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Mars Free Return Trajectories

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For manned missions to Mars, the top priority is a safe return of the crew to Earth. In the case of an emergency, trajectories that naturally return to the Earth with no intervention are preferred. This paper presents the computation of all possible Mars Free Return trajectories from 1995 to 2020.

Introduction

Trajectories to Mars have been separated into various classes by previous authors.[1-3] These classes include Opposition, Sprint, Conjunction, Free Return, and Cycler. Opposition class missions are characterized by a high-energy trajectory and a relatively short Mars stay time (<3 months). The name stems from the fact that the Earth leaves opposition with Mars at the Mars arrival. The total mission duration for this class ranges from 1 to 2.5 years. Also, the Opposition class trajectories have high arrival velocities at both Mars and Earth. Since the periapsis of the transfers can be lower than the orbit of Venus, the addition of a Venus swingby can improve mission performance. A subset of the Opposition class is the Sprint class. This class has a mission duration of approximately 1 to 1.4 years with a 30 day stay time. Sprint missions are intended for piloted missions because of their short flight times, but they have higher ΔV requirements. The most traditional mission class is the Conjunction class. In this class the Earth is moving into conjunction with Mars at the time of Mars arrival. These missions are characterized by low-energy trajectories and have a relatively long stay time (0.8 to 1.5 years). They can be used during the early exploratory phase where many tasks need to be done on the planetary surface.

For initial piloted missions, the Free Return class will most likely be the mission of choice since these trajectories do not require a deterministic maneuver to return the spacecraft to the Earth in the event of an emergency (e.g., Apollo 13). A subset of the Free Return class is the Cycler class,[3] which includes VISIT (Versatile International Station for Interplanetary Transport) and Up/Down Escalator (Aldrin) cyclers. Both of these cyclers repeatedly re-encounter the Earth and Mars. VISIT–I orbits have a 1.25 year period and re-encounter the Earth every 5 years and Mars every 3.75 years. VISIT–II orbits, on the other hand, have a period of 1.5 years and re-encounter the Earth once every 3 years and Mars every 7.5 years. The Up/Down Escalator (described by Byrnes, Longuski, and Aldrin[4]) cycler is composed of multiple Free Return trajectories to Mars connected by Earth gravity assists. The Earth gravity assist rotates the major axis of the orbit so that the phasing will be correct for a Mars encounter on the next leg. The Escalator orbits have an average Earth to Earth transfer time of 2.14 years, which is the Earth–Mars synodic period.

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Numerical Study

Advanced software developed by Patel[5] allows automated searches for multiple-encounter ΔV gravity-assist trajectories. This automated search algorithm (based on an earlier version developed by Williams[6]) solves the restricted n-body problem using the "patched–conic" theory described by Battin.[7] Breakwell and Perko[8] demonstrate that for interplanetary trajectories the patched–conic theory is reasonably accurate. Williams and Longuski,[9-12] Patel and Longuski,[13] and Patel, Longuski, and Sims[14] demonstrate that this algorithm can not only identify known trajectories, but can be used to discover new trajectories much more efficiently.

Figure 1 shows the result of the search for Mars Free Return trajectories for launch dates ranging from 1995 to 2020 with launch V_∞'s of 4, 5, 6, 7, and 8 km/s. (See Table 1 for legend for launch date plots.) Since the inertial positions of Earth and Mars repeat approximately every 15 years, the search over the 25 year span of launch dates represents all possible Free Return families. Notice that the two sets of

Table 1. Legend for Launch Date Plots

PATH	The sequence of numbers refers to the planetary path of the trajectory, e.g., PATH: 3 4 3 implies Earth–Mars–Earth. (See Figure 1.)
Vinf	The sequence numbers are the V_∞'s of launch. The values of V_∞'s in the plot itself are designated by 0,2,3,... (0 was used in lieu of 1 because it is more easily distinguished). Thus the numeral 3 on the plot refers to a V_∞ of 6.0 km/s.
ALTMIN	This value refers to the minimum altitude allowed in the original file.
L/D	Launch date range in calendar dates where 950101 refers to January 1, 1995. Also the increment of the launch date is shown by "BY 15 DAYS."
TFMAX	This value represents the maximum allowable time of flight.
Search Event No.	Event displayed in the path sequence, e.g., if Search Event No. = 3, then the plot is for Mars Arrival (for the path 3 4 3).
Search Min. Alt.	This value refers to the filtering value of minimum altitude. For example, if the original file was created with ALTMIN=-1500 km, then Search Min. Alt. = 0.0 km would filter out the trajectories with flyby altitudes below 0.0 km.

fast trajectories (TOF $\approx$ 1.4 years) during 2000 and 2002 repeat again during 2015 and 2017 (15 years later). In fact, the launch date/arrival date plots for 2000-2002 (Figure 2) and 2015–2017 (Figure 3) are virtually identical. Many of these trajectories have high launch energy requirements and high Mars arrival V_∞'s (Figure 4). Among these are Opposition class trajectories. Figure 5 shows a typical conic trajectory for this class. Notice that the Mars gravity assist has a large effect on the trajectory; therefore, these orbits cannot be analyzed as small perturbations of collision orbits (which naturally re-encounter the Earth with no gravity assist or deep space maneuver). The analysis of collision orbits (discussed later) will improve the understanding of many of the families, but cannot predict all of them precisely. Also notice that three sets of families appear: TOF $\approx$ 1.5, 2, and 3 years. Refined searches for the 9 launch opportunities between 2000 and 2018 were conducted. Detailed plots of the V_∞ at Mars, outbound orbit period, and inbound orbit period corresponding to the 2015–2017 launch window are shown in Figures 6–8.

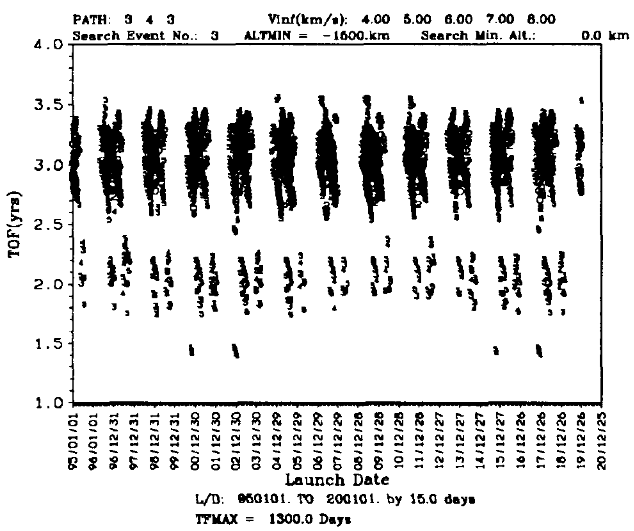


Fig. 1. Mars Free Return (1995–2020)

A Hohmann transfer between the Earth and Mars would be the most desirable (from an energy point of view) because the launch and arrival V_∞'s are minimized. For circular, coplanar orbits a minimum energy transfer between Earth and Mars has an orbital period of 1.42 years. It would therefore be impossible to return to the Earth in a single revolution after such a transfer because the Earth would not be in the correct position. Also since the orbital period does not form an integer ratio with the Earth's period, it would be impossible to return to the Earth after multiple revolutions about the Sun.

Wolf[15] shows that a Free Return trajectory with low launch energy and low arrival V_∞ can exist if the trajectories have orbital periods that are resonant with the Earth and Mars:

$$\begin{aligned} n(\text{Orbital Period}) &= m(\text{Earth Period}) \\ j(\text{Orbital Period}) &= k(\text{Mars Period}) \end{aligned}$$

where n, m, j, and k are small integers. A subset of these trajectories are the VISIT cycler orbits. The VISIT–I orbit $[(n,m,j,k) = (4,5,3,2)]$ is not included in the present analysis because of the long flight time between Earth encounters (5 years). Recall the VISIT–II orbits $[(n,m,j,k) = (2,3,5,4)]$ return to the Earth every 3 years and have a 1.5 year period. As Figure 1 demonstrates, the majority of the

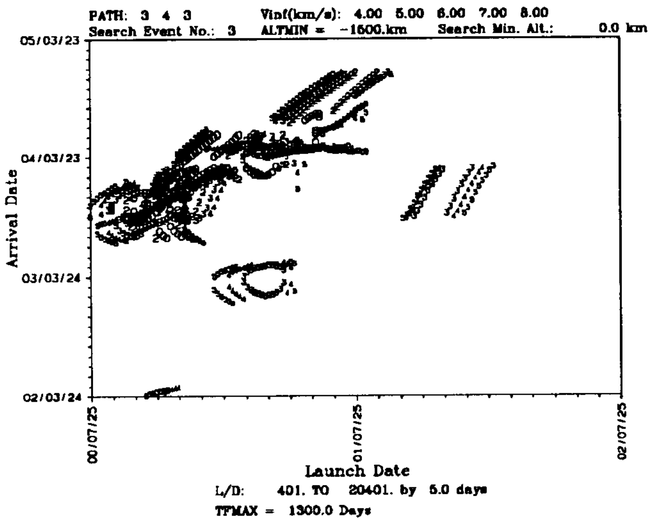


Fig. 2. Mars Free Return (2000–2002)

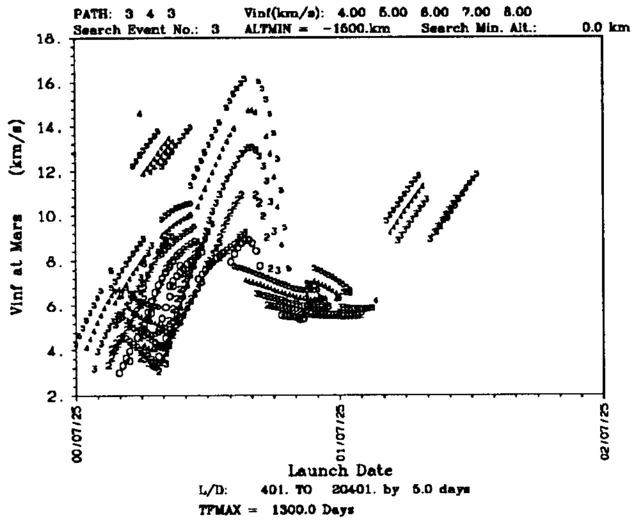


Fig. 4. V_∞ at Mars for Mars Free Return (2000–2002)

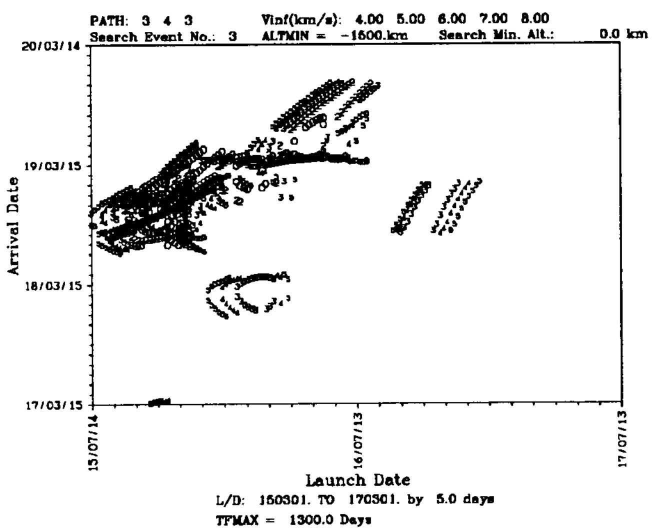


Fig. 3. Mars Free Return (2015–2017)

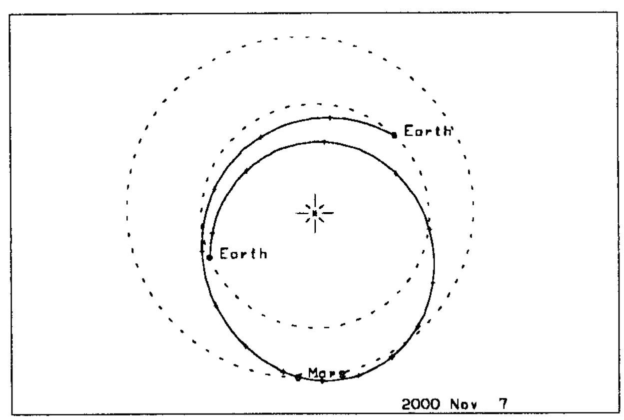


Fig. 5. Mars Free Return–Opposition (TOF=1.4 years)

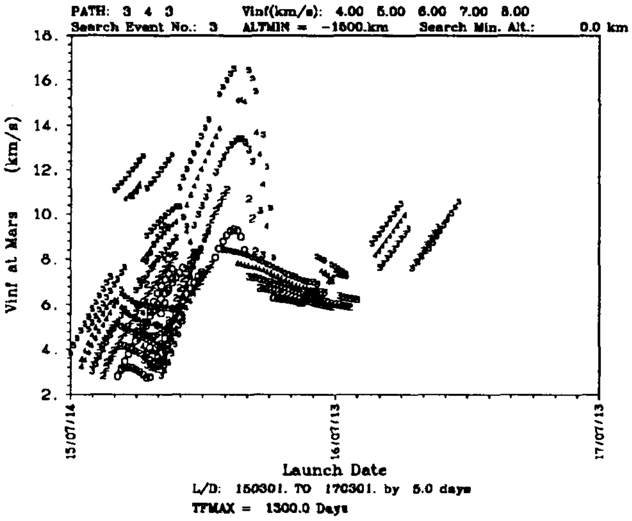


Fig. 6. V_∞ at Mars for Mars Free Return (2015–2017)

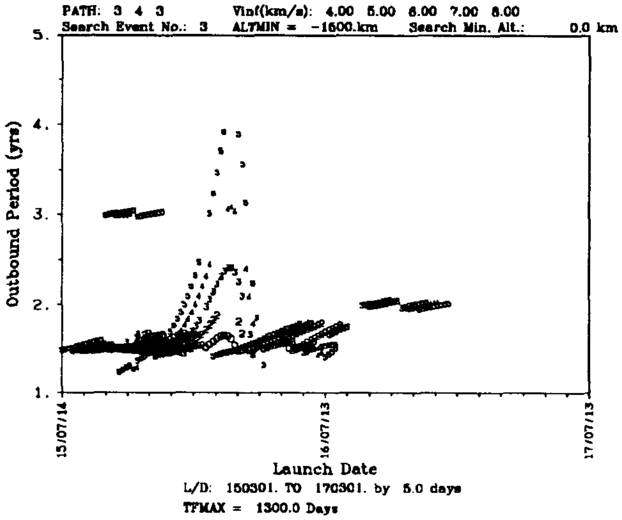


Fig. 7. Outbound Period for Mars Free Return (2015–2017)

Mars Free Return trajectories have flight times near 3 years. Most of these trajectories have either a short outbound/long inbound time of flight or a long outbound/short inbound time of flight. Figures 9 and 10 show the outbound and inbound periods for the trajectories with launch dates between 2000 and 2002. In these two plots, it is apparent that most of the trajectories with flight times of 3 years have orbital periods of approximately 1.5 years for both outbound and inbound legs. Some of these trajectories with low launch energies are, in fact, VISIT–II orbits.

For a transfer orbit with periapsis lower than Earth's orbit and apoapsis higher than Mars' orbit, there are many permutations of encounter positions for a given orbital period. Trajectories completing up to one full revolution can have the following encounter sequences (see Figure 11):

Case 1: E1–M2–E1
Case 2: E1–M2–E2
Case 3: E1–M1–E1
Case 4: E1–M1–E2
Case 5: E2–M2–E1
Case 6: E2–M2–E2
Case 7: E2–M1–E1
Case 8: E2–M1–E2

where 1 refers to encounters (with Earth or Mars) before periapsis and 2 refers to encounters after periapsis. When up to 2 complete revolutions are considered, there are twice as many permutations of the phasing of the Mars encounter. Figure 12 shows a conic trajectory representing the E2–M1–E2 type. Other orbits have a flight time of three years, but also

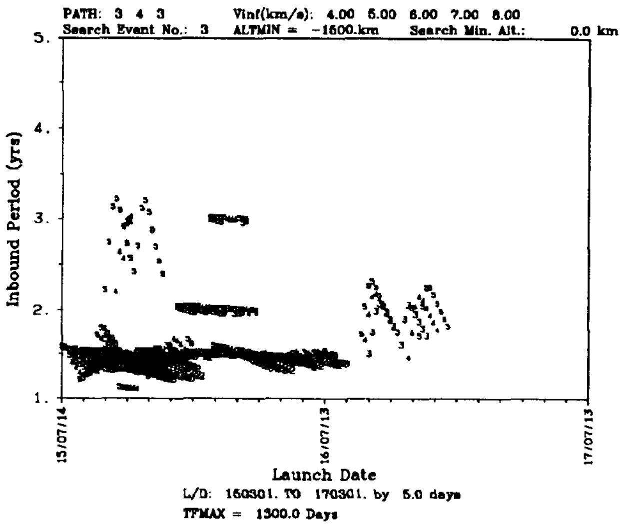


Fig. 8. Inbound Period for Mars Free Return (2015–2017)

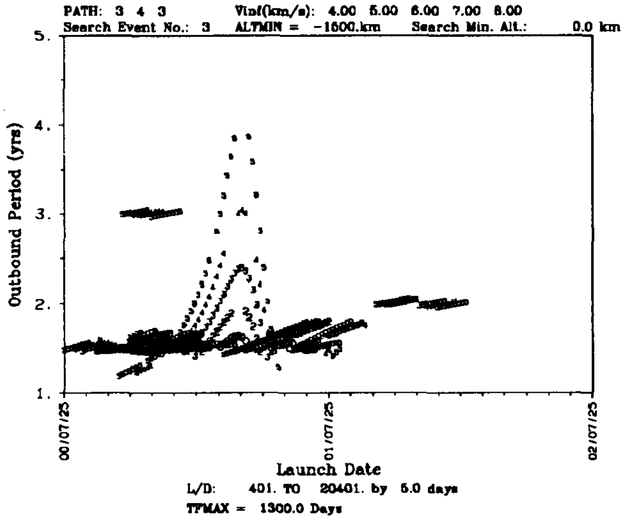


Fig. 9. Outbound Period for Mars Free Return (2000–2002)

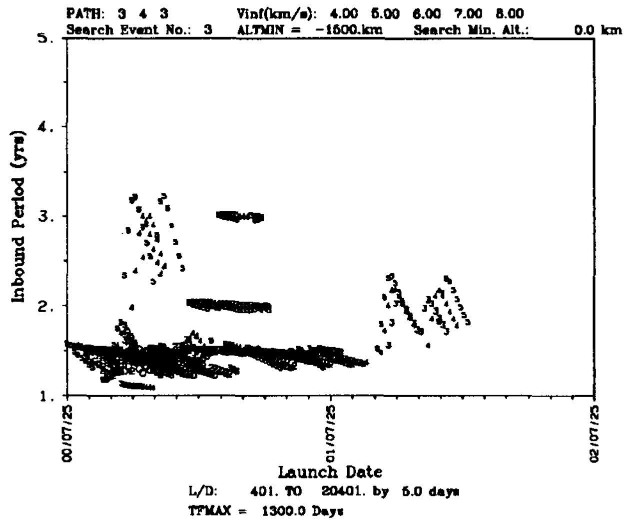


Fig. 10. Inbound Period for Mars Free Return (2000–2002)

have an orbital period of three years. Fewer such trajectories exist than the trajectories with an orbital period of 1.5 years because the spacecraft does not orbit the Sun more than once. (See Figure 13 for a conic trajectory example.) The peak of the V_∞ curve near March 2001 in Figure 4 shows that the orbits with three year periods have very high arrival V_∞'s ($14 < V_\infty < 18$ km/s). These trajectories have large launch energy requirements as well. Another interesting fact about trajectories near the V_∞ peak is that the flyby altitudes become very large. Figure 14 shows the flyby altitudes at Mars for the 2000–2002 Free Return trajectories. The orbits which have large Mars flyby altitudes are in fact the collision orbits predicted by Hénon.[16]

The trajectories with a flight time of two years can also be found in Figure 1. A subset of this family is the Escalator orbit which has an orbital period of about 2.02 years and an Earth V_∞ of about 6.0 km/s. Examples of both Up and Down Escalators are shown in Table 2. The perturbations of the Up Escalator family are shown by the first set of closed contours for launch dates near February 2001 (see Figure 2). The perturbations of the Down Escalator family are shown by the contours near the launch date in October 2001. The Escalator orbits are entirely contained within the launch date/arrival date data (i.e., Trajectories in Table 2 are all included in Figure 1). Maneuvers are sometimes required to maintain the cyclers because the Earth may not provide sufficient bending to shift the line of apsides. Figures 15 and 16 show examples of Up and Down Escalator trajectories.

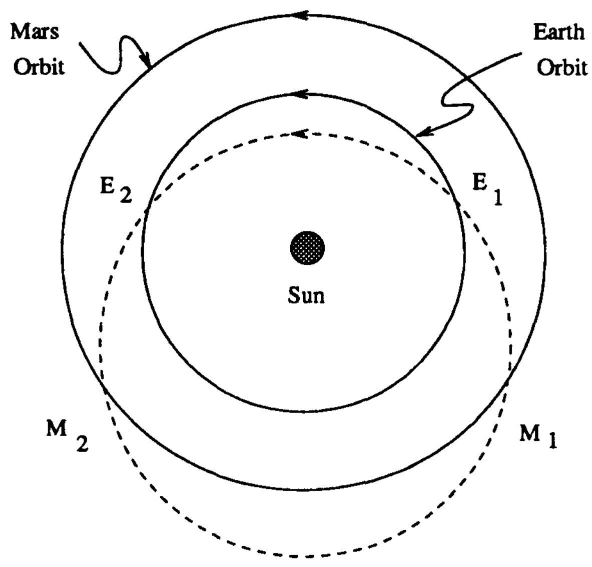
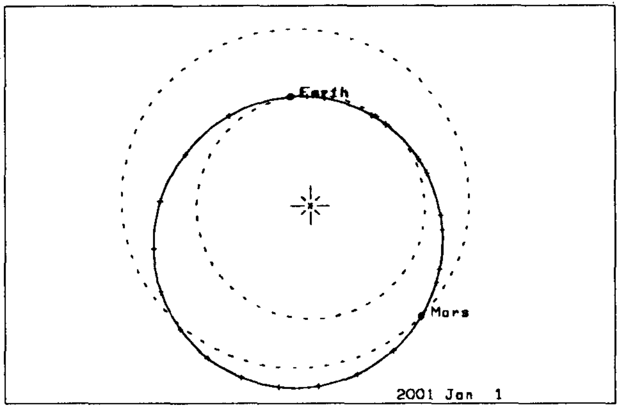


Fig. 11. Mars Free Return Options



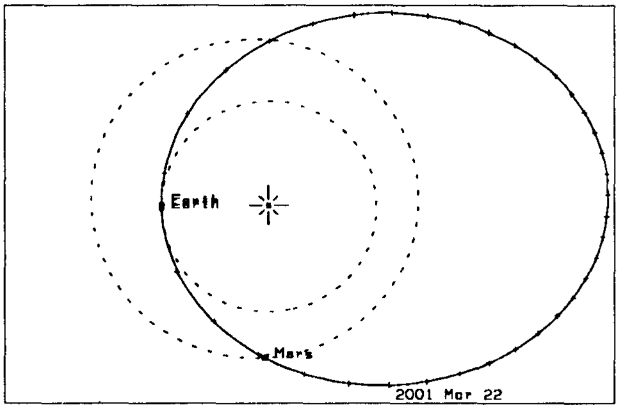


Fig. 12. Mars Free Return–1.5 year Period (TOF=3.0 years)(E2–M1–E2)

Fig. 13. Mars Free Return–3.0 year Period (TOF=3.0 years)

Consecutive Collision Orbits

Free return trajectories with a small body such as Mars are very similar to collision orbits. A collision orbit is an orbit that encounters an object (e.g., the Earth) twice within a certain time. By assuming that a flyby of a second small body (e.g., Mars) only slightly perturbs the orbit, this type of orbit can easily be used to analyze the Mars Free Return problem. An analytical approach to solving the consecutive collision problem is described by Hénon.[16] This work not only addresses the problem of the simple case where the two encounters occur at the same point in space, but also solves the more complicated problem in which the two encounters take place at different points in space. Howell[17] extends this solution to solve for consecutive collision orbits in the elliptic restricted problem. Since the eccentricity of the Earth's orbit is quite small, Hénon's circular approximation is sufficient for this very preliminary comparison of theoretical predictions and numerical results.

A brief summary of the results obtained by Hénon is as follows. Select unit length and time based on the secondary body M_2 (the Earth). Now, from Figure 17, the collision points are denoted P and Q and the time interval between the collisions is 2τ. Taking the middle of the interval as time $t = 0$, the collisions between the Earth and the spacecraft (M_3) occur at $-\tau$ and τ. Different orbit types are described by the following parameters:

$$\varepsilon = \begin{cases} +1 \\ -1 \end{cases} \text{if } M_3 \text{ periapsis} \begin{cases} \text{positive} \\ \text{negative} \end{cases}$$

$$\varepsilon' = \begin{cases} +1 \\ -1 \end{cases} \text{if } M_3 \text{ orbit} \begin{cases} \text{direct} \\ \text{retrograde} \end{cases}$$

$$\varepsilon'' = \begin{cases} +1 \\ -1 \end{cases} \text{if } M_3 \text{ at } t = 0 \begin{cases} \text{periapsis} \\ \text{apoapsis} \end{cases}$$

At the time of collision, the eccentric anomalies of the primary M_2 and M_3 are τ and η respectively. For elliptical transfers, the solution for the transfer orbit can be obtained by solving the following implicit timing equation relating τ and η:

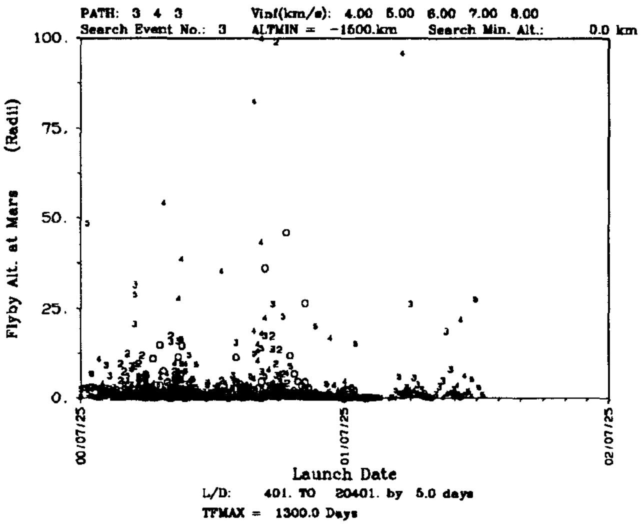


Fig. 14. Flyby Altitude at Mars for Mars Free Return (2000–2002)

Table 2. Up/Down Escalator Orbits (From Byrnes et al.[4])

	Up Escalator		Down Escalator	
Encounter	Date	Approach V_∞ or ΔV (km/s)	Date	Approach V_∞ or ΔV (km/s)
Earth–1	Nov 19, 1996	6.19	Jun 5, 1995	5.88
Mars–2	May 1,1997	10.69	Jan 20, 1997	8.52
Earth–3	Jan 1, 1999	5.94	July 9, 1997	5.95
Mars–4	May 28, 1999	11.74	Mar 7, 1999	7.35
Earth–5	Feb 8, 2001	5.67	Aug 17, 1999	6.01
Maneuver	–	–	Sep 28, 2000	0.27
Mars–6	Jul 6, 2001	10.22	May 15, 2001	6.60
Maneuver	Mar 13, 2002	0.54	–	–
Earth–7	Apr 16, 2003	5.67	Oct 8, 2001	5.88
Maneuver	–	–	Dec 4, 2002	1.11
Mars–8	Sep 12, 2003	7.28	Aug 7, 2003	7.30
Maneuver	May 17, 2004	0.74	–	
Earth–9	Jul 7, 2005	5.87	Jan 2, 2004	5.39
Maneuver	–	–	Feb 2, 2005	0.66
Mars–10	Dec 13, 2005	6.05	Oct 10, 2005	9.96
Maneuver	Jul 23, 2006	0.45	–	–
Earth–11	Sep 6, 2007	5.87	Mar 12, 2006	5.48
Mars–12	Feb 16, 2008	7.43	Nov 19, 2007	11.59
Earth–13	Oct 10, 2009	5.89	Apr 16, 2008	5.96
Mars–14	Mar 28, 2010	8.66	Dec 13, 2009	10.55
Earth–15	Nov 13, 2011	5.81	May 22, 2010	5.93

$$\sqrt{1-\varepsilon\varepsilon''\cos\tau\cos\eta}\,[\eta(1-\varepsilon\varepsilon''\cos\tau\cos\eta)- \sin\eta(\cos\eta-\varepsilon\varepsilon''\cos\tau)]-\tau\left|\sin\eta\right|^3=0 \quad (1)$$

Once τ and η are known the transfer orbit can be determined from

$$a=\frac{1-\varepsilon\varepsilon''\cos\tau\cos\eta}{\sin^2\eta} \quad (2)$$

$$e=\frac{\varepsilon''\cos\eta-\varepsilon\cos\tau}{1-\varepsilon\varepsilon''\cos\tau\cos\eta} \quad (3)$$

Hénon presents numerous tables containing solutions of Equation (1), that is, various combinations of τ and η that solve the timing condition. These tables are used for this analysis. They include the values for η/π (number of revolutions), τ/π (flight time in Earth years), orbit type, a, and e; thus, they represent numerous trajectories that depart from Earth and re-encounter the Earth after a specified time interval.

The Hénon consecutive collision orbits can be used to predict Free Return trajectories to Mars by the addition of a constraint: the aphelion of the collision orbit must be greater than the orbital radius of Mars. Several trajectories obtained from numerical analysis were verified using Hénon's tables;[16] a few of these results are summarized in Table 3. Further research in this area might be fruitful in analytically predicting Free Return trajectories. It is

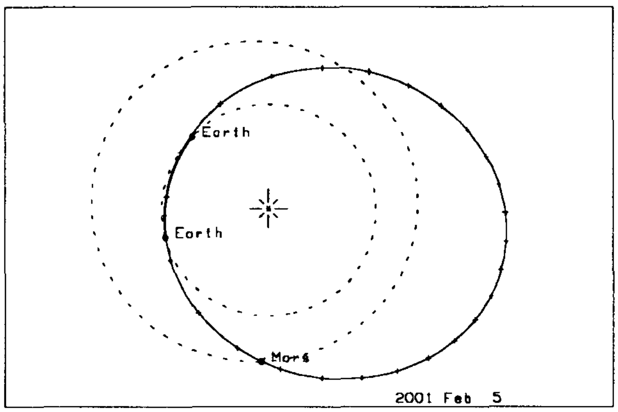


Fig. 15. Mars Free Return–Up Escalator

Table 3. Summary of Hénon's Results

τ/π (yrs)	η/π	Semi-Major Axis (AU)	Eccentricity	Period (yrs)
2.00000	1.00000	1.58740	0.37004	2.00141
3.00000	2.00000	1.31037	0.23686	1.50003
3.00000	1.00000	2.08008	0.51925	3.00083

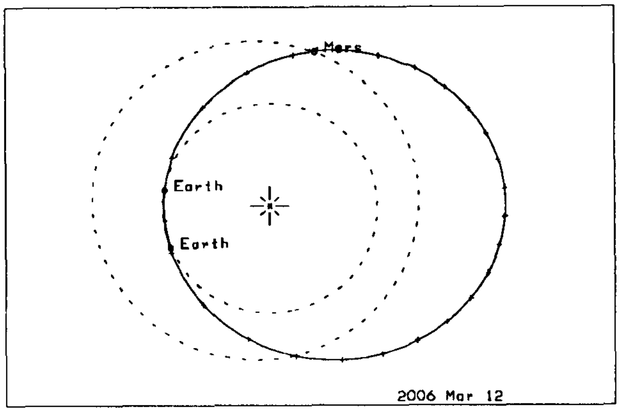


Fig. 16. Mars Free Return–Down Escalator

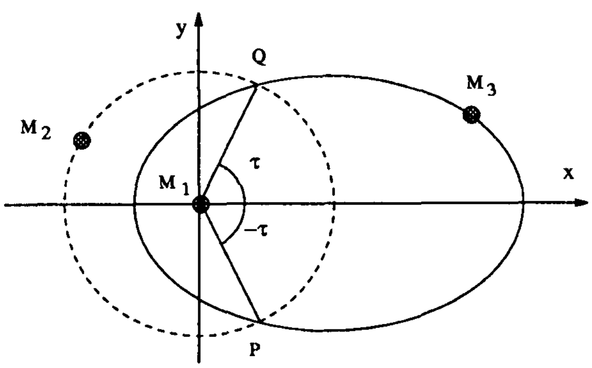


Fig. 17. Collision Orbits

important to note however that the gravity of Mars can cause large perturbations to the trajectories as demonstrated with the fast time of flight trajectories, and thus the analysis might be more useful if generalized to include nonzero values of μ.

Conclusion

Recently developed automated design software, for the analysis of gravity-assist trajectories, has permitted a thorough investigation of Mars Free Return trajectories. The resulting data file contains a variety of well-known trajectory classes. The analysis of Hénon is also used to verify some of the numerical results.

Similar analyses can be performed (for future work) for Mars Free Returns which include Venus as a gravity-assist body.

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