systems, yet any single failure could result in the loss of the crew as emphasized by Lyne and Townsend.⁹ For Mars missions, however, additional capabilities for free returns may significantly increase the total mass, subject the crew to the excessive g-loads upon Earth return, and result in prolonged crew exposure to the interplanetary radiation environment. These assertions led the DRM to the alternative "abort to Mars surface strategy."^{3,4,6}

Cycler Orbits

Mars mission In other scenarios. consecutively repeated free returns may be used for a long-term transportation system. A space station placed in "cycler orbits" provides a safe haven for crew, who may transit to the station using a smaller "taxi" vehicle with small propellant expenditures. Some of the representative cyclers include VISIT (Versatile International Station for Interplanetary Transport)^{1,20} and the up/down escalator cycler (Aldrin cycler).²¹ Both VISIT and the escalator cyclers are considered to repeat endlessly, as the inertial geometry of Earth and Mars repeats itself every 15 years.

VISIT-I cycler with a period of 1.25 years would encounter Earth every 5 years and Mars every 3.75 years. VISIT-II has a period of 1.5 years and encounters Earth once every 3 years and Mars every 7.5 years. The irregular planetary flyby sequences of VISIT result in large variations in Mars stay time. To solve this problem, some concepts proposed placing multiple vehicles VISIT at different orientations to allow more frequent travel opportunities. Yet even with a network of three VISIT vehicles, Mars stay time can vary from 1.6 to 5.9 years.¹

The escalator cycler allows a Mars encounter between every Earth flyby, which occurs every 2 years on average. The cycler is maintained largely by the gravity assists, in which every planetary flyby shifts the major axis of the orbit to allow a vehicle to encounter the next body on an outbound or inbound leg. Compared to VISIT, the escalator cycler imposes higher energy requirements.

Venus Flyby Modes

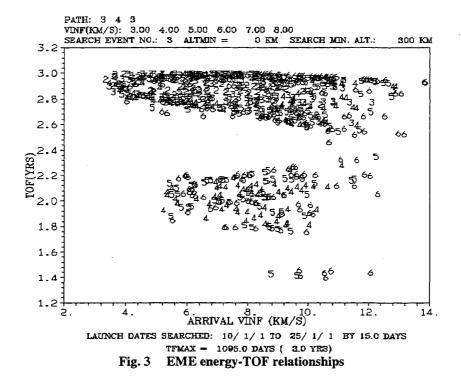
In the initial stage of Mars explorations by humans, a free return may be more useful as a means for the crew to safely return to the Earth in case of emergency. It is known that the high-energy requirements of the Type II transfer between Earth and Mars can be reduced with a close approach to Venus en route. As an extreme case for the short-stay mission is a mission with a zero Mars stay time, the effect of Venus flybys can also be employed for free returns.⁹⁻¹³ Lyne and Townsend⁹ have proposed that such double flybys may be used as a swingby abort for NASA's Mars Design Reference Mission with some modifications. They have shown that if the ERV and the outbound crew vehicle are linked together, the propulsive capability would be enough to perform a powered Mars swingby followed by sequential encounters of Venus and Earth.

Mars Free-Return Trajectories

Between the two planets Hohmann transfers are the most desirable from the energy point of view. For circular, coplanar orbits, minimum ΔV transfers between Earth and Mars have an orbital period of 1.42 years. It is therefore impossible to return to the Earth after a single revolution because the Earth would not be in the correct position. The orbital period of Mars is 1.88 years, making the Earth-Mars synodic period 2.14 years. The launch opportunities for Earth-Mars-Earth (EME) free returns²² are known to have a periodicity of 2.14 years as well. The inertial geometry of the two planets repeats every approximately 15 years.

We use STOUR to compute free-return trajectories for a 15-year period starting in January 2010. Because a typical human Mars mission takes about 2 to 3 years, we limit the search to a total time of flight (TOF) of 3 years. In Fig. 3 TOF is plotted against the arrival V.... The plot shows that local minimum arrival V_∞s occur where the TOFs are approximately 1.4, 2.0 and 3.0 years. Each number on the plot represents a mission identified by STOUR, where the numbers 1, 2, 3, 4, 5 and 6 in the figures correspond to launch V_∞s of 3.0, 4.0, 5.0, 6.0, 7.0 and 8.0 km/s, respectively. We observe that the launch V_∞s of 7 and 8 km/s (the numbers 5 and 6 in the plot) are sufficient to achieve a short TOF of about 1.4 years. In the family of 2-year TOF, the lowest discrete launch V_∞ presented in Fig. 3 is 6 km/s. In other words, to achieve TOF of 2 years the launch V_{∞} must be between 5.00 and 6.00 km/s. But if the TOF is relaxed to 3 years, both the minimum launch and arrival V_{∞} required become less than 4 km/s.

As opposed to the launch velocities, the arrival velocities can be reduced with propulsive maneuvers, aerobraking or a combination of both. Thus the constraints in the arrival V_{∞} could differ significantly from

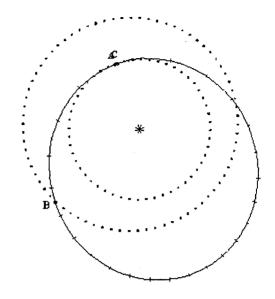


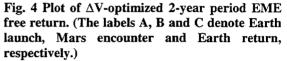
Tuble 1 A v optimized EME field field for (1-2.0						
years)						
Earth launch	01/11/2014	V _∞ =5.01 km/s				
E-to-M transit	135 days					
Mars encounter	05/26/2014	$V_{\infty} = 11.22 \text{ km/s}$				
M-to-E transit	593 days					
Earth return	01/09/2016	V_{∞} =5.02 km/s				
Total TOF	728 days					

 Δ V-optimized EME free return (P=2.0 Table 1

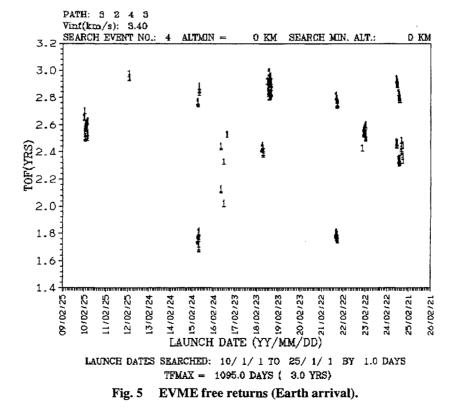
one mission to another. So, although the launch velocities are constrained to a range of 3.40 to 4.80 km/s, STOUR does not restrict the planetary encounter velocities. We note that Fig. 3 shows missions with arrival V_∞s of up to about 14 km/s.

For manned missions, both launch energy and arrival energies should be as low as possible, limiting the TOF of feasible freereturn trajectories to about 2 and 3 years, because variations from these TOFs always result in higher launch or arrival V... The lowenergy requirement for TOFs of integer Earth years is predicted from collision-orbit theory. For example, the lowest-energy mission with a 2-year TOF is a collision orbit with an orbital





period of 2 years. We note, however, that the orbital period of an EME with a TOF of 3.0 years may be either 1.5 or 3.0 years. The 1.5year period provides low $\Delta V \cos t$, while 3.0year period achieves fast-transits for the Type I



portion of the trajectories.

Collision orbits with periods of 1.5, 2.0 and 3.0 years are subsets of the EME freereturn trajectories, and they provide missions with the minimum total ΔVs as TOFs approach 3.0, 2.0 and 3.0 years, respectively. We use MIDAS to find the minimum ΔV solution for a 2-year period case. We find that no deep-space maneuver is required in the optimized solution, so that all of the ΔV is used for launch from Earth. Table 1 provides the dates and V_∞s at Earth launch, Mars flyby, and Earth return. Figure 4 shows the plot of the trajectory. As expected from collision orbit theory, the values of launch and arrival V_{∞} are nearly identical (5.01 and 5.02 km/s, respectively). For EME free returns, two years is the shortest TOF achievable with a typical launch V_∞ range (up to 5.5 km). Although the short outbound transit time of 135 days in our optimized case is highly desirable, the Mars approach speed of 11.22 km/s poses problems for the nominal mission.

Mars Free-Returns via Gravity Assist from Venus

We now use STOUR to search for Mars free-return trajectories with outbound or inbound Venus flybys. Earth-Venus-Mars-Earth (EVME) and Earth-Mars-Venus-Earth (EMVE) launch opportunities are shown in Fig. 5 and Fig. 7, respectively. The initial search assumes a minimum flyby altitude of 0 km to obtain the entire spectrum of trajectories, but our finer analysis and optimization are performed using the minimum flyby altitude of 300 km for Mars and Venus. The initial Earth parking orbit is assumed to be 200 km.

In Fig. 5 the 1s in the plot represent missions with launch V_{∞} of 4.00 km/s. We note that this launch velocity can achieve total TOF of only around 3 years for EME cases (Fig. 3). With the gravity assist from Venus, the same launch energy can achieve the total TOF of less than 1.7 years (Fig. 5), which for EME can be achieved only with a launch V_{∞} of above 6 km/s. When we perform the search with a slightly wider range of launch V_{∞} of 3.40 to 4.80 km/s with an increment of 0.2 km/s), the shortest TOF of 1.3 years occurs in April 2017 (launch $V_{\infty} = 4.60$ km/s). We confirm this result with MIDAS. In the ΔV_{-} optimization, we fixed the TOF to 1.3 years and allowed the launch date to be free (see Table 2 and Fig. 6). As shown in Fig. 6, the gravity assist from Venus raises aphelion from approximately the orbital radius of Earth to that of Mars. We note that slight reduction in total ΔV is possible by inserting a deep-space maneuver at location C in Fig. 6. To facilitate comparison between all free-return cases we omit this maneuver in Table 2.

The EMVE free-return trajectories are shown in Fig. 7. The number 1 in the plot represents each mission with the launch V_{∞} of 4.80 km/s. Missions with TOF less than 2.2 years occur in 2013 through 2014 and in 2024. The number of EMVE mission opportunities appears to be much less than EVME, however

Table 2	ΔV -optimized	EVME fre	e return

Table 2 Av-optimized By MB free feturin					
Earth launch	04/09/2017	V _w =5.00 km/s			
Venus encounter	09/10/2017	V _∞ =9.91 km/s			
E-to-M transit ^a	334 days				
Mars encounter	03/09/2018	V _∞ =6.02 km/s			
M-to-E transit	124 days				
Earth return	07/11/2018	V _∞ =14.08 km/s			
Total TOF	458 days				

^aNo deep-space ΔV maneuver allowed.

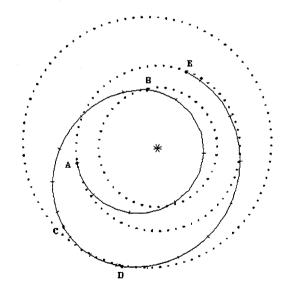
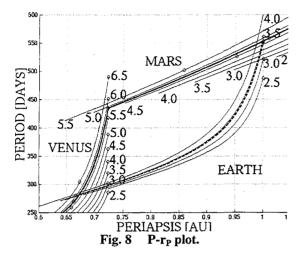


Fig. 6 Plot of optimized EVME free return. (The labels A, B and D denote Earth launch, Mars encounter and Earth return, respectively. The label C represents the location of maneuver to further improve the total ΔV cost.

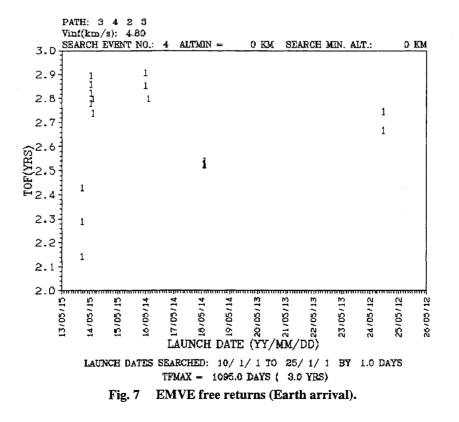


the EVME trajectories often have high arrival V_{∞} , which is undesirable.

The difference in the mission opportunities can be explained in terms of energy of the available orbit by employing a "P-r_P plot,"^{23,24} a graphical representation of Tisserand's criterion. The contours in Fig. 8 represent constant V_{∞} for each planet, assuming circular, coplanar orbits. A gravity assist from Mars or Venus rotates the V_{∞}

vector of the spacecraft along one of these contours to modify the orbit about the Sun. The furthest point to the right on the V_{∞} contour corresponds to alignment of the spacecraft's velocity vector with the planet's velocity. Rotation of the V_∞ vector away from this alignment corresponds to moving from right to left on the V_{∞} contour. A transfer between those planets may exist, where contours of two planets intersect. How far a spacecraft can travel along a contour of P-r_P plot in one flyby is constrained by the minimum flyby altitude allowed. The dots on contours indicate how much an orbit could be shaped in a single gravity assist with a flyby altitude of 300 km by moving either up or down the contour.

Several observations can be made from the plot in Fig. 8. As expected, the minimum energy required to get to Venus is shown to be lower than the minimum energy required to get to Mars.²⁵ The minimum V_{∞} at Mars and Venus required for both EVME and EMVE are dictated by Hohmann transfers, whose V_{∞} contours are shown as solid bold lines. The

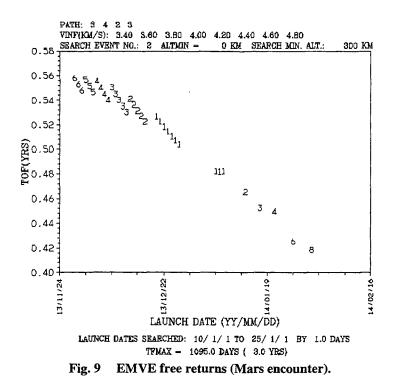


dashed bold line represents the minimum launch (or arrival) V_{∞} contour required for an EMVE (or EVME) free return. The point where the solid bold Mars line and dashed bold Earth lines intersect thus represents the minimum energy launch (arrival) condition. We note that the launch V_{∞} contour represented by the dashed bold line provides a region of EVME launch opportunities, as opposed to a single point for EMVE. This is one of the reasons for the difference in the mission opportunities between these two sequences.

The P- r_P plot does not provide phasing information, however. The planets Earth, Mars and Venus have a composite periodicity of about 6.4 years, and the characteristics of EVME or EMVE are also expected to exhibit a similar periodicity.¹² For example, Fig. 5 shows that similar families of launch opportunities appear every 6.4 years for the chosen time frame. However, the phasing of the three planets becomes sensitive to the eccentricity of Mars² and the repeatability of mission is not obvious as that of EME.¹⁴

Case Study

NASA has been investigating a plan known as the Design Reference Mission (DRM).³ The "split mission" strategy breaks the mission elements into cargo and piloted flights. Cargo will be transferred on a low energy, longer transit-time trajectory, while the crew will be flown on a higher energy, shorter transit-time trajectory. According to the DRM Version 3.0,⁴ two cargo missions in 2011 will consist of one flight containing the Earth Return Vehicle (ERV) and a second flight containing a cargo lander with a propellant production plant, power systems, an inflatable hab and an ascent vehicle. The piloted mission, in which the first human landing on Mars occurs in 2014, spends 130 to 180 days on fast transit trajectory to Mars. After a 500-day or so stopover, the crew would spend another 130 to 180 days on the return trip to the Earth. We assess the applicability of Mars free returns via gravity assist from Venus against NASA's comprehensive DRM.



	Launch	Earth	Transit	Total	Mars	Earth
	Date,	Launch V.,	To Mars,	TOF,	Arrival V.,	Arrival V _∞ ,
	MM/DD/YY	km/s	days	years	km/s	km/s
DRM ^a	01/17/2014	3.69	180	-	5.85	7.04
EME (P=1.5)	12/29/2013	3.30	184	3.00 ^b	6.90	3.30
EME (P=2.0)	01/11/2014	5.01 ^b	135	1.99	11.22 ^b	5.02
EME (P=3.0)	01/18/2014	6.84 ^b	111	3.00 ^b	14.63 ^b	6.85
EMVE	01/14/2014	3.67	169	2.19	6.90	4.56

^aRepresentative values for DRM-class mission consistent with 180-day transit time for both legs. ^bValues exceed constraint guidelines.

Mission Constraints

A mission with free returns must also satisfy the requirements for the nominal mission. Launch energy, trip time and arrival speeds are the major competing constraints for DRM, although none of them are considered "hard".⁵ To minimize effects on crew and consumable masses, trip time should be less than 180 days (6 months). But in case of an emergency, the outbound trajectory must also bring the crew back to the Earth within a reasonable time. The DRM crew flight will contain required consumables for "the Mars transit and surface duration of approximately 800 days (approximately 180 days for transit and approximately 600 days on the surface) as well as all the required systems for the crew during the 180-day transit trip".⁴ In this study therefore an 800-day (or 2.2 yr) TOF is assumed for the mission abort constraint. The atmospheric entry speed should also be low as possible to minimize structural mass, g-load on crew and to maximize entry corridors. Again, in the event of an emergency the same trajectory that satisfies the Mars atmospheric entry speed limit must also satisfy the reentry speed constraint at Earth.

Gravity-Assisted Abort Options

The mission characteristics of the low ΔV cases for EME free returns are summarized as benchmarks in Table 3. The EME collision orbit with a period of 1.5 years (TOF of 3.0 years) requires slightly lower launch and arrival velocities than DRM, yet the 250-day outbound transit time breaks the constraint of the 180-day guideline. In addition, the corresponding 3-year TOF requires additional

consumables. A free-return collision orbit with an orbital period of 2 years satisfies the TOF constraints for transit to Mars for the nominal mission and to the Earth for the emergency case. Yet this high-energy trajectory requires launch and arrival V_∞s of above 5.0 km/s each and Mars encounter V_{∞} of 11.22 km/s. The launch V_∞ and Mars approach speeds for the 3year period EME are unacceptably high in DRM standards. As discussed above, deviations in TOF from 2 or 3 years always require higher launch or arrival V_∞ or both. In summary, the EME abort option for DRM requires costly propellant expenditure and additional systems to safeguard against the relatively improbable event of system failure in flight.

The DRM crew mission launch date, as it turns out, is an extremely favorable year for EMVE free-return trajectories. Fig. 9 shows Earth-to-Mars TOF versus launch dates with

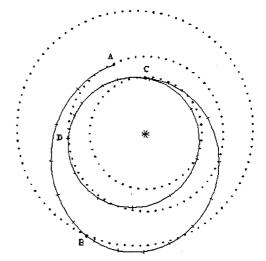


Fig. 10 Plot of optimized EMVE abort option.

the V_{∞} range of 3.40 to 4.80 km/s. The EMVE in January 2014 meets the outbound TOF constraint of 180 days. Some of these trajectories also meet the total TOF constraint guideline of 2.2 years (Fig. 7). The parameters of a case for the EMVE abort option are shown in Table 3.

EVME free-return trajectories are not available in January 2014. They are not considered as a valid abort option for DRMclass mission in any event, because for the given launch energy range, the Type II outbound trajectory does not meet the 180-day TOF constraint.

Although an EMVE free return does expose the crew to higher radiation levels than with the nominally planned mission, it provides a return mechanism for numerous failure scenarios.

Aero-Gravity Assisted Abort Option

The DRM version 3.0 report sizes the entry vehicle for aerocapture.⁴ In this study a triconic aerobrake shape with lift-to-drag ratio (L/D) of 0.6 for a trim angle of 47° is proposed. The shape has the lifting capability to meet all aerocapture and descent-to-surface

requirements. The study team demonstrates that the aerocapture at Mars does not exceed the 5g maximum deceleration limit, a limit necessary to crew safety and performance during the aerobraking maneuver.

Such a vehicle design may enhance the performance of free returns as well. Although it may be small, a lifting force oriented downward would help bend the velocity vector during a planetary flyby. Fig. 11 shows the EMVE aerogravity assisted (AGA) free-return mission opportunities around the DRM crew flight launch dates. The numbers (1, 2, 3, 4, 5, 6, 7, 8)and letters (A, B, C, D, E, F, G) in the plot represent GA and AGA missions with corresponding launch V_{∞} , respectively. The intention here is to show the potential and implication of the vehicle design upon the freereturn capabilities. The L/D and Mars flyby altitude are approximated as constant values of 0.6 and 61 km, respectively. The AGA fills the gaps between two optimal launch windows seamlessly, as can be seen by comparing Figs. 9 and 11. Also the AGA reduces the Earth-to-Mars TOF by more than 10 days.

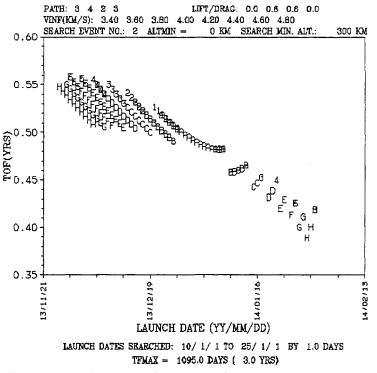


Fig. 11 Aero-gravity assisted free return abort option (Mars encounter).

Conclusions

We have revisited the problem of Mars free returns in the light of current plans for the first human mission. For the likely launch window, Mars alone does not provide an acceptable free return. However, when we consider Venus-Mars or Mars-Venus paths we find that the Mars-Venus free return is acceptable because it can achieve a short transit time to Mars.

Fortuitously our Mars-Venus free-return trajectory fits neatly into the Design Reference Mission for 2014. The free return satisfies all of the DRM constraints concerning launch energy, launch window, flight time to Mars, and total time to return. If the lifting body of the aerocapture vehicle is used for an aerogravity assist, the launch window for the free return opens wider and in many cases required time of flight reduces.

This free return trajectory could easily be adopted in future DRM plans, as it requires no changes in the mission constraints or vehicle specifications and it significantly improves crew safety by granting a practical abort option similar to that of Apollo 13.

The trajectory is only available in 2014 making this launch year particularly important for the first human mission to Mars.

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