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TRANSFERS TO A SUN-EARTH SADDLE POINT: AN EXTENDED MISSION DESIGN OPTION FOR LISA PATHFINDER

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The LISA Pathfinder extended mission may immediately follow the primary mission to a Sun-Earth L_1 libration point orbit. One extended mission concept with scientific appeal is a spacecraft path that includes multiple passes within 100 km of a gravitational equilibrium point. To explore this option, two methodologies are investigated: linking arcs from the circular restricted three-body problem, and propagating natural motion in a four-body model that leverages lunar gravity to complete multiple Earth passes or capture in the system. Potential trajectories are detailed and compared to the mission requirements.

INTRODUCTION

During its 200-day primary mission, the Laser Interferometer Space Antenna (LISA) Pathfinder spacecraft will remain in the vicinity of the Sun-Earth L_1 Lagrange point and demonstrate various technologies to enable detection of gravity waves, the last untested prediction from Einstein's general theory of relativity. Once the primary mission is complete, LISA Pathfinder (LPF) may initiate an extended mission, utilizing any remaining propellant reserves. An extended mission option, one with particular scientific appeal, is a spacecraft path that passes through a gravitational equilibrium point, or "saddle point," which coincides with a saddle in the gravitational potential field. Multiple passes within 100 km of the saddle point may allow LPF to measure the effects of Modified Newtonian Dynamics (or MOND).

Proposed by Mordehai Milgrom in 1983, MOND is an attempt to explain the "missing mass problem," a discrepancy between galactic rotation rates predicted by Newtonian dynamics and the visibly observed mass levels in galaxies to drive such rotations.[1] This discrepancy is commonly explained by allowing large amounts of unseen matter (i.e., dark matter) to affect galactic rotation and satisfy Newtonian dynamics, but MOND suggests that Newtonian dynamics do not apply to galaxies, removing the need for dark matter. As MOND only applies at extremely small levels of acceleration, it is impossible to measure its effects on Earth or at most other locations in the Solar System. Thus, visiting a saddle point offers a unique opportunity to test Milgrom's hypothesis. Such a saddle point exists near Earth, where the gravitational fields from the Sun, Earth, and Moon sum to a near-zero acceleration. Other bodies, such as Jupiter, contribute little to the local gravity field.

Previous work by Trenkel and Kemble demonstrates that small trajectory perturbations on the order of 1 m/s from the LPF primary mission orbit, a Sun-Earth L_1 Lissajous, are capable of reaching

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a transfer trajectory that passes within tens of thousands of kilometers of the saddle point.[2] In principle, additional maneuvers could potentially reduce this pass distance. This investigation builds upon the research by Trenkel and Kemble to identify transfer geometries that meet all mission constraints. The dominant constraint for such a mission is maneuvering capability; the available ΔV for the entire extended mission phase for LPF is less than 5 m/s. Thus, only very low-cost transfers from the primary science orbit to the saddle point are possible. Additionally, the Pathfinder spacecraft must encounter the saddle point at least twice with an Earth-centered inertial velocity between 1 and 2 km/s to obtain sufficient data for the desired measurements. The extended mission duration should not exceed 1 to 2 years to reduce operational costs.

Dynamical systems techniques are leveraged to identify and explore natural transfer arcs between the LPF mission orbit and the saddle point vicinity. Manifold arcs flow from the Sun-Earth L_1 region to the Earth-Moon vicinity. Repetitive motion such as periodic and quasi-periodic orbits in the Earth-Moon system are also investigated as options for multiple saddle point encounters. An end-to-end transfer trajectory is constructed via two methodologies: blending natural motion from circular restricted three-body models, and leveraging the natural dynamics of the bicircular restricted four-body problem. Finally, trajectory designs are transitioned to an ephemeris model and compared with the extended mission requirements.

DYNAMICAL SYSTEMS TECHNIQUES

To explore motion that links the LPF mission orbit to repetitive motion in the saddle point vicinity, dynamical models of varying fidelity are employed. The circular restricted three-body problem (CR3BP) provides autonomous governing equations that rapidly and accurately approximate motion in the Sun-Earth and Earth-Moon systems, including the LPF mission orbit near the Sun-Earth L_1 equilibrium point and periodic motion in the Earth-Moon system that repeatedly encounters the saddle point. These trajectories can later be transitioned to higher fidelity models such as the bicircular restricted four-body problem (BC4BP) and an ephemeris model. The BC4BP incorporates an additional perturbing gravitational body while maintaining simplifying assumptions that facilitate more rapid analysis and corrections processes than can be achieved in an ephemeris model.

Circular Restricted Three-Body Problem

The dynamical model in the CR3BP governs the motion of a relatively small body, such as a spacecraft, in the presence of two larger gravitational point masses (P_1 and P_2) which proceed on circular orbits about their mutual barycenter (B), as depicted in Figure 1. To simplify the governing equations and enable simple visualization of periodic solutions, the motion of the spacecraft is described in a frame ($\hat{x}\hat{y}\hat{z}$) that rotates with the two primaries: $\hat{x}$ points from the larger primary to the smaller primary, $\hat{z}$ is parallel to the primary orbit angular momentum vector, and $\hat{y}$ completes the orthonormal set. By convention, coordinates in this reference frame are nondimensionalized such that the distance between P_1 and P_2 and angular velocity $\dot{\theta}$ are unity.[3] Mass values are nondimensionalized by the total system mass such that the masses of P_1 and P_2 are equal to $1-\mu$ and μ, respectively. Employing these coordinates, the position of the spacecraft, represented by (x, y, z) in the rotating frame, is described by the following scalar equations of motion:

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} \qquad \ddot{y} - 2\dot{x} = \frac{\partial U}{\partial y} \qquad \ddot{z} = \frac{\partial U}{\partial z} \tag{1}$$

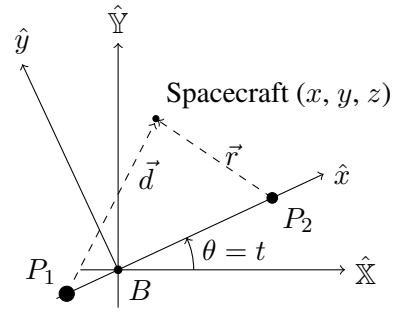


Figure 1: CR3BP System Configuration

where U is the pseudo-potential function, $U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r} + \frac{\mu}{d}$ and r and d represent the distances between the two primaries and the body of interest: $r = \sqrt{(x+\mu)^2 + y^2 + z^2}$ and $d = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$. Because the primaries are fixed in the rotating frame, these equations of motion admit a constant energy-like integral termed the Jacobi Constant, $C = 2U - v^2$ where v is the spacecraft velocity, $v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ relative to the rotating frame. This constant is employed to characterize motion in the CR3BP and offers a basic comparator between solutions.

Periodic Orbits and Manifold Arcs

Motion in the CR3BP is influenced by a system of dynamical structures that includes periodic orbits and their associated center, stable, and unstable manifolds. Well-known periodic solutions include the 3D L_1 and L_2 halo families, as well as the planar Lyapunov families. In fact, various spacecraft have leveraged these libration point orbits in the Sun-Earth system to meet mission goals, including Genesis, Advanced Composition Explorer (ACE), Solar Heliospheric Observatory (SOHO), and Global Geospace Science WIND, among others.[4,5] Transfers to and from such periodic orbits may frequently be accomplished via stable and unstable manifold arcs associated with a periodic orbit; stable manifold arcs asymptotically approach an unstable periodic solution while unstable arcs asymptotically approach the periodic orbit in *reverse* time, and thus depart the orbit in forward time. Because of the asymptotic nature of their approach, transitions from fundamentally unstable periodic solutions to manifold arcs may be achieved with very small perturbations and therefore provide particularly low-cost transfer options between periodic orbits and the surrounding space. Thus, they are attractive options for the LPF extended mission concept.

Resonant Orbits

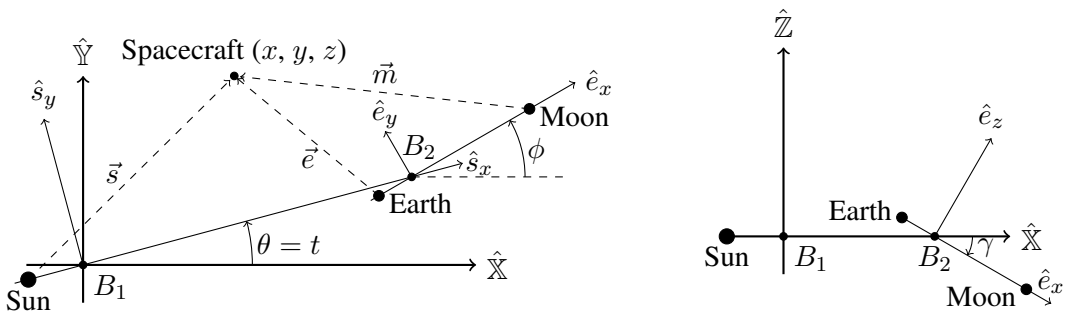
Resonant orbits are periodic solutions with orbital periods in rational ratios as compared to the period of the system in which they reside. For example, a 5:2 resonant orbit in the Earth-Moon system completes five revolutions around the Earth in the same interval that the Moon requires to complete two revolutions. These pure, rational period ratios exist only in the two-body problem. That is, when the gravitational perturbation of the secondary (e.g., the Moon in the Earth-Moon system) is included, resonant orbits must be adjusted to maintain periodicity and the ratio between their period and the system becomes irrational, though it remains near the two-body rational ratio.

Bicircular Restricted Four-Body Problem

Although the CR3BP facilitates accurate and rapid analysis of motion in the Earth-Moon and Sun-Earth systems, a recognition of the dynamical interactions between systems is required to accurately model a trajectory that incorporates motion in both three-body systems simultaneously. The Sun-Earth-Moon BC4BP accomplishes this goal by including the effects of all three bodies, as is apparent in Figure 2. The Sun, Earth, and Moon are modeled as point masses and move along circular trajectories: The Sun and Earth-Moon barycenter B_2 orbit the system barycenter B_1 while the Earth and Moon simultaneously orbit their barycenter B_2. The Earth-Moon orbital plane is inclined relative to the ecliptic plane by a constant angle γ, as depicted in Figure 2b, and does not precess over time. Thus, the motion is not coherent but provides a useful approximation to the true motion of the Sun, Earth, and Moon. Like the CR3BP, a rotating coordinate frame ($\hat{s}_x$, $\hat{s}_y$, $\hat{s}_z$) is employed to simplify the governing equations. Coordinates are nondimensionalized such that the distance between the Sun and B_2 and the angular velocity of the line connecting them ($\dot{\theta}$) are unity. Mass quantities are nondimensionalized by the total system mass such that the masses of the Sun, Earth, and Moon are equal to $1-\mu$, $\mu-\nu$, and ν, respectively. The spacecraft position, represented by (x, y, z) in the Sun-Earth (i.e., Sun-B_2) rotating frame, is governed by the following scalar equations of motion:

$$\ddot{x} - 2\dot{y} = \frac{\partial U_4}{\partial x} \qquad \ddot{y} - 2\dot{x} = \frac{\partial U_4}{\partial y} \qquad \ddot{z} = \frac{\partial U_4}{\partial z} \tag{2}$$

where the pseudo-potential is $U_4 = \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{s} + \frac{\mu-\nu}{e} + \frac{\nu}{m}$, and s, e, and m are the distance from the Sun, Earth, and Moon to the spacecraft, respectively. Due to the motion of the Earth and Moon in this frame, the BC4BP is nonautonomous and the governing equations do not admit an integral of the motion.



(a) Projection onto the ecliptic plane

(b) Projection onto a plane normal to the ecliptic

Figure 2: BC4BP System Configuration

Differential Corrections Scheme

Any end-to-end trajectory can be constructed by linking multiple arc segments, potentially from different CR3BP models, and applying a corrections scheme to enforce constraints such as position, velocity, and time continuity between arc segments. To illustrate this process, consider a series of arc segments such as those depicted in Figure 3. Each segment begins at a node state $\vec{q} = \{\vec{r}, \vec{v}\}^T$ and epoch time T, is propagated for some time τ, and terminates at a final state $\vec{q}_f$. The differential corrections scheme employed in this investigation alters these variables, grouped together in

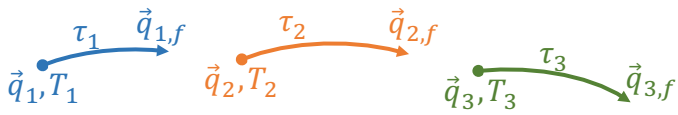


Figure 3: A series of arc segments that require corrections to ensure position, velocity, and time continuity.

a "design variable vector" $\vec{X}$, to meet a set of constraints $\vec{F}(\vec{X})$ by leveraging a first order linear approximation of the relationship between the constraints and the design variables (i.e., Newton's Method). This relationship is represented as

$$\mathbf{J}\Big|_{\vec{X}_n}\left(\vec{X}_n - \vec{X}_{n+1}\right) = \vec{F}(\vec{X}_n) \tag{3}$$

where $\mathbf{J}$ is a matrix of the partial derivatives of $\vec{F}$ with respect to $\vec{X}$. Given a set of design variables $\vec{X}_n$, the "minimum-norm" solution is determined for a corrected set of variables $\vec{X}_{n+1}$, one that ideally reduces the error $\left\|\vec{F}\right\|$ while remaining as similar as possible to the previous design variable vector $\vec{X}_n$. Other solution strategies exist and, thus, the result is not unique. Typically, multiple iterations of this process are required to satisfy all constraints to a desired tolerance. Common constraints include position continuity ($\vec{r}_{i+1} - \vec{r}_{i,f} = \vec{0}$), velocity continuity ($\vec{v}_{i+1} - \vec{v}_{i,f} = \vec{0}$), and time continuity ($T_{i+1} - T_i - \tau_i = 0$), and many other options exist to enforce desirable behavior. Impulsive maneuvers are modeled as instantaneous velocity changes and exist at the node points between segments that do not enforce velocity continuity.

THE SUN-EARTH-MOON SADDLE POINT

A saddle point is located where the gravitational accelerations from all celestial bodies sum to a net-zero acceleration value. Consider the dominant gravitational accelerations near the Earth, i.e., those of the Sun and Earth. The saddle point in this model exists on a line between the Sun and Earth, roughly 259,000 km from Earth. However, the Sun and Earth are not the only celestial bodies to produce significant gravitational accelerations in this region. To investigate the impact of other celestial bodies, consider the additional gravitational accelerations depicted in Figure 4. Each signal in this figure represents the time-varying distance between the Sun-Earth saddle point and the position of the saddle point as computed by incorporating the Sun, Earth, and an additional third body. The greater the distance between the two points, the larger the perturbation due to the additional third body. Because the saddle point exists in a location with net-zero gravitational acceleration, its position is computed by locating the roots (i.e., zeros) of the total acceleration function via a Newton-Raphson process.[6] The positions of all celestial bodies, including the Sun and Earth, are acquired from the Navigation and Ancillary Information Facility (NAIF) SPICE data sets for a roughly five-year duration beginning on Jan 1, 2015.[7] As is evident from Figure 4, the largest effect on the saddle point's position is due to the Moon, with perturbations on the order of hundreds to tens of thousands of kilometers. Clearly, lunar effects must be considered when seeking passes within 100 km of the saddle point. Both Jupiter and Venus perturb the saddle point's position by as much as 10 km, but these perturbations will be ignored for this investigation as their

impacts shift the saddle point within the acceptable pass distance. All other celestial bodies supply comparatively negligible perturbations and are also ignored.

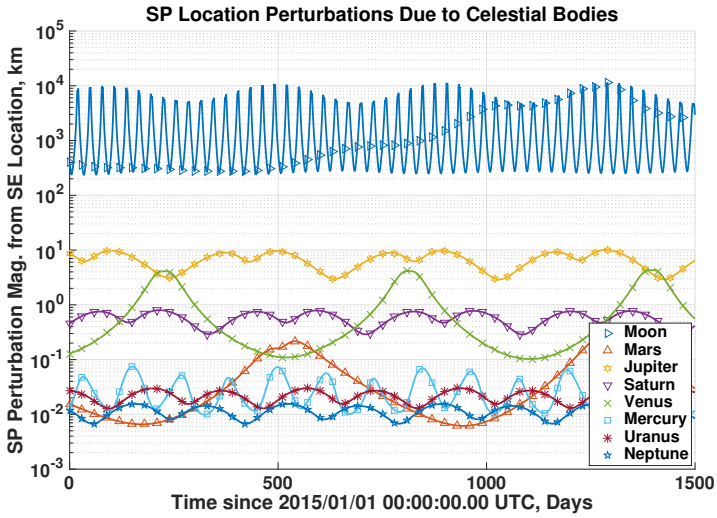


Figure 4: Perturbations of the Sun-Earth-P_n saddle point from the Sun-Earth saddle point position due to various significant celestial bodies P_n

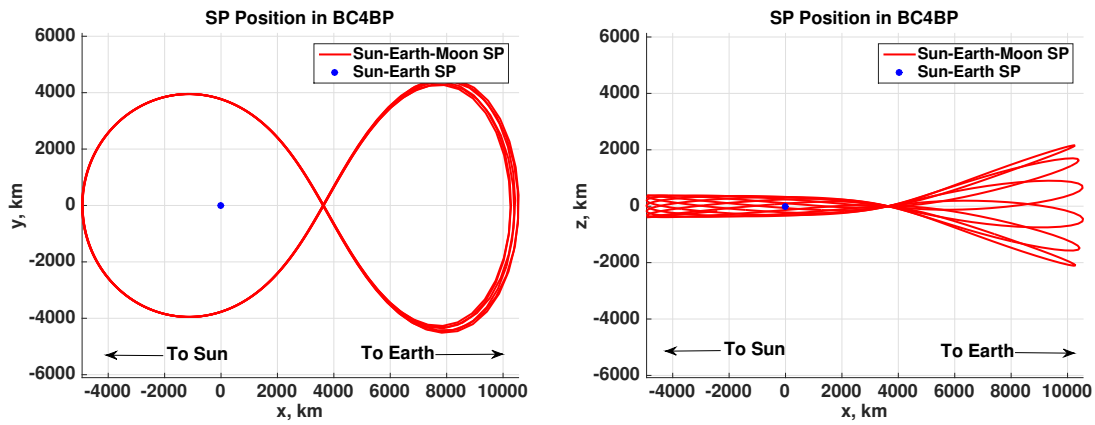
Due to the perturbing effects of lunar gravity, the saddle point's location in the Sun-Earth rotating frame oscillates over time. The location of the Sun-Earth and Sun-Earth-Moon saddle points over a period of 90 days appear in Figure 5 as a blue dot and red arc, respectively. The perturbation relative to the Sun-Earth saddle point approximation due to lunar gravity is represented in Figure 5 as the red oscillatory arc. The addition of the Moon's gravity perturbs the saddle point's location by roughly 5000 to 10000 km in the x direction, 4000 km in the y direction, and approximately 2000 km in the z direction, or normal to the ecliptic plane. The extended mission design for LISA Pathfinder (LPF) must accommodate these perturbations to achieve a pass distance within 100 km of the Sun-Earth-Moon saddle point (hereafter termed saddle point, or SP).

DESIGN METHODOLOGY

The LPF extended mission phase, as defined in this investigation, is initiated immediately following the conclusion of the LPF primary mission in a Sun-Earth L_1 Lissajous orbit. The extended mission path must complete two passes within 100 km of the saddle point at Earth-centered inertial speeds of less than 2 km/s while expending less than 5 m/s total in the impulsive maneuvers. An end-to-end transfer trajectory is constructed via two methodologies: blending natural motion from circular restricted three-body models, and leveraging the natural dynamics in the bicircular restricted four-body problem. Mission constraints are applied to discretized trajectory designs and a differential corrections process is applied to enforce the constraints.

Departing the LPF Mission Orbit

The LPF spacecraft completes its mission in a Sun-Earth L_1 Lissajous trajectory with approximate dimensions close to those of a halo orbit with an in-plane (ecliptic) amplitude of 800,000 km and a 150,000 km out-of-plane amplitude.[2] To simplify initial computations in this analysis,



(a) xy projection

(b) xz projection

Figure 5: Sun-Earth-Moon saddle point location in the Sun-Earth rotating frame relative to the fixed location of the Sun-Earth saddle point, over a period of 90 days

the quasi-periodic Lissajous trajectory is modeled as a periodic halo orbit in the CR3BP. A halo with the appropriate dimensions is easily obtained from software developed by Purdue University and NASA GSFC, i.e., the Adaptive Trajectory Design (ATD) software package.[8] This halo orbit, projected into the ecliptic plane as a black contour, appears in Figure 6. A selection of unstable manifold arcs associated with the halo orbit are plotted in magenta and a representative manifold arc is highlighted in blue for clarity. Each manifold is propagated for 200 days during which many of the arcs pass close to the Earth. Some of the arcs also pass near the saddle point (marked with an arrow) and some may pass sufficiently close to satisfy one of the two required encounters, though this projection does not depict the significant out-of-plane components. Regardless of the pass distance from the saddle point, the unstable halo manifold arcs provide a low-cost transfer option from the Sun-Earth L_1 vicinity to the Earth-Moon (and, therefore, saddle point) neighborhood.

Returning to the Saddle Point

Although the halo manifold arcs may offer one saddle point flyby, at least one additional pass is required to satisfy mission constraints. Periodic motion in the Earth-Moon vicinity, such as resonant orbits, may yield opportunities to repeatedly encounter the saddle point. A large variety of resonant orbits exist, however, so further criteria are employed to identify feasible options. To construct a low-cost end-to-end transfer, the resonant orbit should possess an energy level near the original energy of the Sun-Earth halo orbit and its manifold arcs, i.e., $C = 3.00081826$. Although the Jacobi constant values for an Earth-Moon trajectory and a Sun-Earth trajectory are not directly comparable because they exist in different three-body systems, the Earth-Moon arcs can be transformed into Sun-Earth coordinates and a new Jacobi constant value computed. Since the transformed Earth-Moon arc does not exist naturally in the Sun-Earth system, the computed Jacobi value is not constant in the Sun-Earth system. For the sake of comparison, consider the average value of Jacobi constant corresponding to the transformed Earth-Moon trajectory. A resonant orbit is considered similar in energy to the Sun-Earth halo manifold arcs if the mean Jacobi value, when transformed into the Sun-Earth rotating frame, is approximately equal to 3.00081826.

Among a set of 25 resonant orbit options, the 1:3 resonant orbit possesses an average transformed

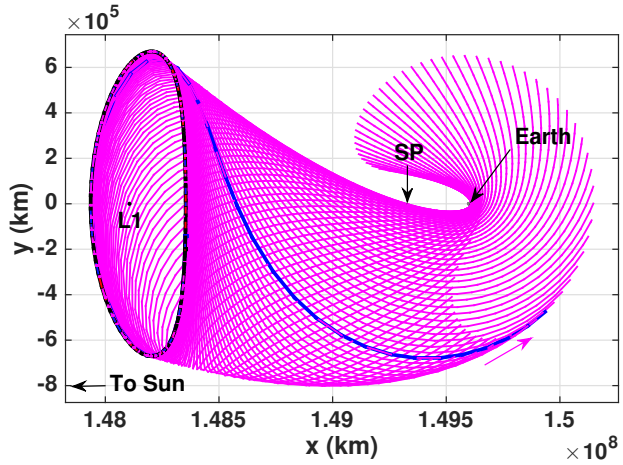
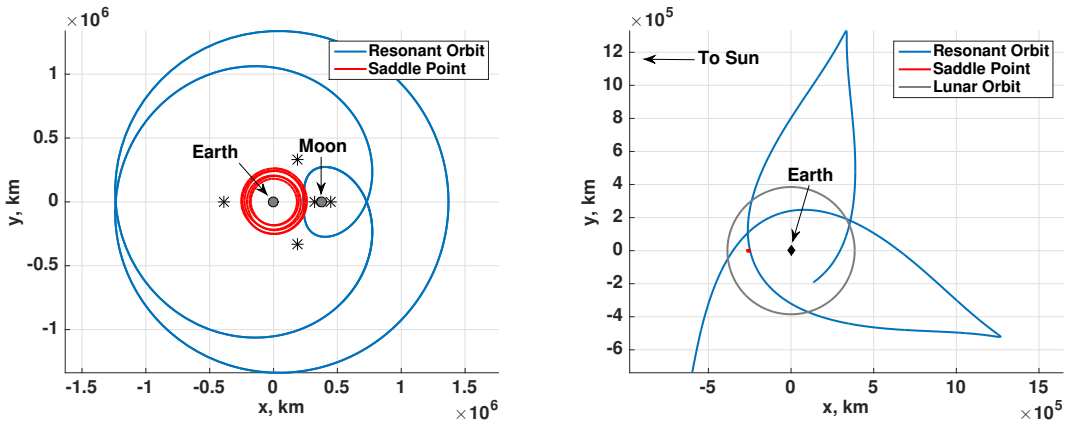


Figure 6: Sun-Earth L_1 Halo orbit (black) and associated unstable manifold arcs (magenta and blue) plotted in the Sun-Earth rotating frame

Jacobi constant value near the manifold arc Jacobi value. Additionally, when an initial epoch is appropriately chosen, this trajectory passes near the saddle point, as apparent in Figure 7. This resonant orbit exists naturally in the Earth-Moon system (Figure 7a) and is transformed into Sun-Earth rotating coordinates for easy comparison to the previously computed halo manifold arcs. The orbit passes through the saddle point trajectory, represented by a red arc, once during a single revolution along the orbit. Although the Earth-Moon representation in Figure 7a clearly demonstrates that the resonant orbit passes through the saddle point trajectory, no information about the synchronicity of the intersection is apparent. Transforming the resonant orbit to the Sun-Earth frame verifies that, given an appropriate choice of initial epoch, the resonant orbit will encounter the saddle point in both space and time.



(a) Earth-Moon Rotating View

(b) Sun-Earth Rotating View

Figure 7: Earth-Moon 1:3 Resonant Orbit

Natural motion in the Sun-Earth three-body problem as well as in the Sun-Earth-Moon four-

body problem can also supply additional opportunities to encounter the saddle point. Though not apparent in Figure 6, a subset of the halo manifold arcs return to the Earth-Moon neighborhood after completing an Earth flyby and reaching apogee. Perturbations from lunar gravity are included in the BC4BP and further affect the manifold arcs and subsequently produce saddle point encounters.

Constructing an End-to-End Transfer

A complete extended mission trajectory is constructed by combining an unstable manifold arc from the Sun-Earth system with an Earth-Moon resonant orbit or with further manifold propagation in the Sun-Earth CR3BP or Sun-Earth-Moon BC4BP. As an example, consider a trajectory constructed from a Sun-Earth manifold arc and an Earth-Moon resonant orbit. To combine these paths, each arc is first transformed into the same coordinate system in a dynamical model that, at minimum, incorporates the forces required to create each phase of the transfer. The Sun-Earth-Moon BC4BP is the most convenient choice as it models the required forces (gravity from the Sun, Earth, and Moon, simultaneously) and is formulated in terms of Sun-Earth rotating coordinates, which facilitate identification of saddle point encounters. The Sun-Earth manifold arcs are easily transitioned into the BC4BP: the variables are non-dimensionalized by the BC4BP characteristic quantities and shifted to a B_2-centered system rather than a B_1-centered system. Any Earth-Moon dynamical structures must be rotated into the correct coordinate frame before non-dimensionalization and coordinate shifting can occur. This rotation is determined by the relative angle between the Earth-Moon line and the Sun-Earth line, nearly a linear function of epoch. In this investigation, each manifold arc is assumed to depart the Sun-Earth L_1 halo orbit on April 18, 2016, an approximate date for LPF end of mission*. The initial epoch along an Earth-Moon resonant orbit must match the final epoch on the connecting halo manifold arc to ensure time continuity, thus, the rotation angle between the Earth-Moon line and Sun-Earth line is uniquely determined by the time-of-flight value for the connecting manifold arc. An end-to-end transfer, plotted in Figure 8, combines a manifold arc (purple) and 1:3 resonant orbit (green). Both transfer segments are depicted in the BC4BP Sun-Earth rotating frame. Each segment is discretized into a series of arcs, each originating at a node (black circle).

Once a complete trajectory is constructed, a differential corrections process is employed to guarantee position, velocity, and time continuity between the discretized arc segments (within some pre-determined tolerance) and to enforce any additional constraints such as saddle point encounters and ΔV budget. A node state coincides with the saddle point by constraining each component of the inertial gravitational acceleration vector at the node to be zero. When this constraint is satisfied, the node will be positioned within a fraction of a millimeter of the saddle point, well within the required 100 km pass distance. Although this precision is not required for the extended mission, other constraint types are less robust and converge much more slowly; the exact saddle point targeting ensures that any perturbations from celestial bodies such as Jupiter and Venus will not violate the 100 km pass distance.

RESULTS

Multiple approaches to construct a feasible transfer have been investigated and three are discussed here. All three methods employ unstable manifold arcs to transfer the LPF spacecraft from its primary mission orbit about the Sun-Earth L_1 point to the Earth-Moon vicinity. Such arcs typically require between 140 and 180 days to reach the Sun-Earth line (i.e., the x-axis) and are then linked to one of three trajectory options to achieve saddle point encounters. The first procedure joins a

*This date is based on an early launch estimate of Nov 1, 2015 and mission duration of 200 days

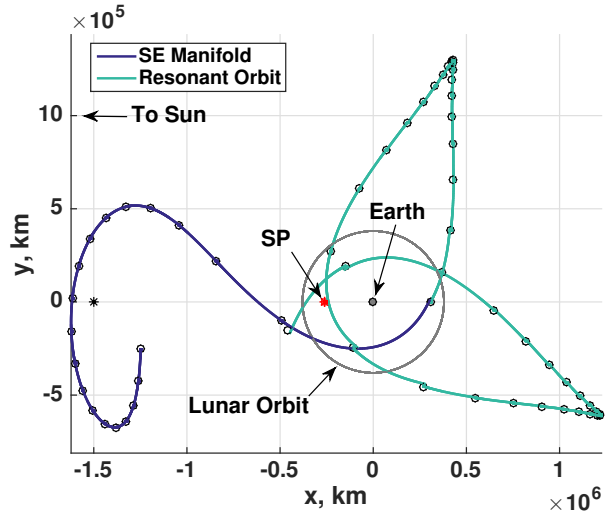


Figure 8: An end-to-end, uncorrected transfer design that incorporates a Sun-Earth halo unstable manifold arc (purple) and Earth-Moon 1:3 resonant orbit (green). The transfer is discretized into arc segments, each of which originates at a node represented by a black circle.

manifold arc to an Earth-Moon resonant orbit that possesses a similar energy level as the manifold arc and encounters the saddle point multiple times. The second approach leverages natural motion in the Sun-Earth-Moon BC4BP propagated forward from the unstable manifold to a second Earth flyby; these transfers are termed "two-pass" transfers. The third method also leverages natural BC4BP dynamics and employs trajectories characterized by a lunar flyby that perturbs the halo orbit manifold into a "captured" trajectory.

Linking Manifold Arcs and Resonant Orbits

The resonant orbits selected for this analysis are nontrivial to reproduce in the Sun-Earth-Moon BC4BP. Resonant orbits that possess energy levels near the halo manifold arc energy value possess high-altitude apogees. Consequently, with apogees far from the Earth and Moon, the path is significantly perturbed by solar gravity. These perturbations might be corrected, but the epoch constraints that are required to enforce time continuity with the manifold arcs remove any flexibility to leverage the solar perturbations constructively. A close-up view of resonant orbit arc segments in Figure 9 at one of the apogees clearly demonstrates visible discontinuities in position and velocity. Although discontinuities are not necessarily prohibitive to the corrections process, the arcs in Figure 9, propagated under BC4BP dynamics, flow in directions inconsistent with the desired behavior. Unless significant maneuvers are placed between arc segments, this resonant orbit, with the same type of geometry, cannot exist at an appropriate epoch and energy level. Other options with alternate geometries may exist but have not been fully explored.

Resonant orbits with lower altitude apogees are clearly less sensitive to solar perturbations as the gravitational accelerations from the Earth and Moon are dominant near the Earth. Unfortunately, the energy values associated with these smaller resonant orbits are inconsistent with the energy values associated with the Sun-Earth manifold arcs that deliver the LPF spacecraft to the Earth vicinity. Overcoming this difference requires a large energy change, likely enabled via a propulsive maneuver or carefully placed flyby; the former is prohibited by mission constraints and the latter is

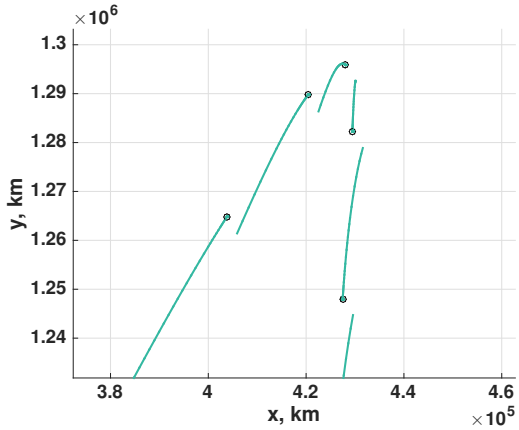
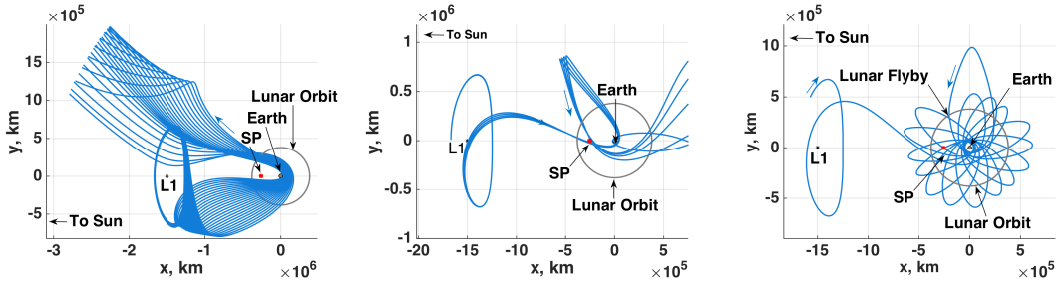


Figure 9: Discontinuities in position and velocity are apparent at an apogee along the transformed Earth-Moon resonant orbit, as displayed in the Sun-Earth rotating frame

not yet facilitated by the corrections scheme employed in this investigation. Alternative geometries are still being explored, but, because of the difficulties, Earth-Moon resonant orbits are poor baseline design options for the LPF extended mission at this time.

Natural BC4BP Motion

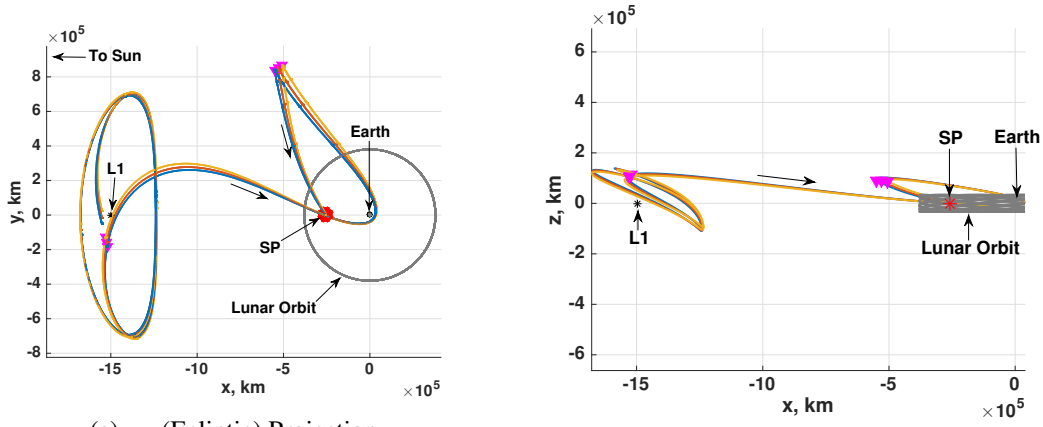
Employing natural motion in the BC4BP avoids the perturbation issues that plague the transitioned CR3BP resonant orbit arcs. A survey of such behavior reveals a variety of geometries; three types are represented in Figure 10 in the Sun-Earth rotating frame. First, a number of natural manifold-like arcs complete a single close Earth pass and then depart the system (see Figure 10a). Some of these arcs pass sufficiently close to the saddle point to achieve an encounter but do not return (at least within any reasonable time frame) and are therefore poor options for the LPF extended mission. A second type of arc, in Figure 10b, remains in the Earth vicinity long enough to complete an additional close flyby. These trajectories pass near the saddle point twice and are promising options for the extended mission. A third set of arcs, those is depicted in Figure 10c, leverage a lunar gravity assist and capture in the Earth-Moon vicinity. These transfer options may encounter the saddle point many times due to their compact structure and relatively short periods. Other types of behavior exist, including arcs that temporarily capture, but only the two-pass and capture geometries are explored further at this time in this investigation.



(a) One-Pass (b) Two-Pass (c) Capture

Figure 10: Three types of natural BC4BP motion displayed in the Sun-Earth rotating frame

Two-Pass Option Though the two-pass arcs exist naturally in the BC4BP, they do not pass within 100 km of the saddle point and must be corrected to meet mission constraints. To offer the corrections process some flexibility, maneuvers are allowed at an early node along the manifold arc and at apogee following the first Earth flyby. Additionally, two nodes are constrained to intersect the saddle point's location. An initial application of the corrections scheme yields end-to-end transfers with total ΔV budgets between 65 and 85 m/s. By applying an additional constraint on the total ΔV and iteratively reducing the allowable budget, three transfer options are constructed. These transfers, plotted in Figure 11 as blue, red, and yellow arcs, require 48, 51, and 57 m/s to achieve saddle point encounters, respectively. Recall that phasing is critical since the saddle point excursion in this model ranges over thousands of kilometers as apparent in Figure 5. Each subsequent saddle point encounter exactly intersects the approximate saddle point position at acceptable velocities: The initial pass occurs at an Earth-centered inertial velocity of approximately 1552 m/s, and the final pass occurs at velocities between 1608 and 1624 m/s. Hence, all mission constraints, except the maximum maneuver budget, are satisfied.

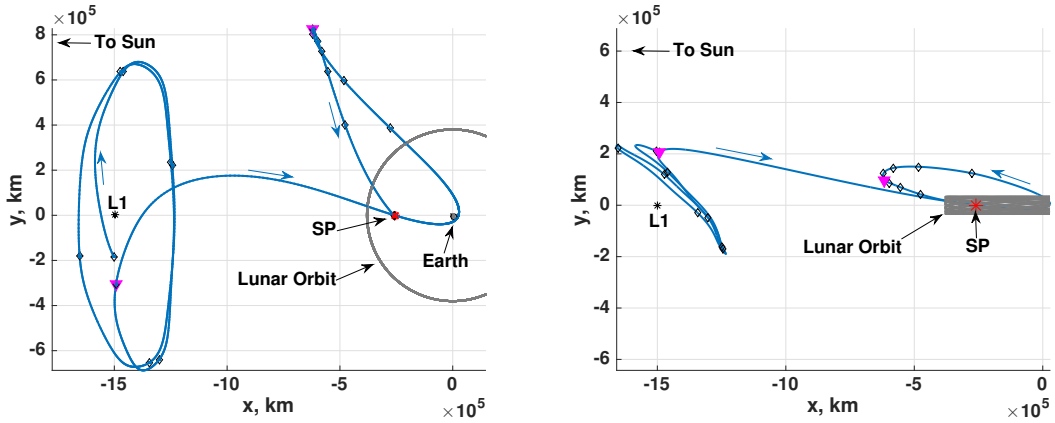


(a) xy (Ecliptic) Projection

(b) xz Projection

Figure 11: Three corrected two-pass BC4BP transfers in the Sun-Earth rotating frame; ΔV's are represented by magenta triangles

As a final step in the corrections process, the solutions converged in the BC4BP model are transitioned to an ephemeris model and re-converged. To illustrate this process, the lowest-cost transfer (the blue arc in Figure 11) is input into ATD's ephemeris corrections module and adjusted under the influence of Earth, Moon, and Sun gravity; solar radiation pressure is not included at this time. As in the BC4BP corrections scheme, two impulsive maneuvers are allowed during the transfer and two nodes are constrained to intersect the saddle point's location. One iteration of the corrections process yields a trajectory continuous in position and time, but with a high ΔV requirement. An additional ΔV constraint is applied to limit the total maneuver cost and iteratively decreased to identify a local minimum. The results from this process are plotted in Figure 12: a transfer with a total maneuver budget of 85 m/s. The transfer geometry remains similar to the behavior in the BC4BP, though the non-circular motion of the Sun, Earth, and Moon have perturbed the solution slightly. Although transitioning this example from the BC4BP to the ephemeris model increases the required ΔV, other solutions demonstrate reduced ΔV's when transitioned to the higher-fidelity model.



(a) xy (Ecliptic) Projection

(b) xz Projection

Figure 12: Ephemeris-Corrected Trajectory; ΔV's are represented by magenta triangles

Further increases in fidelity may be achieved by leveraging quasi-periodic motion to represent the LPF primary orbit. Recall that the LPF spacecraft actually completes its mission in a Sun-Earth L_1 Lissajous path; the Sun-Earth halo orbit employed in this investigation is an approximation of that motion. The ΔV values that are required to implement the extended mission may possibly be reduced by retaining the LPF spacecraft on the Lissajous path until a more opportune epoch or manifold geometry becomes available. An example of an appropriately sized Lissajous (gold) trajectory and its approximating halo orbit (blue) appear in Figure 13. Motion on the quasi-periodic Lissajous orbit differs significantly from the halo orbit geometry and thus results in a different array of manifold arcs, some of which may offer transfers with lower maneuver costs than those considered in this investigation. However, an extended stay on the Lissajous path can significantly increase the extended mission time-of-flight as each revolution along the Lissajous (i.e., when visualized from an xy-projection) requires approximately six months.

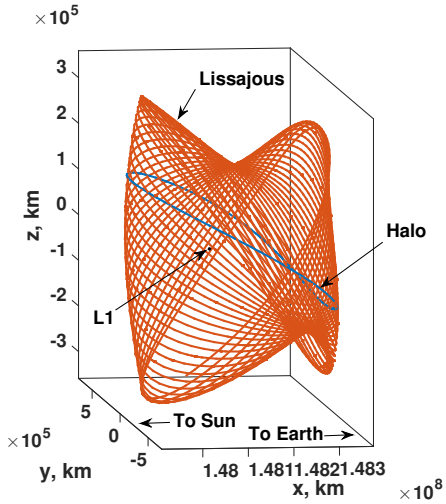
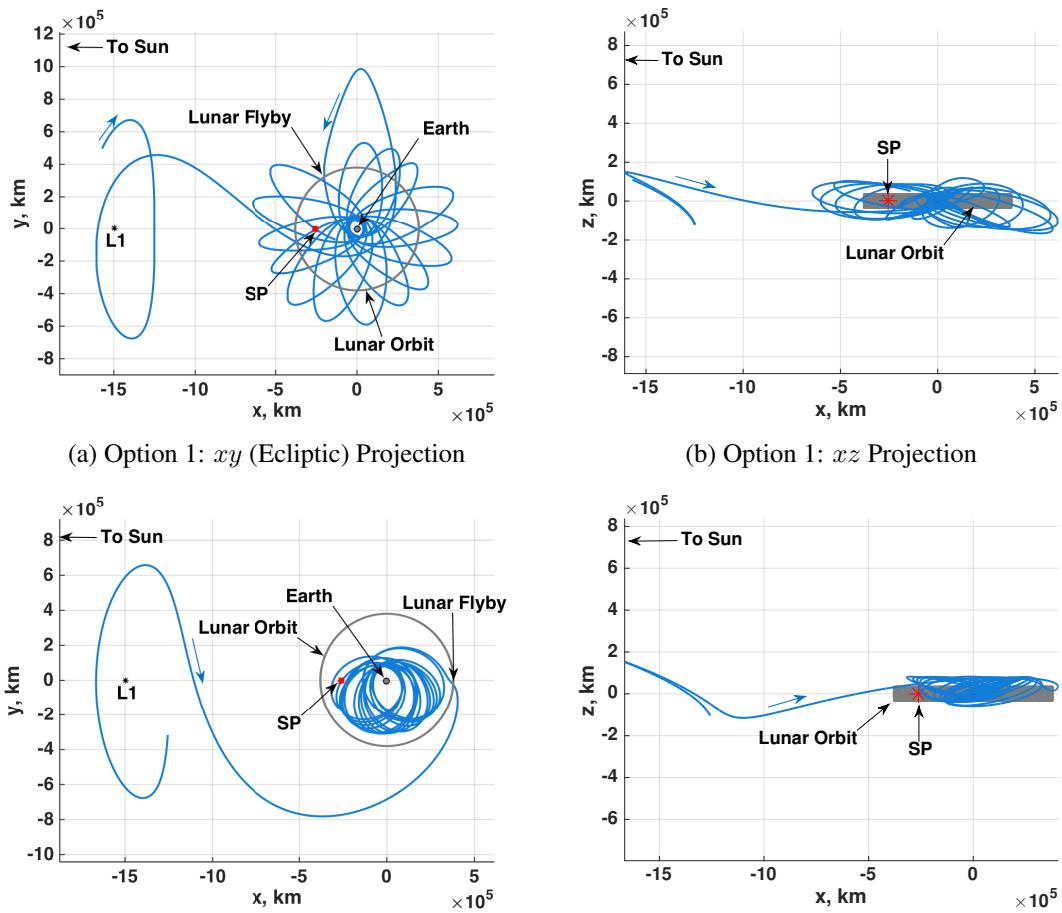


Figure 13: A Sun-Earth L_1 halo orbit (green) and Lissajous orbit (gold); plotted in the Sun-Earth rotating frame; generated with ATD[8]

Capture A final transfer option incorporates natural motion in the BC4BP that leverages a lunar flyby to capture into the Earth-Moon vicinity. Two examples of this type of geometry are displayed in Figure 14. Each transfer in the figure is projected into the xy-plane (ecliptic) and the xz-plane (normal to the ecliptic) for convenient viewing. The saddle point is labeled in all projections and is represented by a red contour and asterisk. Both transfer options depart the Sun-Earth L_1 halo orbit on an unstable manifold arc and approach the Earth-Moon vicinity. The first example, Option 1, moves through an Earth flyby, reaches apogee, and then encounters the Moon and captures within the system. Option 2 reaches a lunar flyby directly from the manifold arc. Though the exact capture conditions vary from example to example, they all offer similar benefits. The compact nature of these arcs provide multiple saddle point encounter opportunities and small maneuvers may facilitate large changes many revolutions about Earth. Additionally, small changes to the lunar flyby state can affect the captured motion to adjust saddle point encounters.



(a) Option 1: xy (Ecliptic) Projection

(b) Option 1: xz Projection

(c) Option 2: xy (Ecliptic) Projection

(d) Option 2: xz Projection

Figure 14: Examples of natural motion in the BC4BP that captures in the Earth-Moon system aided by a lunar flyby; displayed in the Sun-Earth rotating frame

Despite their benefits, capture-type geometries have several drawbacks. Although the compact geometries offer many encounter options, many revolutions may be required to reach the saddle point with the consequence for long times-of-flight. Additionally, the low-altitude perigees common

to these geometries may pass within the orbit of Earth-orbiting satellites or repeatedly pass through the Van Allen radiation belts. Finally, the sensitive lunar flybys facilitate large changes with minimal maneuver requirements, but such downstream effects can be difficult to predict and no additional propellant is available for further adjustments. Further investigation is warranted to leverage these lunar gravity assists and meet mission requirements.

Evaluating Transfer Options

Three transfer design strategies have been explored, each with their own advantages and disadvantages. The first method, which combines Earth-Moon resonant orbits with Sun-Earth halo orbit manifold arcs, leverages knowledge of three-body dynamics to rapidly construct an end-to-end transfer. Motion in each system is well understood and transfer segments can be selected from large families of periodic and quasi-periodic motion. However, the Earth-Moon resonant arcs that match the energy level of the Sun-Earth manifold arc do not transition well into the BC4BP and require large maneuvers to recover the desired geometry. The second approach, which utilizes natural motion in the BC4BP and completes two Earth flybys, clearly leverages three gravitational bodies and is more difficult to predict than three-body motion. Periodic orbits are not available in the BC4BP, and an additional variable, epoch, influences the dynamics. However, because this motion exists naturally, fewer corrections are required. Additionally, the geometry of the transfers is simple (i.e., does not include many "loops" or close flybys) and is easily constrained to encounter the saddle point twice. The final concept, which leverages natural four-body motion and lunar gravity assists, yields trajectories that possess more complex geometry than the double pass transfers. Although the captured motion is natural and does not require corrections for continuity, the differential corrections process struggles to implement meaningful adjustments because of the sensitivity and complexity of the arcs. A more sophisticated strategy is currently under investigation.

The trajectories generated by each strategy offer different strengths and weaknesses as well. Resonant orbits supply simple, planar motion that encounters the saddle point at regular intervals. Unfortunately, constructing the ideal geometry in the BC4BP requires prohibitively large maneuvers. The two-pass transfers are simpler to correct and successfully encounter the saddle point multiple times, albeit while requiring larger maneuvers than are allowed at this time. The final transfer geometry does possess the potential to satisfy all mission requirements - the lunar flybys may be targeted to produce desirable geometries for a minimal maneuver cost - but the sensitive dynamics demand further development of the techniques. Additionally, although the slow precession of the captured arcs guarantees several saddle point encounters, the time-of-flight to reach the saddle point can exceed the desired 1 - 2 year extended mission duration.

Although no transfers have been identified that meet all mission constraints, numerous options remain that may uncover trajectories capable of satisfying the requirements. An extended stay in the Sun-Earth L_1 Lissajous orbit may allow the LPF spacecraft to approach the Earth-Moon vicinity on a variety of different manifold arcs and at a different epochs and, thus, reduce the maneuver costs required to pass within 100 km of the saddle point. A more sophisticated corrections scheme may also leverage lunar flybys to adjust capture arc geometries and reach the saddle point with a minimal propellant cost. Finally, the addition of solar radiation pressure to the ephemeris corrections process can supply additional forces to minimize the total ΔV during the extended mission.

SUMMARY

Three separate design strategies and geometries are considered to investigate the feasibility of an extended mission concept for LISA Pathfinder (LPF). This extended mission aims to redirect the LPF spacecraft from its primary mission orbit around the Sun-Earth L_1 point to a local saddle point in the gravitational potential field. The location of the saddle point is determined by locating the roots of the acceleration function due to the gravity of the Sun, Earth, and Moon; Jupiter and Venus provide small perturbing accelerations and are not considered in this investigation. Dynamical systems theory is employed to identify motion in the CR3BP and BC4BP that provides low-cost transfers from the LPF primary mission orbit to the vicinity of the saddle point and facilitates multiple saddle point encounters. Several design options are identified that leverage natural dynamics in the BC4BP to achieve encounters. So far, no designs have been identified that simultaneously meet the mission constraints on ΔV and saddle point pass distance. However, further investigation may yet identify possibilities by incorporating more complex dynamical structures and update methods.

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