

System Design of a Cold Atom Gyroscope based on Interfering Matter-wave Solitons

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Abstract—We propose a novel implementation of a trapped-atom Sagnac gyroscope based on the interference between matter-wave solitons confined around an optical microring resonator. Our integrated nanophotonic approach to trapped atom interferometry combines the long-term stability and quantum-limited sensitivity of ultracold matter-wave interferometers with the robustness, scalability and low power operation of nanophotonic architectures. The use of optical microresonators for atomic confinement ensures disorder-free symmetric waveguides for the confined atoms, a high degree of vibration insensitivity owing to the reciprocal structure of the waveguide, and enhanced bias and scale-factor stability via concurrent feedback stabilization of the microresonator. We have performed detailed quantum simulations based on demonstrated experimental parameters to confirm stable dispersion-free propagation of matter-wave solitons around the microresonator and the appearance of high contrast interference fringes due to the accrued Sagnac phase shift. We estimate the shot-noise limited rotation sensitivity of this gyroscope to be $0.8 \mu\text{rad/s/Hz}^{1/2}$ for single-loop propagation of the solitons around a microring of radius 1 mm, with the possibility of substantial improvements via multiloop propagation of the solitons, fabrication of microring resonators of larger diameter, and the use of quantum-correlated states such as spin-squeezed quantum states. The proposed device illustrates the benefits of harnessing quantum many-body states such as matter-wave solitons for quantum-enhanced inertial sensing applications.

Keywords—Sagnac Gyroscope, Inertial sensing, Matter-wave solitons, Atom Interferometry, Nanophotonic microring resonators

I. INTRODUCTION

Sagnac gyroscopes detect rotation via an interferometric phase shift accrued by counter-propagating wavepackets around an enclosed area (see [1] for a review). For a closed-path Sagnac interferometer with atoms of mass m , this phase shift takes the form

$$\phi = \frac{4m}{\hbar} \mathbf{A} \cdot \boldsymbol{\Omega} \quad (1)$$

where \mathbf{A} is the area enclosed by the interferometer and $\boldsymbol{\Omega}$ is the rotation rate. To date, atom interferometers based on thermal atomic beams have demonstrated the highest rotation sensitivities of $6 \times 10^{-10} \text{ rad/s/Hz}^{1/2}$, corresponding to $8 \times 10^{-6} \Omega_E / \text{Hz}^{1/2}$, where $\Omega_E = 73 \mu\text{rad/s}$ is the Earth rotation rate [2]. In comparison to thermal atomic beams, Sagnac gyroscopes using dilute and unconfined laser-cooled atoms benefit from marginally increased interaction time and higher intrinsic stability, albeit at the expense of a significantly

lower atomic flux. Such cold atom gyroscopes have made tremendous progress in recent years with projection noise-limited sensitivities around $0.2 \mu\text{rad/s/Hz}^{1/2}$ [3], [4].

In comparison to free-space Sagnac atom interferometers, the realization of compact and integrated trapped-atom gyroscopes offers numerous benefits. In the former case, environmental perturbations and drifts cause fluctuations in the ballistic trajectories of the unconfined atoms and can degrade the long-term stability of the gyroscope. In contrast, confinement of the atoms within well defined, symmetric and reciprocal waveguides ensures stable deterministic paths and close overlap between the interfering wavepackets. Further, in such a reciprocal waveguide, the interfering wavepackets traverse identical paths and thus endow the interferometer with a high degree of common mode rejection to extraneous interactions such as the confining potential, uncompensated electromagnetic fields or vibrations. In addition, modern developments in quantum state preparation and control of ultracold gases have reached a level of sophistication wherein the quantum coherence of the confined gases can be preserved up to several tens of seconds. This degree of control implies that the confined wavepackets can complete several traversals around the interferometer, thereby enhancing the effective enclosed area and sensitivity in a manner analogous to fiber-optic gyroscopes. Lastly, such integrated trapped-atom interferometers also enable the implementation of compact and portable inertial navigation systems for several technological and fundamental applications.

Despite these compelling motivations, the development of trapped-atom Sagnac interferometers has been frustrated by both fundamental and technical constraints. The former factors include the deleterious effects of atomic interactions such as phase diffusion [5] and matter-wave dispersion. On the latter front, heating and loss of atoms due to inhomogeneities and fluctuations in the confining potential, the influence of stray fields and other decohering mechanisms can each lead to a rapid erosion of interference fringe visibility. As detailed in the rest of our paper, our proposal significantly alleviates these limitations by harnessing a combination of dispersion-free quantum many-body states and the use of planar nanophotonic architectures to realize stable, disorder-free and symmetric optical confinement.

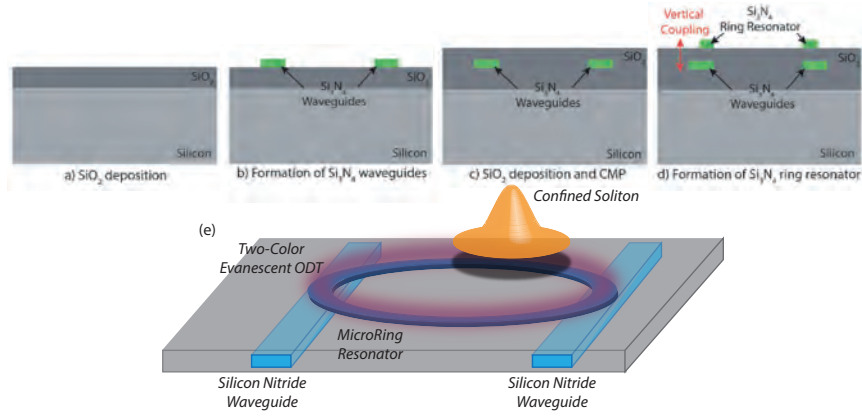


Fig. 1. Top (a,b,c,d). Process flow for the fabrication of multilayer microring resonator for optical confinement of ultracold solitons. This multilayer structure allows the two-color optical dipole trap (ODT), Bragg pulses and other internal state manipulation pulses to be introduced into the microring resonator through embedded waveguides. (e) Schematic of the integrated structure showing a soliton confined in the two-color ODT above the microring.

II. SOLITON MATTER-WAVE INTERFEROMETRY

In order to overcome the decohering effects of atomic interactions, we propose to use harness matter-wave solitons for Sagnac interferometry. Our proposal is based on the insight that quantum degenerate atomic gases with weak attractive interactions (such as Lithium-7 and Rubidium-85) can self-assemble into bright matter-wave solitons wherein the attractive interparticle interactions exactly balance, and annul the quantum kinetic pressure [6], [7]. As such, bright soliton states can be regarded as heavy mass, dispersion-free matter waves that can propagate over macroscopic distances without altering their wavefunction. For similar reasons, unlike Bose condensates confined within a waveguide, bright solitons are also immune to cross-coupling between the radial and longitudinal motion that can arise due to vibrations or trap imperfections. Lastly, the stability window of bright solitons ensures that a coherent splitting of such solitons naturally accomplishes a higher degree of squeezing (and reduced phase diffusion) than their coherent state counterparts.

Recent studies (see, for example [8], [9], [10], [11]) have shown that bright soliton states can be split, recombined and manipulated in a phase coherent manner using optical Bragg pulses and other techniques previously demonstrated in laser-cooled atomic vapors and Bose condensates. These various features suggest that such bright soliton states are a promising candidate for robust, trapped atom Sagnac interferometers - one in which the interparticle interactions are an asset rather than a detriment to the maintenance of a coherent, dispersion-free quantum state.

III. MICRORESONATOR-BASED OPTICAL CONFINEMENT

An equally important ingredient of our proposed gyroscope is the use of an optical microresonator to provide a stable, reciprocal closed-path waveguide for the solitons. Optical microresonators exhibit evanescent whispering-gallery modes (WGMs) at frequencies coincident with its resonant optical frequencies [12]. Optical confinement of solitons around such microresonators is achieved by simultaneously coupling two

optical frequencies on either side of the atomic transition into the ring resonator. This creates a two-color optical dipole trap (ODT) wherein the red-detuned circulating cavity mode creates an attractive potential drawing the solitons towards the surface, and the blue-detuned cavity mode repels the atoms away from the surface. Under the combined influence of both circulating cavity modes, the solitons experience a circularly symmetric, closed-path trap that strongly confines the atoms close to the microring (Fig. 1). Based on the large demonstrated optical quality factors of planar silicon nitride microring resonators [13], [14], we estimate trap depths of 30-50 μK even for input optical powers of a few milliwatts, more than sufficient to enable strong one-dimensional confinement of the soliton states which are nominally at temperatures around 50 nK.

In standard photonics integrated circuits, light is critically coupled into the ring resonator via a coupling waveguide located within the same planar structure. However, in such a configuration, the coupling waveguide will interact with the solitons and distort the symmetry of the trap, leading to heating and particle loss. Here, we propose to use a buried waveguide [15] wherein the coupling waveguide and the planar microring resonator are separated by an oxide spacer gap (Fig. 1).

In addition to providing a stable and symmetric trap for the solitons, the use of high quality optical microring resonators also presents numerous technical benefits in the context of trapped atom interferometry. First, integrated magnetic micro-traps for atoms have been known to exhibit severe trap distortions due to surface roughness and fabrication imperfections [16], leading to rapid heating or loss of atoms. However, high quality-factor microring resonators have measured surface roughness of less than 5 nm. Based on such measurements, we estimate the trap inhomogeneities at the location of the atoms to be less than 1 nK. Second, recent studies of atoms trapped in the evanescent optical fields of nanofibers have pointed to the detrimental role of polarization and intensity fluctuations in the evanescent field [17]. Such fluctuations can lead to heating and loss of atomic coherence. However, the large optical

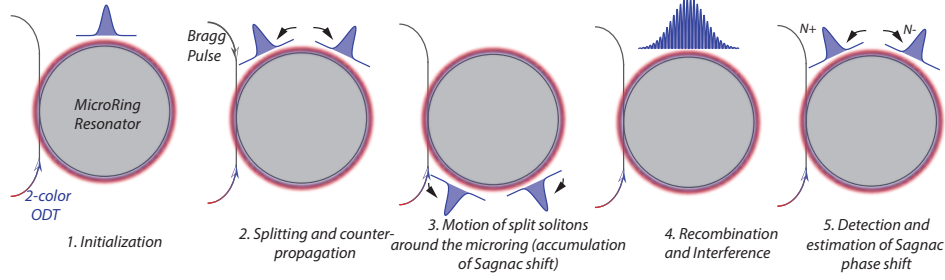


Fig. 2. Schematic description of the Sagnac interferometer showing 1. Initialization of the soliton state in a two-color ODT. 2. Splitting of the soliton using an intra-waveguide Bragg pulse. 3. Dispersion-free propagation of the split wavepackets around the microring resonator. 4. Recombination of the wavepackets using a second Bragg pulse, and 5. Detection of the accrued Sagnac phase shift through imaging and particle counting of the outcoupled channels.

quality factors ($> 50,000$) and the intrinsic birefringence of silicon nitride ensures a large frequency splitting between the optical resonance frequencies corresponding to different polarizations of the WGMs. As such, polarization fluctuations due to coupling between nominally orthogonal polarization states are energetically suppressed within the microring, leading to a corresponding suppression of heating and unwanted atomic transitions in the trapped solitons. Third, in comparison to optical traps created by free-space propagating beams, the microresonator-based optical trap does not suffer from beam-pointing jitter, wavefront distortions or other effects of vibrations and temperature fluctuations, further enhancing the long-term stability of our proposed device. Fourth, the requisite optical pulses for splitting and recombination of the solitons can be introduced into the microring via the coupling waveguide in a manner similar to the two-color ODT (Fig. 2).

Lastly, and perhaps most significantly, the two-color ODT is designed to be in resonance with the microring resonator. As such, continuous monitoring of the transmission channel of the coupling waveguide allows independent and active stabilization of the microring resonance via, for instance, thermal actuation of the substrate. By thus locking the optical resonances of the microring to an independent frequency standard [18], the geometry of the microring (and hence, the trajectories and effective path lengths of the interfering solitons) can be stabilized against temperature fluctuations, strain or other uncontrolled drifts of the microring resonator. This dramatically improves the long-term bias stability and scale factor stability of the proposed device.

IV. SIMULATIONS AND RESULTS

Based on demonstrated experimental and fabrication parameters, we have performed detailed quantum simulations of each aspect of the proposed device. These simulations are based on the truncated Wigner approximation (TWA) and take into account both thermal and quantum fluctuations within the ultracold solitons. For our proposed microring geometry and trap parameters, our simulations confirm stable, dispersion-free propagation of the soliton within the two-color ODT for more than 400 round trips around a 1 mm radius microring.

To further quantify the operation of the soliton-based Sagnac interferometer, we simulated the proposed experimental se-

quence (Fig. 2, 3) to compare the performance of solitons and Bose-condensates within the microring. As seen in Fig. 4, the Bose-condensates exhibit strong dispersion and phase diffusion due to the repulsive interparticle interactions, leading to rapid erosion of the fringe contrast. In contrast, the solitons remain dispersion-free with negligible reduction in the interference fringe contrast even after more than 20 revolutions around the microring.

Next, we simulated the proposed device in the presence of rotation to estimate the linearity of the Sagnac shift and the accuracy of the gyroscope. These results are shown in Fig. 5, further confirming that the strong, stable confinement provided by the microring resonator, and the dispersion-free property of the solitons leads to accurate estimation of the Sagnac phase shifts. Based on these simulations, we estimate the shot-noise limited rotation sensitivity of the proposed device to be $0.8 \mu\text{rad/s/Hz}^{1/2}$ for single-loop propagation around a 1 mm radius microring. As seen from Figs. 4,5, this conservative estimate can be significantly improved by allowing the solitons to propagate multiple revolutions around the ring, thereby increasing the effective enclosed area. Similarly, the fabrication of larger microring resonators will also result in similar performance enhancements. Lastly, the use of collective quantum states such as solitons also enables future enhancements to rotation sensitivity by leveraging quantum correlated states, spin-squeezed states and other quantum-enhanced sensor strategies.

V. CONCLUSION

In conclusion, we describe an integrated Sagnac gyroscope based on the interference of ultracold matter-wave solitons around an optical microresonator. We have performed detailed simulations based on demonstrated experimental parameters to demonstrate stable, dispersion-free propagation of the solitons around the microring and high-contrast interference fringes due to the accrued Sagnac phase. The use of optical microresonators to provide atomic confinement presents numerous advantages including a symmetric and reciprocal trap, insensitivity to various sources of technical noise such as vibrations and pointing fluctuations, and an enhanced scale factor stability via simultaneous stabilization of the microresonator. Estimates of the shot-noise limited sensitivity of this device

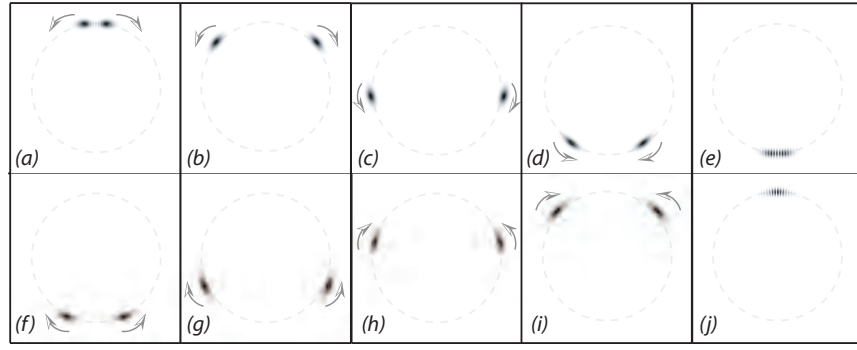


Fig. 3. Simulations of soliton propagation and the Sagnac interferometric sequence showing (a) Splitting of the soliton state using an intra-waveguide Bragg pulse, (b-i) dispersion-free propagation of the solitons around the microring, and (j) interference of the counter-propagating soliton states.

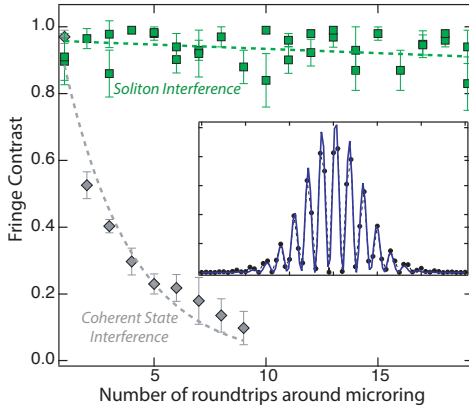


Fig. 4. Simulated Sagnac interference fringe contrast following multiple revolutions around the microring ODT. The solitons (Green data) remain dispersion-free and exhibit negligible reduction of interference contrast. In comparison, Bose-condensates (Gray data) suffer rapid reduction of fringe contrast due to interparticle interactions. Inset: Representative interference fringe of solitons after 10 revolutions around the microring.

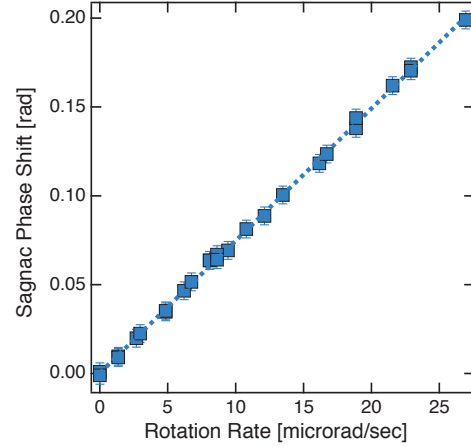


Fig. 5. Simulation results of the soliton gyroscope showing the accrued Sagnac phase vs. rotation rate.

are already on par with state-of-the-art atomic gyroscopes with further substantial improvements possible by straightforward modifications to the proposed design. This proposal highlights the potential benefits of harnessing quantum many-body states for inertial sensing applications.

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