AIAA-94-3769-CP

Analysis of V_∞ Leveraging for Interplanetary Missions

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V_∞ leveraging can significantly reduce the launch energy requirements for interplanetary missions. The objective of this study is to analytically examine this technique and to expand its application. ΔV-EGA (ΔV Earth Gravity Assist) trajectories are used as the first example of V_∞ leveraging. The equations are solved using an iterative algorithm, and the trade-off between V_∞ and turn angle at the Earth gravity assist is examined. Simplifying assumptions reduce the equations to a single function. The ΔV-EGA concept can be extended to other planets, such as Venus, which is used as a second example.

Introduction

Gravity assist is a proven technique in interplanetary exploration, as exemplified by the missions of the Voyager and Galileo spacecraft. The technique can be used to reduce the launch energy requirements for a given mission or to increase the science return by enabling more planetary, satellite, or asteroid encounters. V_∞ leveraging is a method used in conjunction with gravity assists to further reduce the launch energy requirements and total ΔV for a mission. The term *V_∞ leveraging* (coined by Longuski) refers to the use of a relatively small deep space maneuver in order to modify (increase or decrease) the V_∞ at a flyby planet. A typical example of V_∞ leveraging is the ΔV-EGA trajectory (see Fig. 1) introduced by Hollenbeck.[1] Williams[2] applied the term when describing ΔV-EGA type trajectories using Venus instead of Earth; however, the above definition is much more general.

In this paper we first analyze ΔV-EGA trajectories because they represent a straightforward application of V_∞ leveraging and are frequently considered for use in interplanetary missions. Even though the "exact" equations are relatively simple, solving them requires an iterative algorithm because of their transcendental nature. A simplifying assumption allows the energy gain to be computed without iteration but does not allow for the calculation of the ΔV. Further analysis (using variations) results in an estimate for the ΔV. We compare this estimate to one developed by Sweetser,[3] who demonstrates how the ΔV calculation can be made through use of Jacobi's integral. The first method is based on the theory of patched-conic orbits, while Jacobi's integral is associated with the circular restricted three-body problem.

This analysis also provides insight into the trade-off between the magnitude of the relative velocity (V_∞) and the turn angle (of the V_∞ vector) which can be effected by a flyby body for the purpose of optimizing the heliocentric trajectory (e.g., maximum energy or maximum aphelion radius). We also examine aerogravity assists in which the turn angle is not constrained.

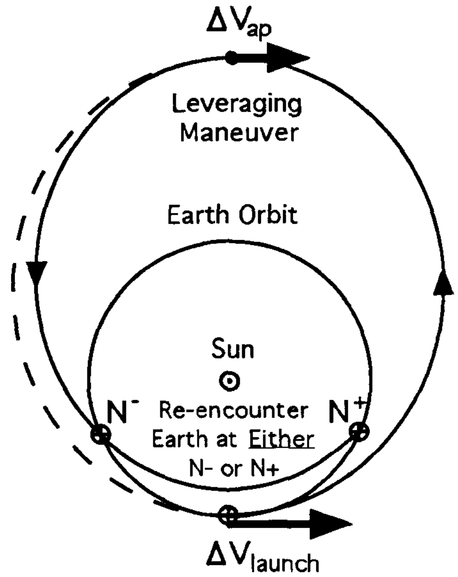

Fig. 1. ΔV-EGA Trajectory.

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An extension of the ΔV-EGA concept is to use other planets instead of Earth. Venus is the most likely candidate. The first step is to consider "launch" from Venus analogous to the ΔV-EGA. The analysis and trajectory types in this case are similar to those with launch from Earth. The next step is to consider launch from Earth to Venus followed by one or more Venus flybys. We then include a V_∞ leveraging maneuver between Venus flybys. This type of trajectory has been considered for the Cassini mission to Saturn. Acceptable candidate trajectories of this type initially took weeks to discover. Using recently enhanced automated design software, Patel and Longuski[4] generated estimates for launch dates for certain categories of these trajectories. The present analysis leads to the development of a more methodical procedure to find the most efficient of these trajectories.

We close this paper by discussing more general types of V_∞ leveraging, including non-ideal ΔV-EGAs, interior "ΔV-EGA," and reverse "ΔV-EGA," and drawing some conclusions.

ΔV-EGA

In a ΔV-EGA trajectory as examined by Hollenbeck, a spacecraft is launched from Earth into a heliocentric orbit with a period slightly greater than an integer number (N) of years and perihelion radius equal to 1 AU (assuming circular Earth orbit). At aphelion a (tangential) retrograde ΔV is applied to lower the perihelion in order to intercept the Earth non-tangentially. This maneuver enables the Earth to be used as a gravity-assist body to increase the heliocentric energy. As shown in Fig. 1, the re-encounter with Earth can occur either before (N^-) or after (N^+) perihelion of the new orbit.

Figures 2 and 3 show the aphelion radius of the orbit after the Earth flyby as a function of total ΔV. Total ΔV includes launch from an Earth parking orbit (circular, 185 km altitude) and all post-launch ΔVs: at aphelion and, when appropriate, after the Earth flyby. The minimum flyby altitude is 200 km; however, near the low end of the curves, maximum aphelion radius is attained with higher flyby altitudes. Figure 3 shows that N^+ trajectories have a larger maximum final aphelion radius and better performance near the maximum. We note, however, that N^- trajectories require less total ΔV for smaller final aphelion radii. This observation, which could be important in some applications, is not apparent in the work by Hollenbeck.

The direct launch curve is shown for comparison. As the figures show, ΔV-EGAs can significantly reduce the total ΔV required to reach a given aphelion radius. This reduction in total ΔV

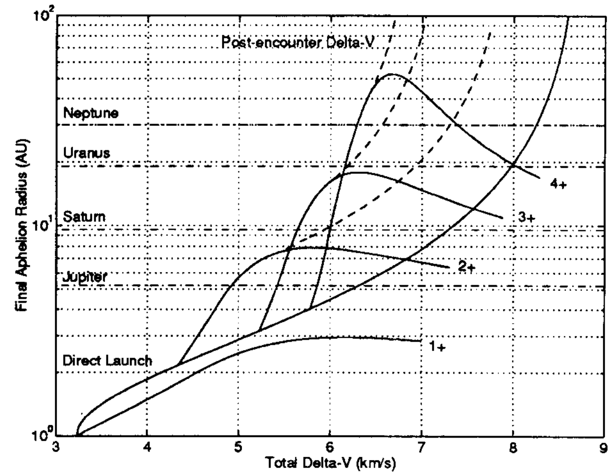

Fig. 2. ΔV-EGA Performance.

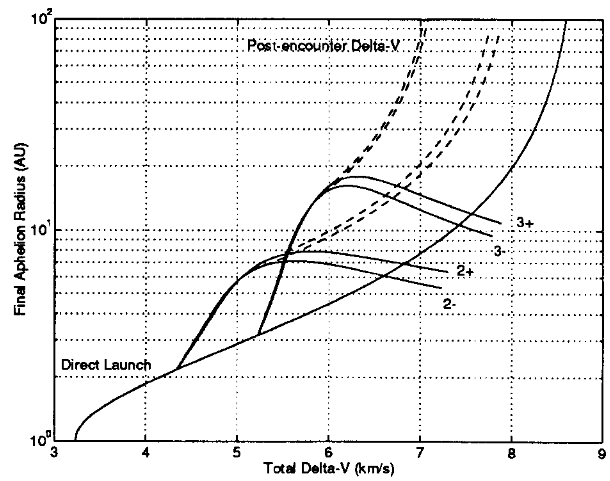

Fig. 3. Comparison between N^+ and N^- ΔV-EGAs.

comes with the cost of a longer flight time and larger post-launch ΔV.

The effectiveness of V_∞ leveraging is shown in Fig. 4, where the increase in V_∞ at Earth is plotted versus the aphelion delta-V, ΔV_{ap}. The launch V_∞ ranges from about 5.1 to 5.5 km/s for N=2 and 6.9 to 7.2 km/s for N=3. (See Fig. 5.) To achieve a given aphelion radius, an N^+ trajectory requires a slightly higher launch V_∞ and a slightly smaller ΔV_{ap} than an N^- trajectory. In many applications, minimizing the magnitude of the deep space maneuver may be a more important consideration than minimizing launch energy or total ΔV, which includes launch ΔV.

We see from Fig. 4 that the V_∞ at Earth return increases continuously as ΔV_{ap} is increased, but from Figs. 2 and 3 we see that the final aphelion radius

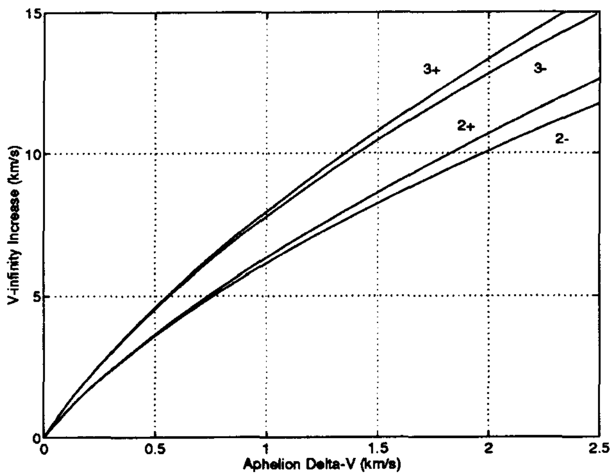

Fig. 4. Effectiveness of V_∞ Leveraging for ΔV-EGA Trajectories.

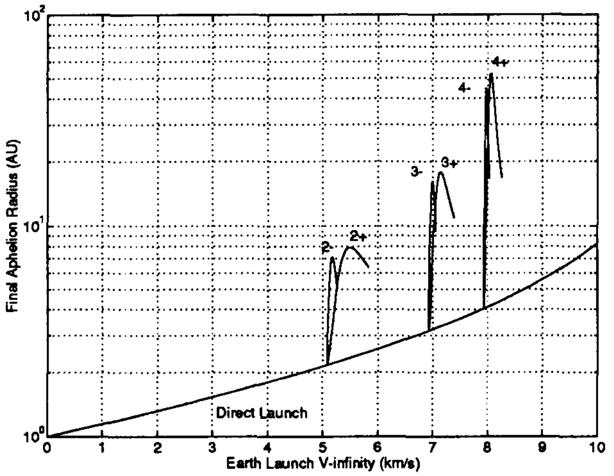

Fig. 5. Launch V_∞ Requirements for ΔV-EGA Trajectories.

reaches a maximum and then decreases if no ΔV is applied after the flyby. The reason for this phenomenon is that the amount the $\mathbf{V}_\infty$ vector can be turned by the gravity of a flyby body decreases as the V_∞ increases.

Geocentric and heliocentric views of a typical Earth gravity assist are shown in Fig. 6. The superscripts "−" and "+" indicate quantities before and after the flyby, respectively. The turn angle, δ, is determined from

$$\sin(\delta/2) = 1/e_{fb} \qquad (1)$$

where $e_{fb} = 1 + r_{pfb}V_\infty^2/\mu_E$ is the eccentricity relative to the Earth, r_{pfb} is the perigee radius, and μ_E is the gravitational parameter of the Earth. As equation (1) indicates, a larger V_∞ gives a larger e_{fb} and, hence, a smaller δ.

The velocity vector diagram of the Earth gravity assist is presented in Fig. 7 for the 2^+ ΔV-EGA with three different launch energies. A higher launch energy results in a larger γ^- and therefore a larger V_∞ at the re-encounter with Earth. In Fig. 7a, the launch energy is relatively small, and so the V_∞ is relatively small. If the flyby altitude were 200 km, the $\mathbf{V}_\infty$ vector would be "over turned" (i.e., turned beyond parallel to $\mathbf{V}_E$). In Figs. 2 and 3 we assume that the $\mathbf{V}_\infty$ is turned parallel to $\mathbf{V}_E$ when it can be, since this produces the largest aphelion radius and the largest V^+. Figure 7b shows a case in which the launch energy is relatively large. The resulting V_∞ is large, and so the turn angle and gravity-assist delta-V, ΔV_{ga}, are small. Figure 7c shows the case which provides the $\mathbf{V}^+$ with the largest magnitude. (*Maximum V^+ is equivalent to maximum heliocentric energy.* For a given V_∞, the largest V^+ and aphelion radius occur when $\mathbf{V}_\infty$ is turned as close to parallel to $\mathbf{V}_E$ as possible. In general, however, the aphelion radius depends not only on V^+ but also on γ^+, so the overall maximum V^+ for a given type of ΔV-EGA is not

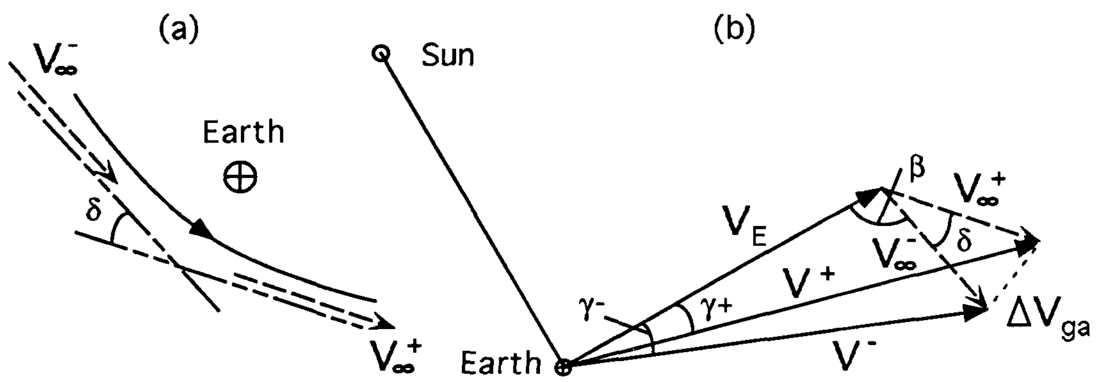

Fig. 6. Earth Gravity Assist: a) Geocentric View and b) Heliocentric View.

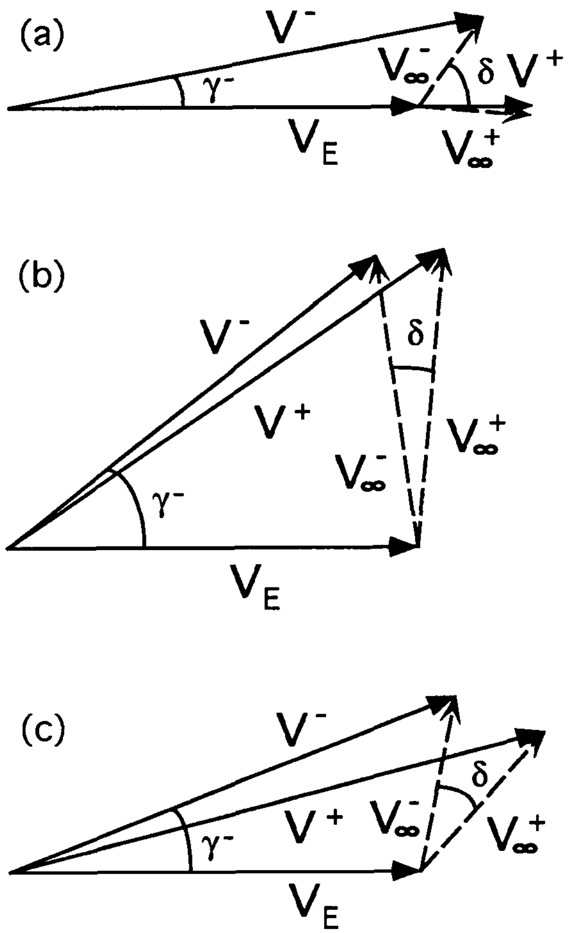

Fig. 7. 2$^+$ ΔV-EGA Earth Gravity Assist: a) Low Launch Energy (Maximum ΔV_{ga}), b) High Launch Energy, and c) Medium Launch Energy (Maximum V^+).

equivalent to the overall maximum aphelion radius. Nevertheless, in our applications the conditions resulting in the overall maximum aphelion radius are extremely close to those resulting in the overall maximum V^+.) The locus of V^+ is plotted in Fig. 8. Figures 7 and 8 illustrate the trade-off between V_∞ and turn angle resulting in the maximum energy orbit and the maximum aphelion radius.

The problem of maximizing V^+ (or aphelion radius) is not the same as the simpler problem of maximizing ΔV_{ga} for a flyby at a given radius. From Fig. 6b we see that

$$\Delta V_{ga} = 2V_\infty \sin(\delta/2) \qquad (2)$$

Substituting equation (1) into (2), differentiating with respect to V_∞, and setting the resulting expression equal to zero leads to the following conclusion: the V_∞ that maximizes ΔV_{ga} is equal to $(\mu_E/r_{pfb})^{1/2}$, which is the local circular speed. (This result is presented in Ref. 5 also.) Substituting this value of V_∞ into equations (1) and (2) gives

$$(\Delta V_{ga})_{max} = (\mu_E/r_{pfb})^{1/2} = V_\infty \qquad (3)$$

$$\delta = 60^\circ \qquad (4)$$

At Earth with $r_{pfb} = 185$ km we have $(\Delta V_{ga})_{max} = 7.79$ km/s $= V_\infty$. This case is shown in Fig. 7a for the 2$^+$ ΔV-EGA. For ΔV-EGAs with $N > 3$, this situation does not occur because the launch V_∞ is greater than 7.9 km/s. Obviously, maximizing ΔV_{ga} does not maximize heliocentric energy.

One method of circumventing the turn angle limitation is to use aerogravity assist (AGA).[6] In this method, a lifting body flies through the atmosphere of

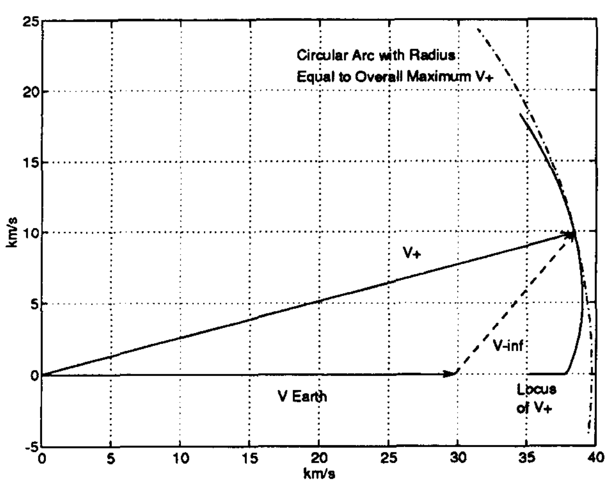

Fig. 8. Locus of V^+ for 2$^+$ ΔV-EGAs.

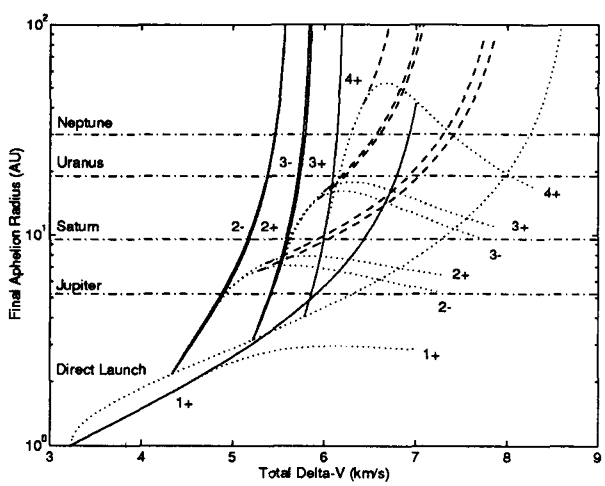

Fig. 9. ΔV-EAGA (Aerogravity Assist) Performance Compared to Gravity Assist Only (Figs. 2 and 3).

the planet (Earth in this case) to turn the V_∞ in any desired direction. Figure 9 shows the tremendous advantage possible using aerogravity assist. (For this analysis we have assumed an infinite lift-to-drag ratio.) Even $1^+\Delta V$-EAGAs provide an advantage over direct launch for total ΔV beyond about 5.4 km/s. Although a 2 ΔV-EAGA requires a lower total ΔV to reach a particular aphelion radius than a 3 ΔV-EAGA, the post-launch (aphelion) ΔV is larger.

After launching from Earth with a given launch energy, an iterative algorithm is used to determine the ΔV required to re-encounter the Earth. We want to be able to quickly analyze the potential of other ΔV-EGA type trajectories. Specifically, we would like analytic expressions for r_a and V^+. A critical observation that leads to approximate analytic results is that the speed before the flyby, V^-, is nearly constant for all ΔV-EGAs with the same N. An equation for this speed is

$$V^- = [\mu_S(2/r_E - 1/a_{re})]^{1/2} \qquad (5)$$

where μ_S and r_E are constant and a_{re} (semi-major axis of the return orbit) varies only slightly, particularly for N^+ ΔV-EGAs. Assuming that V^- is a constant (taking $a_{re} = a_{nom}$ for a nominal N-year period orbit), we are able to write the final aphelion radius, r_a, and V^+ as functions of a single variable, γ^- in this case. We could write an expression for r_a in terms of γ^- and known constants (including V^-) only, but the resulting equation is very cumbersome and reveals little information on the interdependence of the various parameters involved. Instead, we choose to show more explicitly how each variable is determined.

Assuming V^- is a known value, for a given γ^- we have, from Fig. 6,

$$V_\infty = [V^{-2} + V_E^2 - 2V^-V_E \cos(\gamma^-)]^{1/2} \qquad (6)$$

where V_E is the speed of the Earth (assuming circular Earth orbit). Once V_∞ is determined, we can calculate β from

$$\sin(\beta) = (V^-/V_\infty)\sin(\gamma^-) \qquad (7)$$

and δ from equation (1). The heliocentric velocity and flight path angle after the flyby are computed from

$$V^+ = [V_E^2 + V_\infty^2 - 2V_E V_\infty \cos(\beta + \delta)]^{1/2} \qquad (8)$$

$$\sin(\gamma^+) = (V_\infty/V^+)\sin(\beta + \delta) \qquad (9)$$

For the new heliocentric orbit we have

$$a = 1/[2/r_E - V^{+2}/\mu_S] \qquad (10)$$

$$e = [(r_E V^{+2}/\mu_S - 1)^2 \cos^2(\gamma^+) + \sin^2(\gamma^+)]^{1/2} \qquad (11)$$

and finally

$$r_a = a(1 + e) \qquad (12)$$

Equations (1) and (5) - (12) result in approximate values for r_a and V^+. The values are approximate only because we use an approximate value for V^-; the equations themselves are exact for two-body orbits.

Table 1 presents the maximum values of V^+ and r_a for 2^+ ΔV-EGAs from the exact analysis (iterative procedure) and from equations (8) and (12) using the nominal value for V^-.

Table 1 Comparison of Maximum V^+ and r_a

	Exact	Approx.	% Diff.
Max V^+ (km/s)	39.67	39.74	0.18
γ^- at max V^+ (deg)	21.7	22.2	2.30
Max r_a (AU)	7.90	8.18	3.54
γ^- at max r_a (deg)	21.9	22.4	2.28

The simplification of constant V^- removes the connection to the initial orbit and the ΔV_{ap} necessary to achieve the return orbit. So we need to find a means to estimate the total ΔV associated with a particular γ^-. We are able to do this by taking a variation of the nominal values for an N-year period orbit with perihelion at 1 AU. The eccentricity of the return orbit is determined from equation (11) with V^+ and γ^+ replaced by V^- and γ^-, and we compute

$$\delta e = e - e_{nom} \qquad (13)$$

Starting with the equations

$$r_a = a(1 + e) \qquad (14)$$

$$V_a^2 = \mu(2/r_a - 1/a) \qquad (15)$$

the variation with constant r_a provides

$$\delta a = -[a/(1 + e)]\delta e \qquad (16)$$

$$\delta V_a = (\mu/2V_a a^2)\delta a \qquad (17)$$

We can now compute the total ΔV by

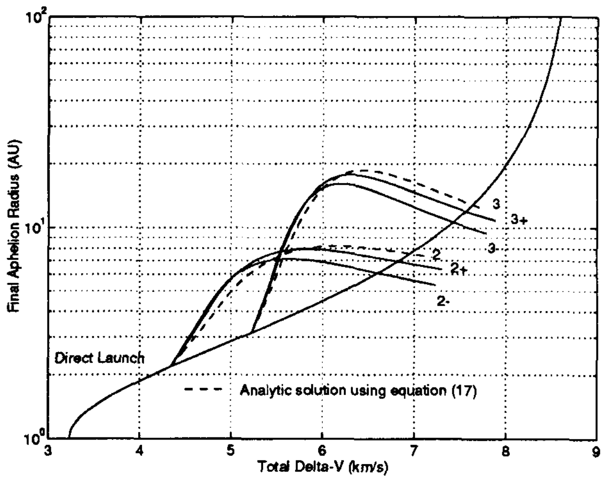

Fig. 10. ΔV-EGA Analytic Solution Using Equation 17.

$$\Delta V_{total} = \Delta V_{launch} + \delta V_a \qquad (18)$$

where ΔV_{launch} is computed for the nominal orbit. The results of this analysis are presented in Fig. 10 where the approximate curves are plotted with the exact curves. The approximations are quite reasonable and may be used for an initial estimate of the potential of other V_∞ leveraging trajectories.

So far we have been using the theory of patched-conic orbits. In some cases we can get a better approximation for ΔV_{ap} by following Sweetser[3] and considering ΔV-EGA trajectories as a three-body problem. For the circular restricted three-body problem Jacobi's integral is a constant given by

$$C = -v^2 + \omega^2\rho^2 + 2\mu_E/r_E + 2\mu_S/r_S \qquad (19)$$

where v is the relative speed in the coordinate frame rotating at the angular rate ω, and ρ, r_E, and r_S are distances from the spacecraft to the Earth-Sun barycenter, Earth, and Sun, respectively. Sweetser substitutes $v + \Delta v$ for v in equation (19) to obtain

$$-\Delta C = 2v\Delta v + (\Delta v)^2 \qquad (20)$$

Writing equation (20) at aphelion and perigee and equating, we have

$$2v_p\Delta v_p + (\Delta v_p)^2 = 2v_a\Delta v_a + (\Delta v_a)^2 \qquad (21)$$

Using nominal values for v_a and v_p, for a given γ^- we can determine V_∞ (and hence Δv_p) from equation (6) and then solve for Δv_a.

In Fig. 11, results from both approximations of ΔV_{ap} are plotted along with the exact solutions. Sweetser's approximation predicts the V_∞ for a given ΔV_{ap} very well, especially for small values of ΔV_{ap}. Unfortunately, his analysis cannot predict the V^+ or final aphelion radius because it has no direct connection with the flyby geometry. But we can incorporate his approximation for ΔV_{ap} into our previous analysis which assumed constant V^- to obtain the results shown in Fig. 12. The approximate curves are now much closer to the exact curves for the lower total ΔVs. We can infer from Sweetser's analysis that the most efficient deep space maneuver is at aphelion and parallel to the velocity at that point, which is how we have been applying our deep space maneuver.

Returning to Figs. 2 and 3, an important point on the ΔV-EGA curve is where it becomes more efficient to apply ΔV after the Earth flyby. Mathematically, we want to know at what point the initial slope of the extended curve (with ΔV after the flyby) equals the slope of the curve with no ΔV after the flyby. Since γ^+ is relatively small at this point (<15° from numerical results), we have

$$r_a \approx r_E^2 V^{+2}/(2\mu_S - r_E V^{+2}) \qquad (22)$$

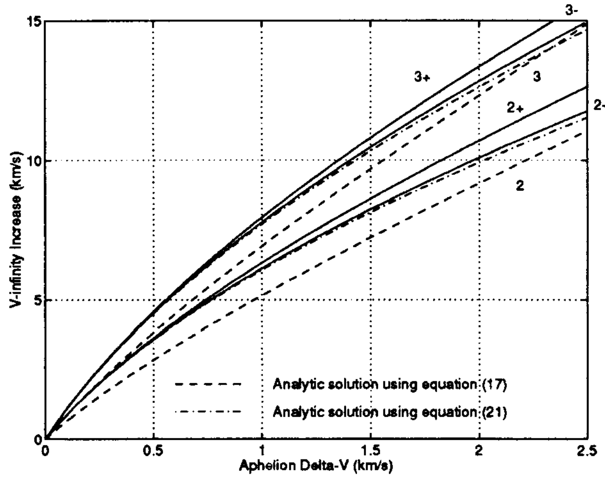

Fig. 11. Comparison between Analytic and Exact Solutions of ΔV-EGAs.

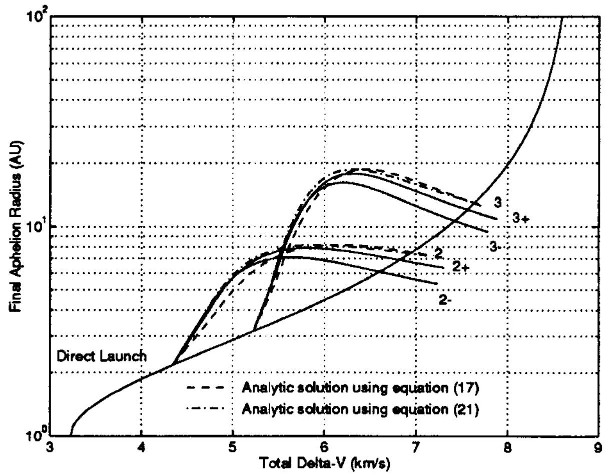

Fig. 12. ΔV-EGA Analytic Solutions Using Equations 17 and 21.

The post-encounter ΔV is added tangentially to V^+, so the initial slope can be approximated by differentiating equation (22)

$$dr_a/dV \approx 4\mu_S r_E^2 V^+/(2\mu_S - r_E V^{+2})^2 \qquad (23)$$

For the 2^+ ΔV-EGA trajectory, equation (23) gives 3.033 AU/(km/s) for the initial slope, while the curve has a slope of about 3.036 AU/(km/s) at the point where it becomes more efficient to apply ΔV after the flyby. So equation (23) provides a good estimate for this point.

ΔV-VGA

The ΔV-EGA concept is not limited to heliocentric orbits with Earth launch and gravity assist. It can be applied using other planets orbiting the Sun or even in systems with a different primary body (e.g., the Earth-Moon system). Whether or not a particular N ΔV-EGA type trajectory provides any advantage depends on the size of the orbit of the secondary about the primary and the relative masses of the two bodies. The orbit and mass of Venus are similar to those of Earth, indicating that ΔV-VGA trajectories may be useful.

The potential of ΔV-VGA trajectories (corresponding to launch from Venus) is shown in Fig. 13. Since launch ΔV is not as relevant, we also show final aphelion radius versus aphelion ΔV in Fig. 14. The integer N in this case refers to Venus years. One Venus year lasts 0.615 Earth years, so, for example, the Venus-Venus leg of a 6 ΔV-VGA would take about 3.7 Earth years, less than a 4 ΔV-EGA. From these figures we see that a 3 ΔV-VGA can reach Jupiter and a 5 ΔV-VGA can reach Saturn with no post-encounter ΔV. The V_∞s at Venus necessary to initiate these trajectories are presented in Table 2.

Table 2 V_∞ at Venus for N ΔV-VGA

N	V_∞
2	5.97
3	8.15
4	9.32
5	10.07
6	10.60

VGA

We now step back and consider trajectories launched from Earth to Venus. Figure 15 shows the V_∞ at Venus versus the Earth launch V_∞ directed opposite to Earth's velocity (assuming circular coplanar orbits for Earth and Venus). Figure 16 shows the potential of one or more Venus gravity assists (250 km minimum flyby altitude) after launch from Earth. (Multiple Venus encounters in Fig. 16 are not phased, i.e., the time of flight problem has not been solved.) As the Earth launch energy increases, the V_∞ at Venus increases. The solid line in Fig. 16 represents the final aphelion radius if the V_∞ can be turned parallel to the velocity of Venus, V_V. This can be accomplished with a single aerogravity assist for any V_∞ at Venus. However, if we consider gravity assist only, then each flyby can turn the V_∞ a limited amount. This turn angle decreases as the V_∞ increases, as previously described. The shapes of the curves are similar to those for ΔV-EGAs. As the launch energy increases and the corresponding V_∞ at Venus increases, a point is reached at which a single flyby can no longer turn the V_∞ parallel to V_V, and the single flyby curve in Fig. 16 leaves the solid curve.

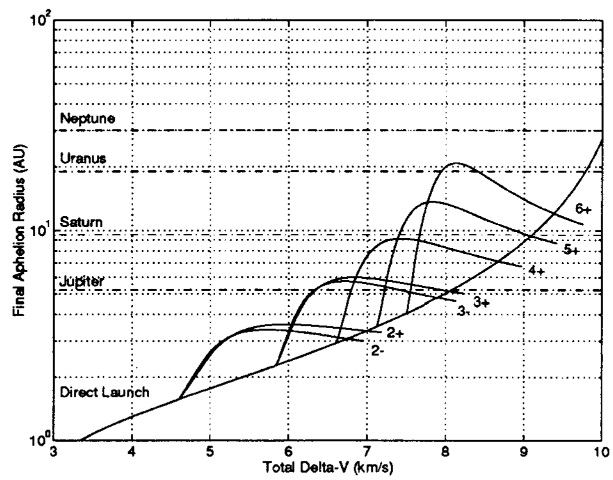

Fig. 13. ΔV-VGA Performance.

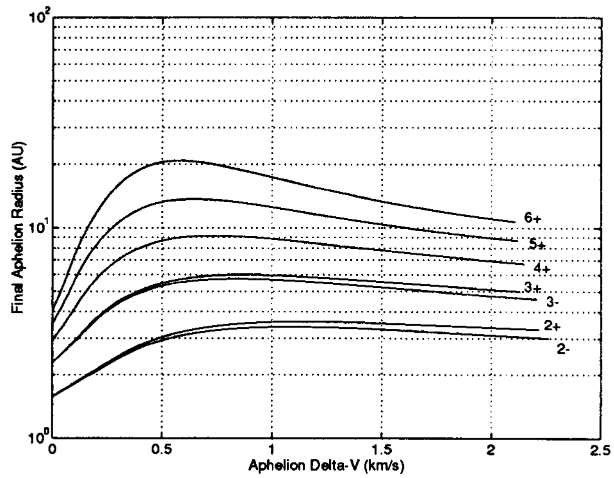

Fig. 14. ΔV-VGA Performance Versus Aphelion ΔV.

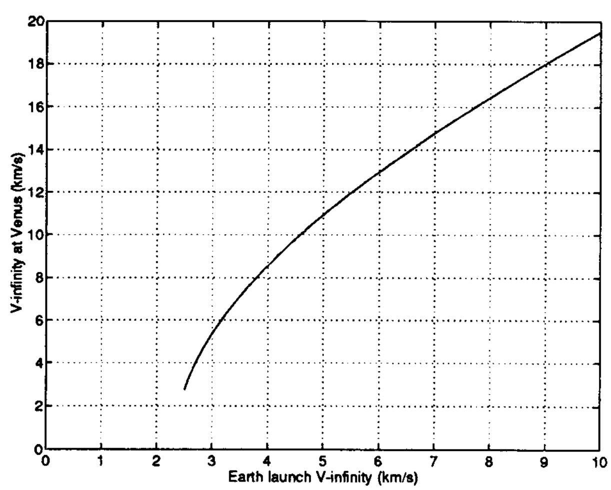

Fig. 15. V_∞ at Venus Versus Earth Launch V_∞.

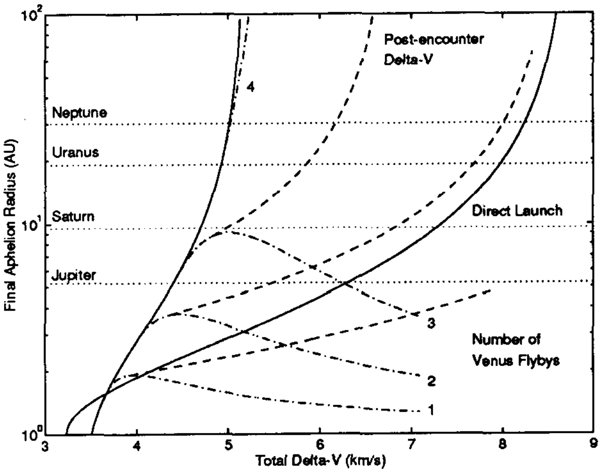

Fig. 16. Venus Gravity Assist Potential. (Multiple Venus Encounters without V_∞ Leveraging.)

The final aphelion radius then reaches a maximum and decreases. The trade-off between V_∞ and turn angle causes this phenomenon as it does for ΔV-EGAs. Multiple (n) Venus flybys increase the effective turn angle by a factor of n, and the curve shapes are similar to those for a single Venus flyby.

EV-ΔV-VGA

We are now prepared to consider launch from Earth to Venus followed by one or more ΔV-VGAs. Since we assumed earlier that the ΔV-VGA trajectories have "launch" perihelion at Venus, this initial analysis is limited to small Earth launch energies. A single Venus flyby can turn the $\mathbf{V}_\infty$ parallel to $\mathbf{V}_V$ for Earth launch C_3 up to 10 $(km/s)^2$. The V_∞ at Venus for this launch energy is 6.0 km/s, and so a 2^- ΔV-VGA can be flown. After the second Venus flyby, we use a 3^- ΔV-VGA and then a 5^+ ΔV-VGA. We have constrained the Earth launch and the 2^- and 3^- ΔV-VGAs so that the resulting $\mathbf{V}_\infty$ at Venus can be turned parallel to $\mathbf{V}_V$. Although the ΔV-VGAs in this case are not used to their fullest potential in maximizing heliocentric energy, this method allows us to use the previous analysis for each ΔV-VGA and quickly estimate the potential of multiple Venus flyby trajectories.

The trajectory described above flies by Venus four times and can reach the orbit of Saturn in 9.8 years with Earth launch $C_3 = 10\ (km/s)^2$ and post-launch $\Delta V < 1.0$ km/s. An additional ΔV of 0.7 km/s after the final Venus flyby decreases the total flight time to 9.1 years. References 2 and 4 each present a triple Venus flyby trajectory to Saturn. The trajectory in Ref. 2 includes a 2^- ΔV-VGA followed by a 4^+ ΔV-VGA. This trajectory, which has not been optimized, reaches Saturn in 8.2 years with Earth launch $C_3 = 19.4\ (km/s)^2$ and post-launch $\Delta V = 1.7$ km/s. The optimized trajectory in Ref. 4 includes 2 Venus flybys followed by a 4^- ΔV-VGA. The trajectory reaches Saturn in 9.1 years with Earth launch $C_3 = 25.0\ (km/s)^2$ and post-launch $\Delta V = 0.92$ km/s. Both of these trajectories include a maneuver after the final Venus flyby.

More General Types of V_∞ Leveraging

The ΔV-EGA and ΔV-VGA trajectories analyzed in this paper are a special case of V_∞ leveraging. The trajectory initiates at perihelion, and the leveraging maneuver is performed at aphelion and is applied along the velocity vector. These conditions allow a quick analysis of the potential and characteristics of these types of trajectories. They can be relaxed for a more general, and complex, analysis.

If the ΔV-EGA type trajectory is run backward (recall Fig. 1), the heliocentric energy and V_∞ can be decreased. Yen[7] proposes this type of trajectory, which she calls the "reverse ΔV-EGA" process, to reduce the ΔV required for capture into orbit around Mercury. The analysis and numerical values (including ΔVs) are the same whether the trajectory is run forward or backward.

Instead of using orbits larger than the Earth (or other secondary body), we can use smaller orbits. For example, a spacecraft is launched from Earth into a heliocentric orbit with a period slightly less than 1/2 year and aphelion radius equal to 1 AU. At perihelion a ΔV is applied to raise the aphelion in order to intercept the Earth non-tangentially. This enables the Earth to be used as a gravity-assist body. Referring to this type of trajectory as an "interior" ΔV-EGA, Sweetser[3] analyzes a reverse interior ΔV-EGA type trajectory in the Earth-Moon system which can reduce the total ΔV required to transfer from low Earth orbit to Lunar orbit.

The nominal orbit period does not have to be an integer multiple of the orbit period of the secondary body, as long as interception occurs in an integer number of periods. For example, the nominal orbit for a ΔV-EGA trajectory can have a period of 1.5 years, in which case the interception will take place after 3 years. Some trajectories in Ref. 7 include reverse ΔV-M(ercury)GAs in which the ratio of spacecraft orbit period to Mercury orbit period is 3/2, 4/3, or 6/5. The flight times in this case are reasonable because the orbit period of Mercury is short (88 days). These ratios would result in extremely long flight times for trajectories of this type involving other planets. For interior ΔV-EGAs the increased flight time is not significant.

V_∞ leveraging trajectories can reduce the total ΔV even more if the deep space ΔV is obtained without propellant expenditure by using a gravity assist with another body. A Venus-Earth gravity-assist (VEGA) trajectory is one example; this trajectory can be considered an interior ΔV-EGA with the Venus gravity assist providing the ΔV.

V_∞ leveraging trajectories can be used to modify not only the energy of the orbit but also the inclination. Bender[8] analyzes ΔV-EGAs which provide the high V_∞s necessary to achieve high inclinations with respect to the solar equator.

Conclusions

V_∞ leveraging is a powerful technique that can be used to reduce the launch energy requirements and total ΔV for interplanetary missions. ΔV-EGA, and the analogous ΔV-VGA, trajectories represent a straightforward application of V_∞ leveraging. An analytic procedure has been developed which can be applied to other types of V_∞ leveraging trajectories.

Acknowledgements

The authors thank Sylvia L. Miller and Steven N. Williams for their assistance and continuing support. This work has been supported by National Aeronautics and Space Administration Grant NGT-51129 (NASA Technical Officer John Lynch, JPL Program Administrator Carol S. Hix, and JPL Technical Monitor Steven N. Williams).

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