AIAA TP 82-1467

A82-40017

AIAA-82-1467

A Parametric Study of the Behavior of the Angular Momentum Vector During Spin Rate Changes of Rigid Body Spacecraft

J.M. Longuski and T. Kia, Jet Propulsion Lab, Pasadena, CA

AIAA/AAS Astrodynamics Conference

August 9-11, 1982
San Diego, California

A PARAMETRIC STUDY OF THE BEHAVIOR OF THE ANGULAR MOMENTUM VECTOR DURING SPIN RATE CHANGES OF RIGID BODY SPACECRAFT

A82-40017

James M. Longuski*
Tooraj Kia**
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

Abstract

During a spin-up or spin-down maneuver of a spinning spacecraft, it is usual to have not only a constant body-fixed torque about the desired spin axis, but also small undesired constant torques about the transverse axes. This causes the orientation of the angular momentum vector to change in inertial space. Since an analytic solution is available for the angular momentum vector as a function of time, this behavior can be studied for large variations of the dynamic parameters, such as the initial spin rate, the inertial properties and the torques. As an example, the spin-up and spin-down maneuvers of the Galileo spacecraft was studied and as a result, very simple heuristic solutions were discovered which provide very good approximations to the parametric behavior of the angular momentum vector orientation.

I. Introduction

The study of the parametric behavior of the angular momentum vector during spin-up and spin-down maneuvers of rigid body spacecraft is greatly facilitated by the analytic solution, $\vec{H}(t)$. While it is a simple matter to obtain the solution numerically by integrating the equations of motion on a computer, it is not so easy to find out how the final conditions vary as each of the dynamic parameters involved are perturbed. This is because a separate simulation is required for each perturbation and this can be expensive and time consuming. In this study, a computational algorithm was written based on the analytic solutions[1] for Euler's equations of motion, the Eulerian angles and the angular momentum vector so that the final angular momentum vector could be found quickly and efficiently without numerical integration.

The primary interest is focused on the variation of $\vec{H}$ $(t_f, \omega_{xo}, \omega_{yo}, \omega_{zo}, M_x, M_y, M_z, I_x, I_y, I_z)$ as each of the ten parameters is varied separately from its nominal value. Thus, plots are displayed which show the functional behavior of $\vec{H}_f$ in terms of $\Delta\omega_{xo}$, in terms of $\Delta\omega_{yo}$, etc. Specific numerical parameters from the Galileo spacecraft spin-up and spin-down maneuvers were used as an example. The perturbations corresponded mainly to the 3σ(standard deviations) values or larger. Using a deterministic performance analysis approach, an estimate of the $\vec{H}$ vector variations from its nominal final orientation is found. Since some of the perturbations, such as M_x and M_y, exhibited a linear variation of $\vec{H}_f$, these were examined for much greater ranges than the 3σ values. This approach revealed simple heuristic relations for the error model. Finally, the results of Monte Carlo simulation are compared to the heuristic estimates and conclusions are drawn for this particular and similar examples.

II. Solution of Euler's Equations of Motion

Euler's equations of motion of a rigid body are

$$M_x = I_x\dot{\omega}_x + (I_z-I_y)\omega_y\omega_z$$
$$M_y = I_y\dot{\omega}_y + (I_x-I_z)\omega_z\omega_x$$
$$M_z = I_z\dot{\omega}_z + (I_y-I_x)\omega_x\omega_y \qquad (1)$$

An accurate approximate analytic solution is obtained for near-symmetric rigid bodies subject to arbitrary constant moments by assuming

$$\omega_z = M_z t/I_z + \omega_{zo} \qquad (2)$$

When $I_x = I_y$, then the equation is exact as in the Bodewadt[2] solution but when $I_x \approx I_y$, the approximation provides very useful accurate solutions, particularly when ω_x and ω_y are small, which is usually the case for spin stabilized spacecraft. The solution for ω_x is given in the reference.[1]

III. Approximate Solution of the Eulerian Angles

The kinematic equations of motion for a Type 1: 3-1-2 Euler Angle Rotation are[3]

$$\dot{\phi}_x = \omega_x \cos\phi_y + \omega_z \sin\phi_y$$
$$\dot{\phi}_y = \omega_y - (\omega_z \cos\phi_y - \omega_x \sin\phi_y) \tan\phi_x$$
$$\dot{\phi}_z = (\omega_z \cos\phi_y - \omega_x \sin\phi_y) \sec\phi_x \qquad (3)$$

For the case of symmetric rigid bodies subject to constant moments, Bodewadt[2] proposed a solution of Eqs. 3 (and all other versions of Euler Angle Rotations) which has been shown to be valid only for a very restricted set of conditions and incorrect in general.[4]

*Senior Engineer, Guidance and Control Section, Jet Propulsion Laboratory
**Member of Technical Staff, Guidance and Control Section, Jet Propulsion Laboratory

A highly accurate approximate analytic solution for the Eulerian angles for a near-symmetric rigid body has been found corresponding to Case 3 of Bodewadt. The main restrictions in the solution are that two of the Eulerian angles must remain small and the parameter, $|\omega_z| / \sqrt{|\dot{\omega}_z|}$ must remain large.

IV. Solution of the Angular Momentum Vector

With the analytic results for the angular velocities, ω_x, ω_y, and ω_z and the Type 1:3-1-2 Euler angles, ϕ_x, ϕ_y and ϕ_z, the approximate analytic solution for the angular momentum vector in inertial space can be easily constructed:

$$\begin{Bmatrix} H_X \\ H_Y \\ H_Z \end{Bmatrix} = A \begin{Bmatrix} I_x\omega_x \\ I_y\omega_y \\ I_z\omega_z \end{Bmatrix} \qquad (4)$$

where A is the transformation matrix based on the Eulerian angles.

V. An Example of Parametric Behavior of the Angular Momentum Vector

In this section we will study the parametric behavior of the angular momentum vector during spin up and spin down maneuvers for a specific spacecraft, the Galileo. The purpose here is to obtain practical results which may be used in probabilistic error models. It will be demonstrated that simple heuristic solutions can be used to express the orientation of the final angular momentum vector and that perturbation of these solutions provide the secular effects. In addition it is shown that the periodic effects can be closely modeled by a ring distribution. The final result is a probabilistic error model which can be used to assess dispersions of the angular momentum vector during spin up and spin down maneuvers. The result is confirmed by the classical Monte Carlo analysis.

Galileo is a dual-spin dual purpose spacecraft. Scheduled for a 1986 launch toward Jupiter, it will release a probe into the Jovian atmosphere and then orbit the planet for about twenty months gathering and transmitting scientific data. The spacecraft is usually in dual-spin mode, but during ΔV maneuvers the mode is changed to single-spin by locking the rotor and stator together. It is also spun-up to 1. rad/sec, from a nominal of 0.306 rad/sec, prior to an axial ΔV burn. The procedure is reversed following the burn. This increases the stability margin and the accuracy of the maneuver. The thruster configuration is illustrated in Figure 1 where S2A and -S1A are the primary spin-up and spin-down thrusters, respectively and S2B and -S1B are backups.

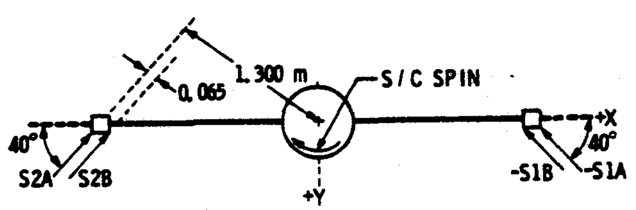

Figure 1. Spin Thruster Configuration

The nominal $\vec{H}$ pointing during the spin-up maneuver is shown in Figure 2 which was generated from Eq. 4. Since the angles involved are very small, the quantities H_Y/H_Z and H_Y/H_Z are used to describe the orientation of the angular momentum vector in inertial space. The analytic solutions were tested by employing the ACSL (Advanced Continuous Simulation Language) to do a very precise numerical integration by the Runge-Kutta method. It was found that the analytic solution for the $\vec{H}$ pointing produced errors no greater than order of 10^{-5} radians throughout the maneuver duration.

It is very interesting to note that the radial distance of the spiral path exhibited in Figure 2 from its center can be accurately approximated, in this particular case, by the heruistic relation

$$\rho(t) = (M_x^2 + M_y^2)^{1/2} / I_z\omega_z^2 \qquad (5)$$

where

$$\omega_z \approx M_z t / I_z + \omega_{zo}$$

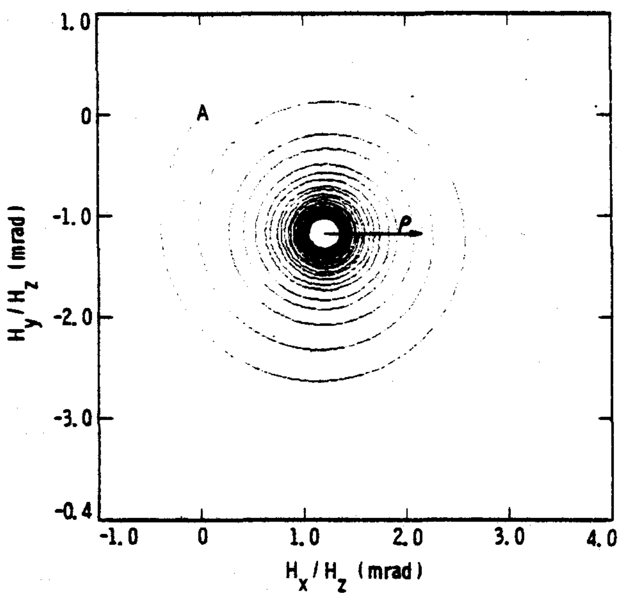

Figure 2. Nominal Orientation of the Angular Momentum Vector in Inertial Space During Spin-up Maneuver

The center point of the spiral is also approximated by simple heuristic relations:

$$x = -M_y / I_z \omega_{zo}^2$$
$$y = M_x / I_z \omega_{zo}^2 \qquad (6)$$

These solutions were inspired from the case of $M_z = 0$, in which they can be derived for small angles ϕ_x and ϕ_y. In this situation of constant spin rate, the angular momentum vector processes about the direction given by x and y, in a closed curve.

Now the parametric behavior of the final angular momentum vector orientation will be examined. For the purpose of this study it was assumed that the correct final spin rate is always achieved. This can be accomplished by using appropriate sensors such as star trackers or sun sensors in a feedback control system. An alternative approach was also studied involving a controlled burn time which is open loop, but these results are not presented.

Let $\vec{H}_f$ represent the nominal angular momentum at the end of the spin rate change maneuver and let $\vec{H}_f$ $(\Delta\alpha)$ represent the vector when a parameter α is perturbed by $\Delta\alpha$. Using ACSL Galileo spin-change dynamics was simulated.[1] The simulation was based on the analytic solutions derived from Eqs. 1-4. The method of deterministic performance analysis was employed to calculate and plot the variational behavior of the angular momentum vector $\vec{H}_f(\Delta\alpha) - \vec{H}_f$ as $\Delta\alpha$ is varied from extreme negative to extreme positive values. A few sample cases for spin-up and spin-down modes are shown in Figures 3 through 13.

These results indicate that parametric behavior can be described as either periodic, secular or a combination of periodic and secular effects. Perturbation of the transverse torques resulted in a secular perturbation of the final angular momentum vector while perturbation of I_z, M_z and ω_z produced combined periodic and secular variations with periodic effects being the dominant factor except for ω_z in the spin-up mode. Perturbations of I_x and I_y resulted in very little variations. Variation of the transverse angular velocities produced a linear (or secular) effect on the final angular momentum vector.

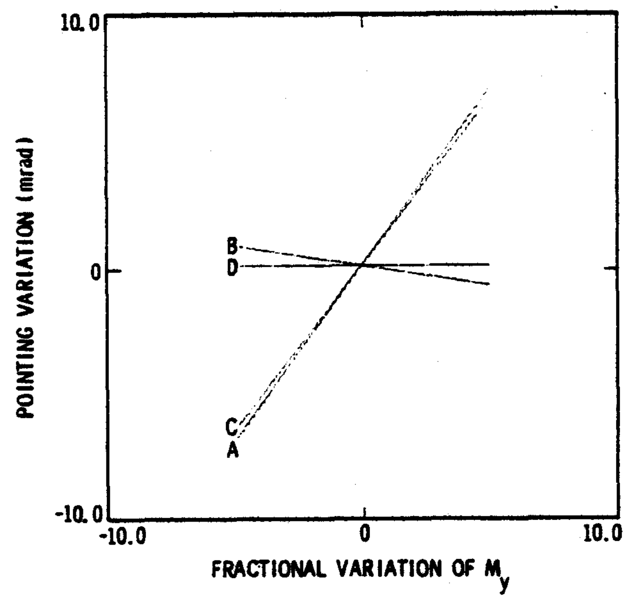

Figure 3. Variational Orientation of $\vec{H}_f$ During Spin-up for $M_y \pm 500\%$ (A) x-axis, (B) y-axis, (C) x-axis heuristic, (D) y-axis heuristic

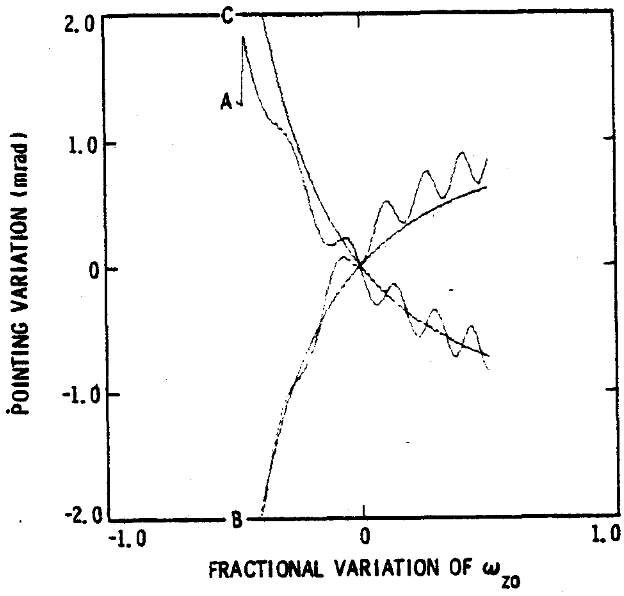

Figure 4. Variational Orientation of $\vec{H}_f$ During Spin-up for $\omega_{zo} \pm 50\%$. (A) x-axis, (B) y-axis, (C) x-axis heuristic and (D) y-axis heuristic

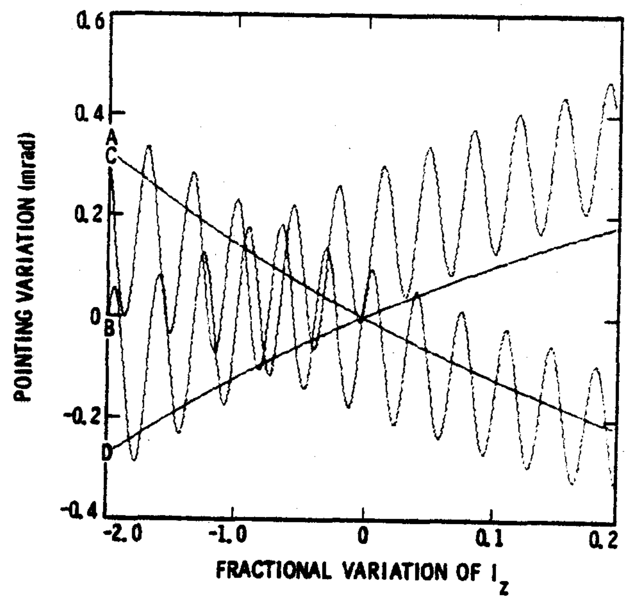

Figure 5. Variational Orientation of $\vec{H}_f$ During Spin-up for $I_z \pm 20\%$ (A) x-axis, (B) y-axis, (C) x-axis heuristic and (D) y-axis heuristic

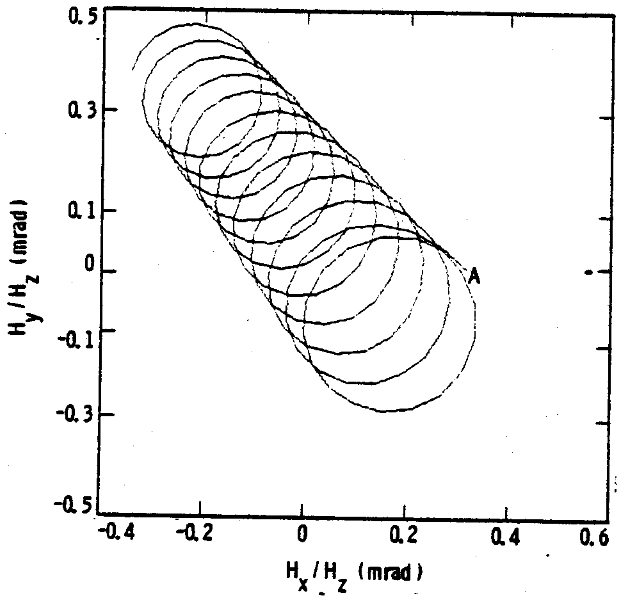

Figure 6. Variational orientation of $\vec{H}_f$ During Spin-up for $I_z \pm 20\%$

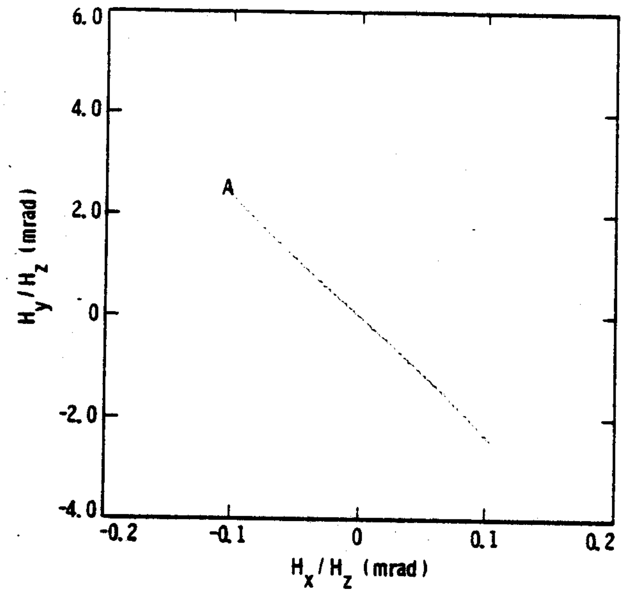

Figure 8. Variational Orientation of $\vec{H}_f$ During Spin-up for $\omega_{xo} \pm .0005$ m radians per second

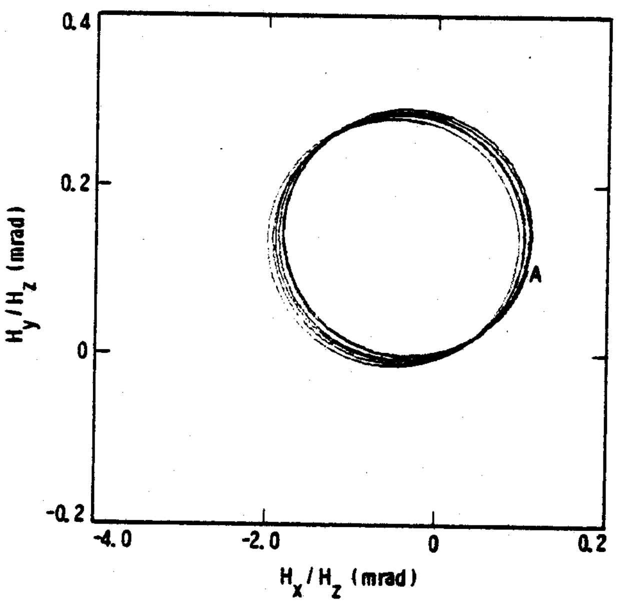

Figure 7. Variational orientation of $\vec{H}_f$ During Spin-up for $M_z \pm 50\%$

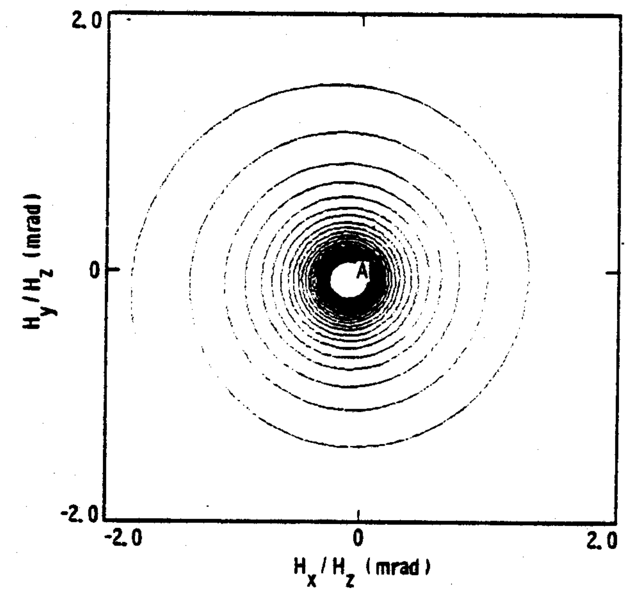

Figure 9. Nominal Orientation of the Angular Momentum Vector in Inertial Space During Spin-down Maneuver

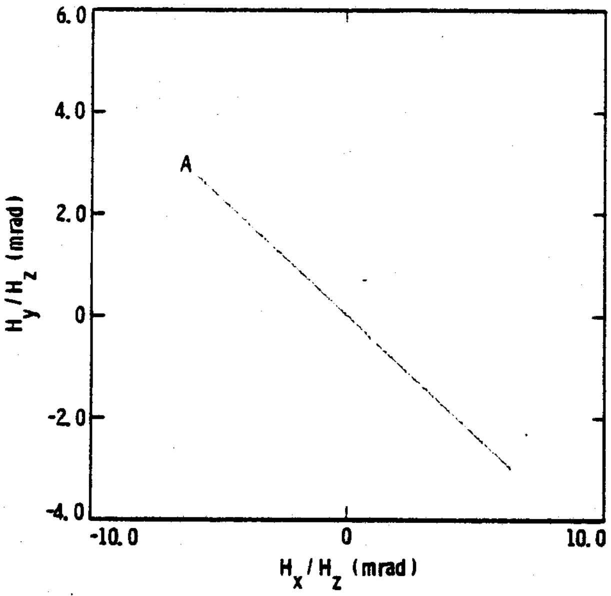

Figure 10. Variational orientation of $\vec{H}_f$ During Spin-down for $M_y \pm 500\%$

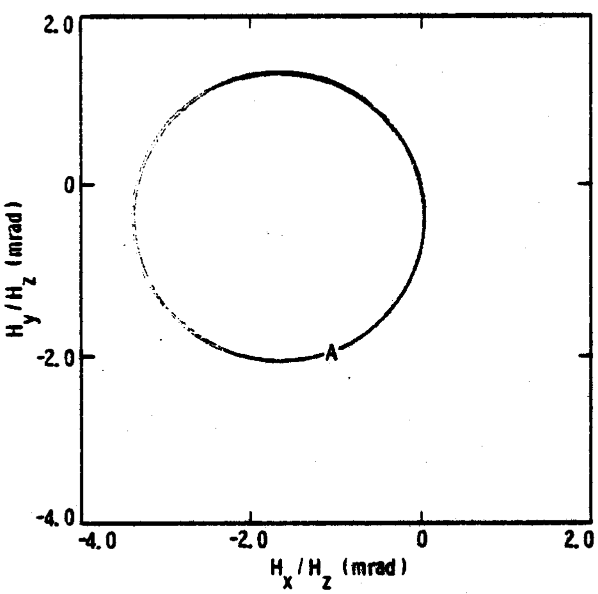

Figure 11. Variational Orientation of $\vec{H}_f$ During Spin-down for $\omega_{zo} \pm 50\%$

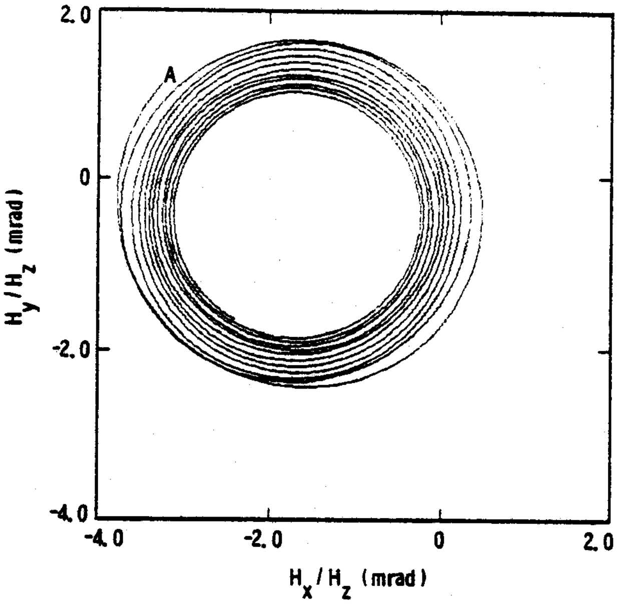

Figure 12. Variational Orientation of $\vec{H}_f$ During Spin-down for $I_z \pm 20\%$

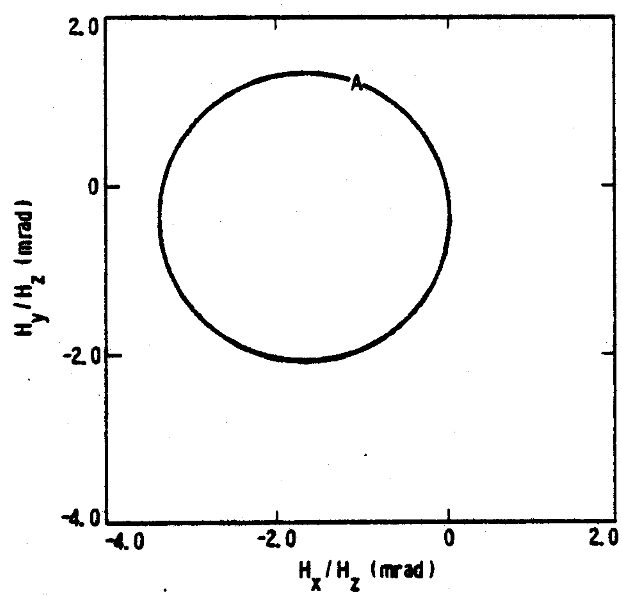

Figure 13. Variational Orientation of $\vec{H}_f$ During Spin-down for $M_z \pm 50\%$

Figures 3 through 5 have the heuristic solutions, as given by Eq. 6, superimposed on the plots. For zero initial transverse velocities the heuristic solution provides a good approximation of the secular and average periodic effects. The effect of nonzero initial transverse angular velocities is to produce an initial nutation angle, ξ, which is defined as the angle between the z axis and the angular momentum vector. The initial nutation angle is given by

$$\tan \xi = (I_x^2 \omega_{xo}^2 + I_y^2 \omega_{yo})^{1/2} / I_z \omega_{zo} \qquad (7)$$

Normally the transverse velocities are nearly zero and there is no nutation angle. Perturbations of ω_{xo} and ω_{yo} result in a linear perturbation of $\vec{H}_f$. This is shown in Figure 7 for variations in ω_{xo}.

Since Eq. 5 provides a good estimate for the average pointing error, it may also be used to estimate the variance due to the secular terms. Let $\varepsilon_x = dx/x$ signify the error in x, then,

$$\varepsilon_{\rho o} = d\rho_o/\rho_o = dM/M - dI_z/I_z - 2d\omega_z/\omega_z \qquad (8)$$

where

$$M = (M_x^2 + M_y^2)^{1/2}$$

Hence, assuming that the parameters are independent the secular uncertainty about the average may be estimated by the following equation.

$$\sigma_{\rho o} = (\sigma_M^2 + \sigma_{Iz}^2 + 4\sigma_{\omega z}^2)^{1/2} \qquad (9)$$

The simulation results for the periodic errors show that the final momentum vector due to these parameter variations will trace a circle about an average point which is given by Eq. 6. This result suggests a narrow width ring distribution model for the variations of the periodic effects.

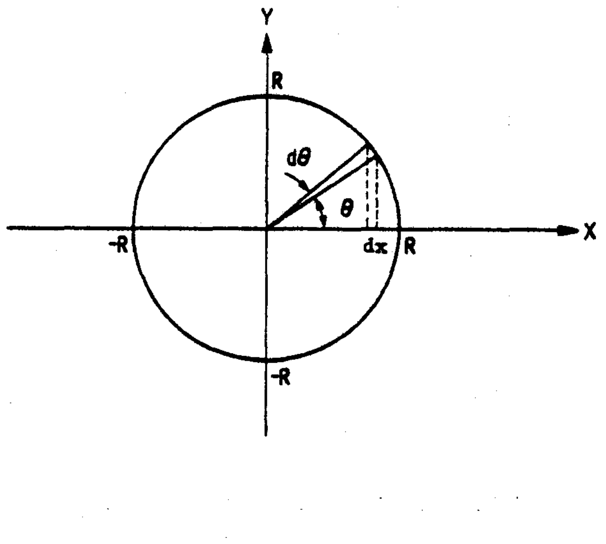

Figure 14. Zero Width Ring Distribution

To simplify the analysis a zero-width ring distribution, Figure 14, is assumed, which implies that the probability density function with respect to r is impulsive.

$$f(r) = \delta\,(r - R) \qquad (10)$$

Furthermore a uniform distribution in θ is assumed, i.e.,

$$f(\theta) = \begin{cases} 1/2\pi & 0<\theta<2\pi \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

These assumptions imply that the momentum vector points, with equal probability, to any location on the perimeter of a circle the center of which is given by Eq. 6 and the radius is R. This in effect defines a cone about the average pointing direction, given by Eqs. 5 and 6, the trace of which in the x-y plane is a circle that may be estimated as follows.

It may be shown that the probability density function in terms of one of the cartesian coordinates, say x, will be given by

$$f(x) = \begin{cases} (R^2 - x^2)^{-1/2}/\pi & -R<x<R \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

Where mean and variance for this distribution are given as zero and $R/\sqrt{2}$ respectively. The distribution plot is shown in Figure 15.

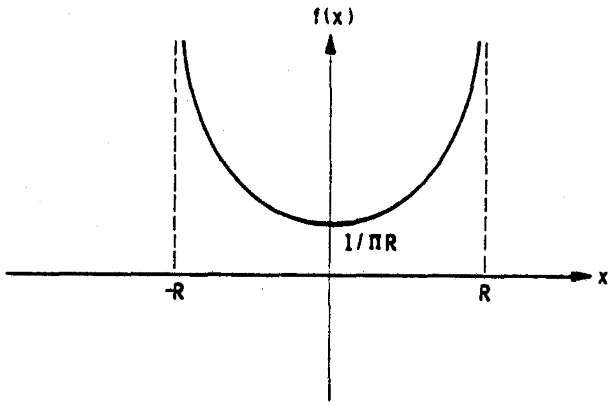

Figure 15. Zero Width Ring Distribution In Cartesian Coordinates

In the application to the Galileo spacecraft we are interested in the 3σ dispersions of the angular momentum vector, which corresponds to 99.7% level of confidence for a Gaussian distribution. For the ring distribution it is easily shown that the 99.7% level of confident will occur at x = R. The heuristic solution, Eq. 5, gives a good estimate of R for any $\omega_z(t)$. The circle of interest is the trace of the momentum vector H(t)at $t = t_f$. Hence Eq. 5 with $\omega_z = \omega_{zf}$ gives a good approximation to the 99.7% confidence circle of uncertainty.

By combining the above results we obtain a simple probabilistic model for the pointing error, Figure 16. The final angular momentum vector will point, with 99.7% probability, within a circle of radius R_t about the average pointing vector ρ_o where,

$$R_t = 3\sigma_{\rho o} + R \qquad (13)$$

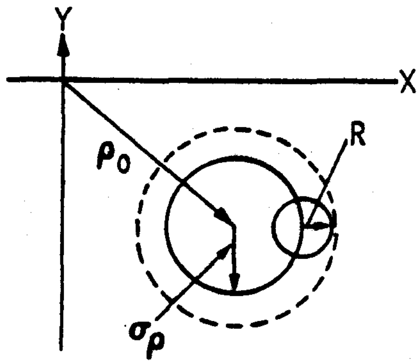

Figure 16. The Pointing Error Model for the Spin-change Maneuver

The numerical values of the parameters used in the simulation and the result of the deterministic performance analysis are summarized in Table I. Using these parameter values the estimated ring radius is calculated as 0.146 mrad, in close agreement with the results given in Table I.

VI. Conclusions

A highly accurate approximate analytic solution is given for the angular momentum vector for near-symmetric rigid bodies subject to constant moments. The solution applies when two of the Eulerian angles are small and a certain parameter is large. This analytic result permits the parametric behavior of the angular momentum vector to be studied during spin rate change maneuvers such as occur in the Galileo mission to Jupiter. For this particular case, simple heuristic formulas were discovered which aid greatly in obtaining quick numerical results. While it would not be wise to extend these heuristic results to the general case, the basic approach can be used to study a variety of interesting cases using the same analytic solution, without which such studies would be prohibitively expensive and time consuming due to the many computer simulations required.

Table 1. Deterministic Performance Analysis Results

ERROR SOURCE	NOMINAL SOURCE VALUE	SOURCE ERROR	RESULTANT SECULAR ERROR (mrad)	PERIODIC RADIUS OF UNCERTAINTY (mrad)
I_x	3012 kg-m^2	2.1%	0.00381	-
I_y	2716 kg-m^2	2.4%	0.00479	-
I_z	4627 kg-m^2	1.5%	0.04	0.15
M_x	-0.4757 N-m	5.1%	0.0585	-
M_y	-0.5669 N-m	5.1%	0.0699	-
M_z	13.0 N-m	5.1%	-	0.147
ω_{z0}	0.306 rad/s	6.8% (1)	0.225	-
ω_{zf}	1.047 rad/s	17% (1)	0.07	0.135
TOTAL ERROR			0.247	0.15

Note-1 Uniform distribution

Finally, a Monte Carlo simulation was performed. For the parameters producing periodic effects the Monte Carlo simulation resulted in 1.69 mrad mean and 0.1079 mrad 1σ values. These values compare favorably to 1.708 mrad and 0.1032 mrad estimated mean and 1σ values.

VII. Acknowledgements

The authors thank W. Breckenridge for his helpful suggestions.

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

VIII. References

(1) J. M. Longuski, "Solutions of Euler's Equations of Motion and Eulerian Angles for Near Symmetric Rigid Bodies Subject to Constant Moments," AIAA/AAS Astrodynamics Conference, AIAA Paper 80-1642, Danvers, Mass. August 11-13, 1980.

(2) U. T. Bodewadt, Der symmetrische Kreisel bei zeitfester Drehkraft. Math Z.55, 310-320 (1952).

(3) J. R. Wertz, Spacecraft Attitude Determination and Control, D. Reidel Publishing Co., Dordrecht, Holland, 1980.

(4) J. M. Longuski, "Comments on the Leimanis Solution of Self-Excited Rigid Body," AAS/AIAA Astrodynamics Conference, Paper 81-104, Lake Tahoe, Nevada, August 3-5, 1981. Accepted for publication by the Journal of the Astronautical Sciences.

(5) E. Leimanis, The General Problem of the Motion of Coupled Rigid Bodies About a Fixed Point, Springer-Verlag, New York, 1965.

IX. Nomenclature

I_i	moment of Inertia about the i-axis
M_i	applied torque about the i-axis
ω_i	i-axis angular rotation rate
t_f	spin-change duration
$\vec{H}$	angular momentum vector
ω_{io}	i-axis rotation rate at t=0
ϕ_i	Eulerian angles
ρ	radial distance from the center of spiral
ρ_o	distance from origin to the center of spiral
Δx	variations about x
ξ	nutation angle
$f(x)$	probability density function
ε_x	percent x-variable error
σ_x	x-variable error variance