EXPERIMENTAL VERIFICATION OF INTERNAL FRICTION AT GHZ FREQUENCIES IN DOPED SINGLE-CRYSTAL SILICON

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ABSTRACT

This paper reports on the experimental verification of the intrinsic loss mechanisms present in degenerately doped single-crystal silicon. Previous work reported at the Hilton Head Workshop 2010 [1] experimentally showed the dominant acoustic loss mechanism in a 3.72-GHz silicon resonator (Fig. 1) to be Landau-Rumer phonon-phonon dissipation as seen from the $1/T^4$ temperature dependence for temperatures as low as 77K. This work extends these measurements to lower temperature (1.5K) and finds that free electron scattering due to degenerate doping affects the *Q* of silicon micromechanical resonators at low temperatures and in some cases, such as in the current resonator design, becomes comparable to anchor loss even at room temperature.

INTRODUCTION

Steady advances in the field of RF MEMS resonators have resulted in devices operating at gigahertz frequencies with quality factors (*Q*) exceeding 10,000 [2-4]. As continued research pushes both frequency and *Q* of such resonators to ever higher values, it is becoming more and more important as designers to have a good understanding of what factors limit their performance. While there is in theory no limit to the resonant frequency of MEMS resonators, practical implementations are limited by the manufacturing tolerances of a given technology. The quality factor, a measure of the energy loss in a resonator, also has similar limits that arise from practical implementation, including but not limited to anchor loss, viscous damping, etc. While it may be argued that these losses can be alleviated by clever design, in addition to these mechanisms, however, there exist internal loss mechanisms which are material dependent and unavoidable. Thus, efforts to increase the quality factor must take these internal losses into account.

Recent work in MEMS resonators has started to utilize some of the electrical properties of the resonator material by degenerate doping. Reasons for this include the fact that motional impedances are decreasing to the point where interconnect and tether resistance are starting to appear as significant factors [5]. Other research groups have used degenerate doping as a means of reducing the temperature coefficient [6]. In light of these advances, the affects of degenerate doping on the mechanical properties of the material must be investigated, in particular, the effect of electron-phonon interactions and their effects on quality factor.

This work makes use of previously developed high-*Q*, gigahertz frequency resonators as a vehicle for experimentally investigating these phenomena. In particular, we add to our knowledge of acoustic loss mechanisms in the aforementioned resonators by presenting experimental evidence of electron-phonon dissipation. We will first discuss the theory developed regarding acoustic damping in doped semiconductors [7-12] and then compare experimental data with this theory to confirm its validity and investigate the regimes in which electron-phonon dissipation becomes significant.

MECHANISMS OF INTERNAL FRICTION

An acoustic wave in a resonator can be thought of as a highly excited acoustic phonon mode containing a number of phonons much larger than the thermal equilibrium value, given by the Bose-Einstein distribution. Due to various scattering processes, phonons in this highly excited mode will decay into other modes, resulting in attenuation of the acoustic wave. The various theories surrounding this attenuation were extensively developed by work starting in the 1950s [7-12]. In crystalline solids where the volume to surface-area ratio is large (i.e., surface scattering is negligible), the different mechanisms can be categorized based on the scattering source of the acoustic wave. In the context of degenerately doped single-crystal silicon, the sources of scattering include thermal phonons, charge carriers, and point defects in the crystal lattice. This section will explain each of these sources of acoustic damping and relevant signatures that would denote their presence in degenerately doped silicon micromechanical resonators.

Point Defect Scattering

Point defect scattering can be due to isotopic mass variations in the silicon itself or variations in both the mass and interatomic potential resulting from the presence of dopant ions in substitutional lattice sites [9]. Due to the low occurrence rate (4.67%) of ^{29}Si – the second most abundant isotope of silicon – and its small mass difference to ^{28}Si (3.57%), we will neglect isotope mass variations as a significant source of point defect scattering.

We are then left with dopant ions as the main source of phonon scattering due to point defects. Intuitively, one may suspect that this source of acoustic dissipation may be significant due to the large number of dopant ions present in degenerately doped silicon. The predicted *Q* due to this type of phonon scattering is given by Eq. (1). Impurity scattering also exhibits a $\sim 1/\omega^2$ dependence and a linear temperature dependence at gigahertz frequencies which may be used as indicators when interpreting experimental results.

Table 1: Analytical expressions for predicted quality factor of relevant phonon scattering mechanisms

Impurity Scattering [9]	
	$Q_{imp} = \frac{\pi n^2 c_D{}^3}{3N_D}\left[\left(\frac{M}{\Delta M}\right)^2 + 2\left(\frac{G}{\Delta G}\right)^2\right]^{-1} \frac{1-e^{-\hbar\omega/k_B T}}{\omega^3}$ (1)
Thermal Phonon Scattering [11-12]	
Akhiezer Effect	$Q_{AKE} = \frac{\rho c_D{}^2}{c_v \gamma^2}\frac{1+(\omega\tau_{th})^2}{\tau_{th}}\frac{1}{\omega T}\ ,\ \tau_{th} = \frac{3\kappa}{c_v c_D{}^2}$ (2)
Landau-Rumer Effect	$Q_{L-R} = \frac{15\rho c_l{}^5 \hbar^3}{\pi^5 \gamma^2 k_B^4}\frac{1}{T^4}$ (3)
Free Electron-Phonon Scattering [7-8]	
$q < 2K_F$	$Q_{ep,q<2K_F} = \frac{2\pi\rho c_l{}^2 \hbar^4 \omega}{m_e^{*2} E_D^2 k_B T}\left(\ln\left(\frac{1+e^{\eta^* - N/T - PTx^2 + x/2}}{1+e^{\eta^* - N/T - PTx^2 - x/2}}\right)\right)^{-1}$ $N = \frac{m_e^* c_D^2}{k_B}\ ,\ P = \frac{k_B}{8 m_e^* c_D^2}\ ,\ \eta^* = \frac{\pi^2\hbar^2 (3N_m/\pi)^{2/3}}{2 m_e^* k_B T}$ (4)
$q > 2K_F$	$Q_{ep,q>2K_F} = \frac{\hbar^3 \rho q^5 a^3}{185 N_m m_e{}^{*2} E_D^2}\omega$ (5)
ρ = density	c_v = volumetric heat capacity
n = atomic density	E_D = dilatational deformation potential
N_D = dopant density	γ = Grüneisen parameter
M = host atomic mass	κ = thermal conductivity
ΔM = mass difference	m_e^* = effective electron mass
G = host crystal interatomic force constant	N_m = metallic electron density $\approx N_D$ (in this work)
ΔG = interatomic force constant difference	q = acoustic wave vector magnitude
c_l = longitudinal velocity	
c_D = Debye velocity	a = effective Bohr radius

Thermal Phonon Scattering

Scattering from thermal phonons – or what is also known in the literature as phonon-phonon scattering – can be manifest in a number of ways depending on the resonant mode shape of the structure, the resonant frequency, and the thermal phonon lifetime of the material (which exhibits a strong temperature dependence). A macroscale thermodynamic analysis leads to what is known as thermoelastic damping. This type of damping is significant in flexural mode resonators operating in the megahertz range where the motion results in heating and cooling of different faces of the resonator [13]. When this occurs, thermal phonons travel from the hotter region to the cooler region in an attempt to restore thermal equilibrium. When the resonant period is long enough such that this transfer of thermal energy can take place, entropy is produced drawing energy from the acoustic wave. For bulk mode resonators with typically higher resonant frequencies, the resonant periods are usually much shorter than this thermal time constant, and thus thermoelastic damping is not a significant source of acoustic loss [13].

At these higher frequencies, a quantum mechanical analysis must be used, which leads to two regimes known as the Akhiezer and Landau-Rumer regimes [11-12]. Akhiezer damping can be thought of as the quantum mechanical analog of thermoelastic damping: the strain wave in the lattice causes a change in the local thermal phonon distribution, which is eventually brought to equilibrium by scattering of phonons from the original acoustic wave to the thermal phonon bath [11]. This restoration to equilibrium only occurs when the thermal phonon bath can redistribute faster than the change in strain field in the lattice. This redistribution takes a time roughly on the order of the thermal phonon lifetime τ_{th}, thus indicating that the Akhiezer effect describes phonon-phonon scattering only when $\omega\tau_{th} \ll 1$. For higher frequencies where this relationship does not hold, the system is said to be in the Landau-Rumer regime, where the attenuation can be described as the scattering of phonons from the acoustic wave to individual phonon modes [12]. The expressions for Q in these two regimes are shown in Eq. (2) and (3). Note especially the independence of Q with frequency in the Landau-Rumer regime. This characteristic indicates the potential for silicon micromechanical resonators capable of high Q at mm-wave frequencies. Also note the strong temperature dependence in this regime ($1/T^4$), which can be used to identify this type of phonon dissipation in experimental data.

Electron-Phonon Scattering

Phonons can also be scattered by charge carriers present in the crystal lattice. This may be true particularly in the case of degenerately doped silicon due to the large number of carriers present.

Before further discussion of electron-phonon scattering, the temperature dependence of carrier concentration must be clarified. In the case of non-degenerately doped silicon, the donor/acceptor states can effectively be represented as single states close to the conduction/valence bands. The carrier concentration can then be represented by the Boltzmann approximation to the Fermi-Dirac distribution. In degenerately doped silicon, however, there is a high enough concentration of donor/acceptor ions that they come close enough in the lattice to interact, forming "bands" around the original single donor/acceptor states. For high enough doping levels, this "band" of donor/acceptor states merges with the conduction/valence band, resulting in semi-metallic behavior and a non-zero carrier concentration even at zero Kelvin [14-15]. For the devices used in this work, the doping concentration is roughly 1×10^{19} cm^{-3}, which is above the experimentally verified Mott critical concentration of 3×10^{18} cm^{-3} [15], signifying semi-metallic behavior. Therefore, this work assumes a free electron density roughly equal to the dopant density even at low temperatures, which has also been used by others in the study of low temperature thermal conductivity, a phenomenon also intimately related to electron-phonon scattering [16-18].

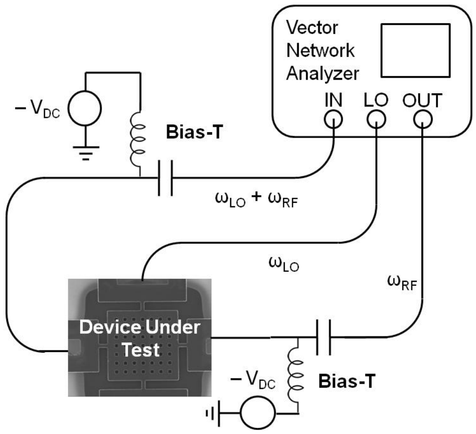

Figure 1: Mixing measurement setup for accurate extraction of mechanical Q. Operation of DUT is described in [2].

This clarification is important because electron-phonon scattering depends largely on the state of the electrons in the system, i.e., whether they are free or bound to their dopant atoms [7-8]. For this work, we assume that all dopants are ionized even at low temperatures, therefore allowing us to consider only free electron scattering. This type of scattering can be modeled in one of two regimes based on the relationship between the acoustic wave vector magnitude, q, and the Fermi wave vector magnitude of the electron distribution, K_F. If $q < 2K_F$, then Eq. (4) describes the Q of an acoustic wave due to scattering with the free electron gas. For $q > 2K_F$, the Q is predicted by Eq. (5). For degenerately doped silicon, the Fermi wave vector is typically very large (i.e., the Fermi level is close to or even inside the conduction/valence band edge), so it is most likely the case that $q < 2K_F$. At a doping level of 10^{19} cm^{-3}, $K_F = 6.67\times10^8$ m^{-1}, which corresponds to a longitudinal acoustic wave traveling in the <100> plane of approximately 90 GHz. Therefore, we find that only free electron scattering for the case of $q < 2K_F$ is relevant for this work.

EXPERIMENTAL DATA AND DISCUSSION

In this work, we utilize a mixing measurement setup in order to accurately measure the mechanical Q of the micromechanical resonator. The experimental setup is shown in Fig. 1. This technique uses the inherent nonlinearity in capacitive actuation to apply the input stimulus at a frequency greater than that of the resonant frequency and obtain the output at the resonant frequency by using an external carrier. Such methods enable the application of a purely mechanical force without the effects of capacitive feedthrough and Q loading due to electrical resistance. This allows for accurate measurement of mechanical Q even in cases where $k_t^2 \cdot Q < 1$ [19].

The data obtained from this setup is shown in Fig. 2. We see that at high temperatures (T ~ 300K), the data follows a 1/T^4 trend, which is expected from devices that are limited by Landau-Rumer thermal phonon scattering. At lower temperatures, the Q seems to plateau to a value of

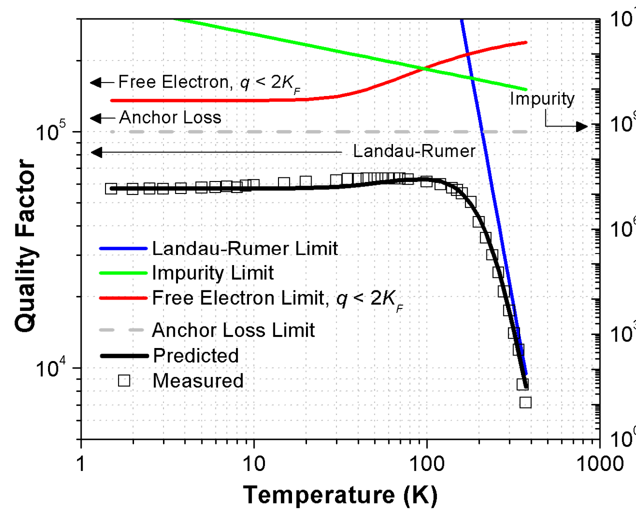

Figure 2: Temperature vs. Q predicted by Eq. (1), (3), and (4) and the total Q predicted by the combination of said factors. The Q limited by impurity scattering follows the y-axis on the right side of the figure. Measured data is included and shows good fit with predicted Q.

roughly 63,000 at T < 50K, below which point the Q starts to gradually drop to around 57,000 at 1.5K. In previous work which verified operation of these resonators in the Landau-Rumer regime [1], we initially thought that this peak in the measured Q was simply due to the dominance of temperature independent anchor losses. However, with data now available at lower temperatures, we are now able to verify that electron-phonon scattering will also be significant.

To better illustrate the impact of all the discussed scattering mechanisms, we also plot in Fig. 2 results obtained from numerical integration for n$^+$-Si and <100> longitudinal acoustic resonance at the fundamental frequency of 3.72 GHz. Also included is an estimate of the anchor loss and the overall predicted Q. From this plot, we can verify that only free electron-phonon scattering for $q < 2K_F$ can adequately describe the reduction in Q at T < 50K. This is consistent with other experimental work which concludes that electron-phonon scattering is the limiting mechanism for low temperature thermal conductivity [16-18].

One note must be made here about the exact magnitude of electron-phonon and phonon-phonon limited Q. Here, we use the dilatational deformation potential and the Grüneisen parameter as tunable parameters to fit the predicted Q to the experimental data. We have chosen these parameters largely because of a lack of consistent experimentally extracted values in the literature. To be fair, extraction of such parameters is not simple and is highly sensitive to measurement uncertainties. Using values of $E_D = 0.15$ eV and $\gamma^2 = 0.2116$, we extract the anchor loss limited Q to be approximately 100,000 and achieve reasonable quantitative agreement with the data throughout the temperature range of interest. The deformation constant used may seem smaller

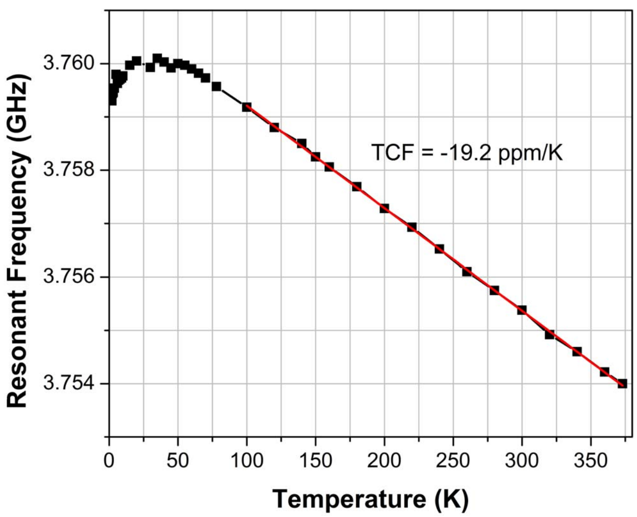

Figure 3: Resonant frequency as a function of temperature. The anomalous behavior at T<50K is consistent with that observed in p-type Si [10].

than what is commonly observed for Si, but it is still within the same order of magnitude as those reported in the literature for low temperature thermal conductivity measurements of doped semiconductors [16-17]. The Grüneisen parameter is also similar to values reported in the literature [20].

Fig. 2 also shows that the free electron limited Q is ~136,000 at 1.5K and rises up to ~240,000 at high temperature, which is comparable to the extracted anchor loss and indicates that electron-phonon scattering may not be negligible as was previously understood. This, of course, is highly dependent on resonator and anchor design and becomes important only in cases of low loss anchors.

The temperature dependent frequency shift and its relationship with doping is also phenomenologically related to electron-phonon scattering, as both are a consequence of the change in the electronic band structure as a function of strain. This relationship has been recently studied in detail to obtain an extremely small temperature coefficient of frequency (TCF) [6]. In Fig. 3, we plot the resonant frequency as a function of temperature. For T > 100K, we see a roughly linear dependence with a TCF of -19.2 ppm/K. At T ~ 50K, the TCF approaches zero and for lower temperatures it exhibits a positive value. This is consistent with measurements of highly doped p-type Si performed by Mason and Bateman [10] and confirms our conclusion that electron-phonon interactions are significant for acoustic phonons at low temperatures (T < 50K).

CONCLUSION

This work presents temperature dependent measurement results of resonant quality factor for a single crystal silicon micromechanical resonator using pn-diode electrostatic transduction. We add to previous experimental work which verified Landau-Rumer acoustic damping in these resonators and observe that the degenerate doping leads to significant electron-phonon scattering that significantly influences the mechanical properties for T < 50K and even appears at room temperature. This is exhibited both in the decrease in quality factor and resonant frequency at T < 50K. These findings indicate that degenerate doping plays a relatively larger role than previously expected – comparable to anchor loss for well-designed anchors. The understanding of the behavior and limitations of micromechanical resonators at low temperatures presented here will be crucial for applications such as quantum-limited detection and ground state preparation.

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