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Theory and Experimental Verifications of the Resonator Q and Equivalent Electrical Parameters due to Viscoelastic, Conductivity and Mounting Supports Losses

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Abstract—**A novel analytical/numerical method for calculating the resonator Q, and its equivalent electrical parameters due to viscoelastic, conductivity and mounting supports losses was presented. The method presented will be quite useful for designing new resonators, and reducing their time and costs of prototyping. There was also a necessity for better and more realistic modeling of the resonators due to miniaturizations, and the rapid advances in the frequency ranges of telecommunication. We present new three-dimensional finite elements models of quartz resonators with viscoelasticity, conductivity, and mounting support losses. For quartz the materials losses attributed to electrical conductivity and acoustic viscosity were obtained from Lee, Liu and Ballato[1], and Lamb and Richter[2], respectively. The losses at the mounting supports were modeled by perfectly matched layers (PML's). The theory for dissipative anisotropic piezoelectric solids given by Lee, Liu and Ballato [1] was formulated in a weak form for finite element applications. PML's were placed at the base of the mounting supports to simulate the energy losses to a semi-infinite base substrate. FE simulations were carried out for free vibrations and forced vibrations of quartz tuning fork and AT-cut resonators. Results for quartz tuning fork and thickness shear AT-cut resonators were presented and compared with experimental data. Results for the resonator Q and the equivalent electrical parameters were compared with their measured values. Good comparisons were found. Results for both low and high Q AT-cut quartz resonators compared well with their experimental values. A method for estimating the Q directly from the frequency spectrum obtained for free vibrations was also presented.**

An important determinant of the quality factor Q of a quartz resonator is the loss of energy from the electrode area to the base via the mountings. The acoustical characteristics of the plate resonator are changed when the plate is mounted onto a base substrate. The base affects the frequency spectra of the plate resonator. A resonator with a high Q may not have a similarly high Q when mounted on a base. Hence, the base is an energy sink and the Q will be affected by the shape and size of this base. A lower bound Q will be obtained if the base is a semi-infinite base since it will absorb all acoustical energies radiated from the resonator.

***Index Terms*— Piezoelectric resonators, tuning forks, quartz AT-cut, *Q* factor, equivalent electrical parameters, energy** highsink **method.**

I. INTRODUCTION

QUARTZ, due to its properties of zero temperature cuts, and very low intrinsic acoustic loss, is the one of the most preferred material in the manufacture of frequency control devices, sensors, filters and many other applications. One of the important parameter in the design of the quartz resonators is the quality factor Q. This is especially true for miniaturized and MEMS devices where the problem is usually that of low Q.

The intrinsic Q of quartz is very high; however, the same may not be true of the resonator made of quartz. A high Q resonator has to be well designed so that there is a minimum loss of energy. Some of the energy losses are unavoidable such as those associated with the material damping and conductivity. There are some losses that could be avoided namely those associated with the leakage of energy to, for example, the mounting supports, base substrates, and electrode leads. Yet the resonator is oftentimes designed as a standalone structure minus its mounting supports and base.

There is as yet no formal analytical method for calculating the resonator Q without resorting to prior assumptions of various damping factors in the resonator. Often the resonator Q is assumed and the I-V (current (I) versus voltage (V)) curves are then calculated. There is thus no theoretical basis for comparing the quality or suitability of various frequency device designs before the devices were made. It is difficult to determine which resonator dimensions can lead to a high Q value when mounted onto a base.

In this paper, we propose an "energy sink method"[5] for estimating the Q of a resonator when mounted onto a base. The energy sink method will provide some estimation of the Q for a particular resonator design and mounting configurations.

This will allow the weeding out of poor resonator and mounting designs before the actual prototyping. It will also allow the numerical testing of novel resonator designs for the obtainable Q and equivalent electrical parameters.

In order to incorporate the material losses in a piezoelectric materials, Lee, Liu and Ballato [1] presented the 3-D equations of linear piezoelectricity with quasi-electrostatic approximation which include losses attributed to mechanical damping in solid and resistance in current conduction. The theory took into consideration the viscosity losses which were first measured for quartz by Lamb and Richter [2], by assuming that the dissipative stresses in an anisotropic solid were due to the linear viscosity tensor η_{ijkl} . Although conductivity losses in quartz were negligible compared to its viscosity losses, it was useful to include conductivity losses for piezoceramics such as barium titanate, or materials with strong electromechanical coupling such as lithium niobate. The values of the conductivity tensor for the AT-cut quartz and barium titanate were estimated by Lee, Liu and Ballato [1].

In this paper, we present three-dimensional finite element analyses for piezoelectric resonators which incorporates the materials losses attributed to the acoustic viscosity η_{ijkl} and electrical conductivity σ_{ik} [4]. The theory for dissipative anisotropic piezoelectric solids given by Lee, Liu and Ballato [1] was formulated in a weak form for finite element applications. The resulting FE model gives the Q value without prior assumptions of various damping factors and impedance. The frequency spectrum obtained from the free vibration analysis using the dissipation theory helped in understanding the interaction of the main mode with the spurious modes. The Q obtained from a free vibration analysis was compared to the Q obtained from a forced vibration analysis, and it was found to be similar. We also present a method for obtaining the equivalent electrical parameters from the finite element admittance curve.

II. Governing Equations for 3-D Piezoelectricity with Dissipation

The 3-D equations of linear piezoelectricity with dissipation [1] are summarized as follow:

Strain-displacement relationship:

$$E_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad \text{Eq(1)}$$

Constitutive stress-strain relationship:

$$T_{ij} = C_{ijkl}\,E_{kl} + e_{pij}\,\phi_{,p} + \eta_{ijkl}\,\dot{E}_{kl} \qquad \text{Eq(2)}$$

Electrostatic constitutive equation:

$$D_i = e_{iq}E_q - \varepsilon_{ik}\phi_{,k} \qquad \text{Eq(3)}$$

$$J_i = -\sigma_{ik}\phi_{,k} \qquad \text{Eq(4)}$$

Stress equation of motion:

$$T_{ij,j} = \rho\ddot{U}_i \qquad \text{Eq(5)}$$

Charge equation of motion:

$$\dot{D}_{i,i} + J_{i,i} = 0 \qquad \text{Eq(6)}$$

In these equations, J_i is the current conduction, η_{ijkl} is the viscosity coefficient and $\dot{E}_{kl}$ is the rate of change of strain. The weak form of the above set of equations was formulated and implemented in the finite element software COMSOL Multiphysics.

One of the most common methods of determining the Q using standard FE software is by performing a frequency response analysis. Currently most of the FE software takes into account the dissipative media by assuming various damping factors such as the Raleigh damping parameters. However, the set of equations from this section takes into account the material dissipation values through the viscosity tensor and the resistance in current conduction, resulting in a more accurate FE model. These equations are used to obtain a frequency response for forced vibrations without resorting to any assumptions of damping.

III. Modal Analysis of Quartz Resonators

It is well known that the addition of a mounting base onto a standalone resonator changes the acoustical characteristics of the standalone resonator. We could demonstrate this by comparing the acoustical characteristics of a standalone resonator with its resonator-base counterpart. For our demonstration purposes we chose a 2-D model of a very high frequency inverted mesa AT-cut quartz resonator vibrating at a fundamental thickness shear frequency of about 970 MHz.

The top figure of Fig. 1 shows the typical mode shape of the fundamental thickness shear of the 2-D model of the inverted mesa AT-cut resonator itself, while bottom figure shows the resonator mounted on a base substrate. The eigenfrequencies calculated as a function of the length to thickness (a/b) ratio is known the modal frequency spectrum or simply frequency spectrum. Fig.2 shows the comparison of the frequency spectrum of the resonator itself (left graph) with the frequency spectrum of the resonator mounted on a base substrate (right graph). The graphs show the frequency spectrum of the mesa plate resonator mounted on a base substrate is much richer the the frequency spectrum of the standalone resonator. Hence well designed frequency devices ought to include the interactions of the resonator with its base mountings and substrate. When a resonator is mounted on a base there will be more interactions with spurious modes that lead to lower *Q* factors.

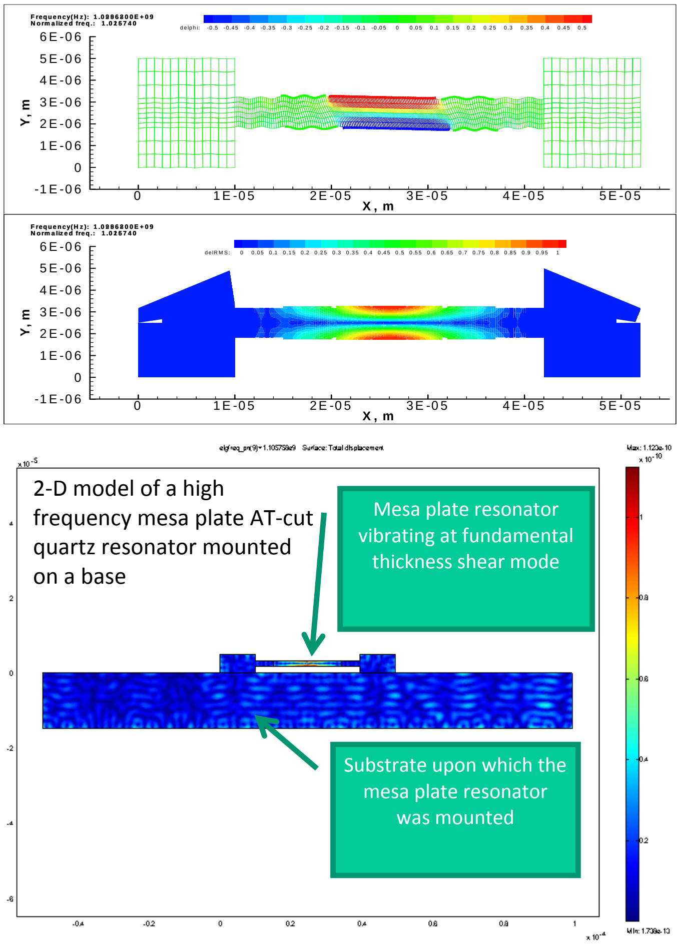

Figure 1. (top) Mode shape of the 2-D inverted mesa AT-cut quartz resonator and (bottom) resonator mounted on its base substrate.

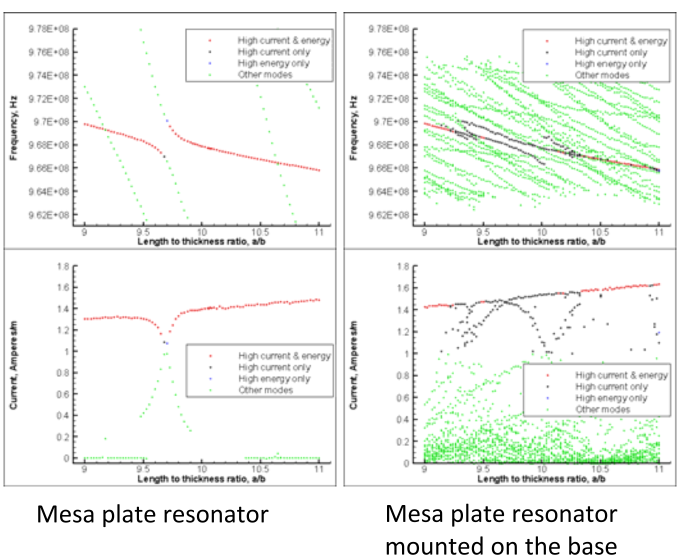

Figure 2. Comparison of the frequency spectra obtained from the resonator itself with that obtained from the resonator mounted on a base.

A. Calculation of resonator Q from modal analysis

While the frequency response analysis is useful for obtaining the Q, and other electrical parameters of the resonator, it can be carried out for only one resonator configuration at a time; hence it is very time consuming for determining an optimal resonator design. It is not convenient for studying the characteristics of effects due to parameters such as resonator dimensions, and electrode placements and dimensions. Free vibration analyses which provide the frequency spectra as a function of these parameters are much more useful for designing resonators. The frequency spectra can be used as resonant mode charts for determining the best resonator designs for high *Q*. The effects of the spurious modes on the main mode could also be studied using the frequency spectra.

The frequency spectrum obtained without taking into consideration the material losses gives a qualitative analysis of the interaction of the main mode with the spurious modes and hence helps in selecting the best resonator configuration. However, this frequency spectrum cannot provide the *Q* of the modes. Hence, we present a technique of determining the *Q* directly from free vibration analysis by taking into consideration the dissipation of energy in a lossy material.

The complex resonant frequencies obtained from a free vibration analysis with dissipation correspond to the resonant frequencies of harmonic waves with real wave numbers and amplitudes decaying exponentially with respect to time.

The eigenvalue analysis with dissipation yields complex eigenvalues.

$$\omega = \omega_R + j\omega_I \qquad \text{Eq(7)}$$

where,

ω_R is the real part which represents the resonant frequency of the lossless, piezoelectrically stiffened free vibration mode.

$$\omega_R = k_m v_m \,;\; v_m = \sqrt{\frac{c_m}{\rho}} \qquad \text{Eq(8)}$$

where,

k_m represents the real wave number,

v_m represents the piezoelectrically stiffened phase velocity,

c_m is the effective elastic constant, and

ω_I is the imaginary part of the resonant frequency for free vibrations with dissipation. The imaginary part is the temporal decay of the free vibration mode. It is a function of the real frequency, effective viscosity, effective conductivity and effective elastic constant of the free vibration mode [1].

$$\omega_I = \omega_R^2 \frac{\eta_m}{2c_m} + \frac{\sigma_m}{2c_m} \qquad \text{Eq(9)}$$

where,

η_m is the effective viscosity coefficient, and

σ_m is the effective conductivity coefficient.

The ratio of the imaginary part of the resonant frequency to its real part represents the damping ratio ζ in the system.

$$\zeta = \frac{\omega_I}{\omega_R} \qquad \text{Eq(10)}$$

The Q is then calculated using the damping ratio as follows,

$$Q = \frac{1}{2\zeta} \qquad \text{Eq(11)}$$

A free vibration analysis with material dissipation can provide directly the *Q* value of resonant modes in the frequency spectrum. Since we could also include dissipations/losses due to the mounting supports and base substrate, the *Q* value from Eq. (11) would represent the total *Q* of the resonator-base structure, Q_{total}.. As an example, we demonstrate the calculation of the *Q* in a modal analysis of a 10 MHz At-cut plate shown in Fig. 3 along with its frequency spectrum and resonator *Q*. The *Q* for the thickness shear mode was calculated to be about 1.36×10^6 which compared reasonably with the value 1.4×10^6 from Warner's[3] equation for AT-cut resonators.

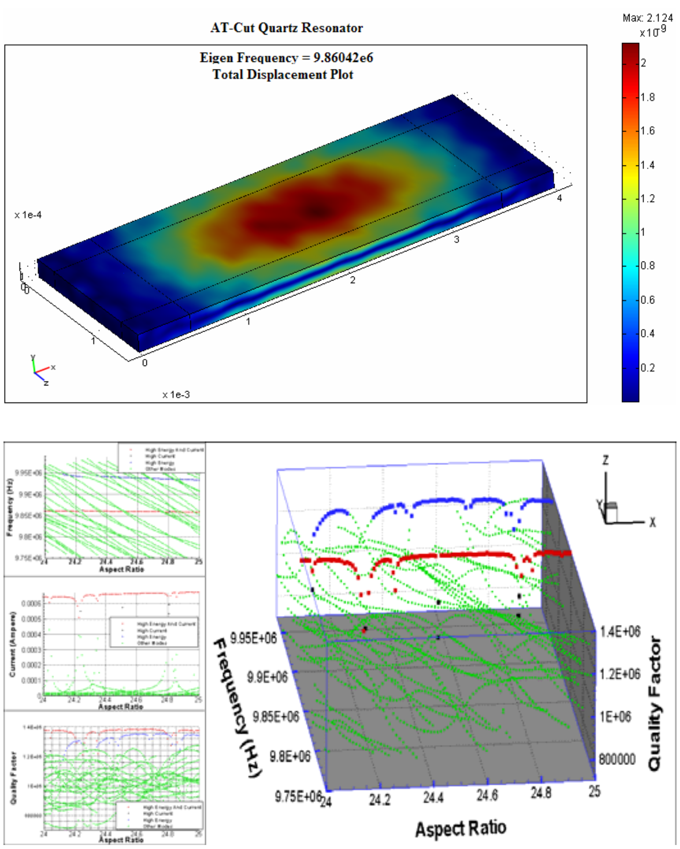

Figure 3. 10 MHz AT-cut quartz plate resonator (top) and its modal frequency spectrum along with the calculated *Q* (bottom).

IV. Frequency Response Analysis of Quartz Resonators

Frequency responses obtained from a forced vibration analysis of a resonating system with no dissipation resulted in an infinite *Q* value. Thus, it was necessary to introduce energy loss in the form of mechanical damping and resistance to obtain a practical *Q* value.

Fig. 4 shows the comparison of the frequency response obtained from forced vibration analyses of a 10 MHz thickness shear AT-cut quartz resonator with material damping and without any damping. It could be observed that without damping the imaginary part of current (top graph) tends to infinity, while the real part of current (bottom graph) is zero. With the introduction of material damping, the imaginary and the real part of current have a value reflecting the magnitude of viscosity and resistance existing in the material. The *Q* obtained from frequency response analysis was 1.45e6 which compares well with the free vibration analysis results of Fig. 3 and the results obtained by Lee, Liu and Ballato [1].

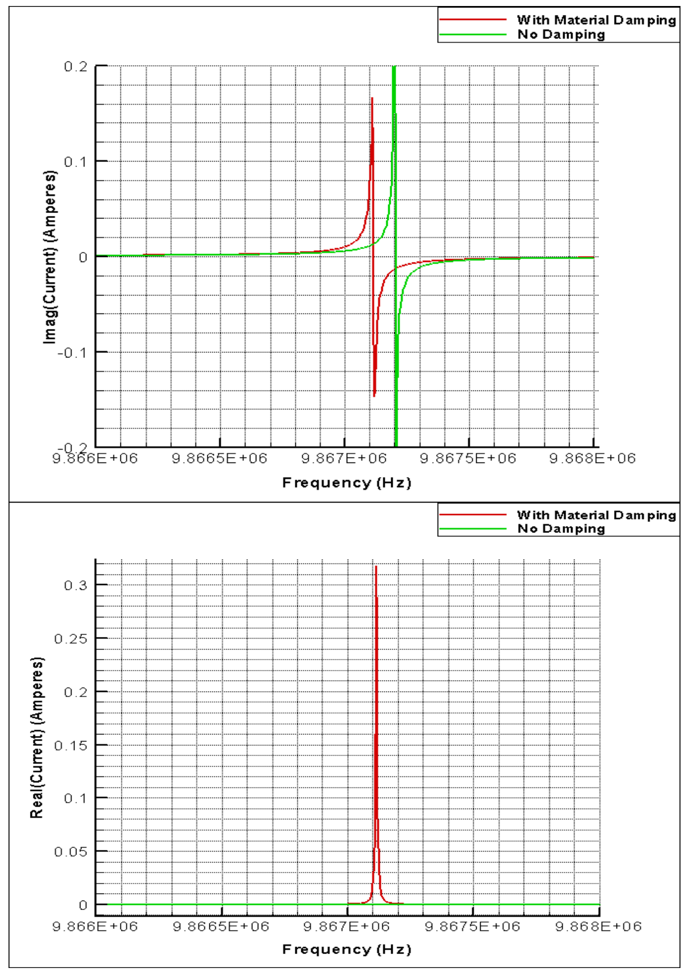

Figure 4. Frequency response of the 10 MHz AT-cut plate of Fig. 3. Bottom electrode was grounded, top electrode was +/- 1V. Plots of the imaginary current (top) and real current (bottom) versus driving frequency .

A. Algorithm for extracting the equivalent electrical parameters from the finite element admittance curve

The finite element frequency response analysis will yield an admittance curve of the resonator. Good quality quartz resonators tend to have very high *Q* and the resulting admittance curve would have a very narrow bandwidth at the resonance frequency. The finite element admittance curve would need small frequency increments of 5 to 10 Hz in order to capture accurately the frequency response in the vicinity of the resonance frequency since a frequency increment greater than the bandwidth may miss the resonance frequency altogether. Hence, for a high Q resonator, it is not only time consuming and computationally intensive to compute its admittance curve, it is also difficult to calculate accurately its Q, and its equivalent electrical parameters. We needed an algorithm for extracting the equivalent electrical parameters that was not sensitive to the frequency increments.

1) Butterworth Van Dyke model for the quartz resonator

The Butterworth Van Dyke model is generally a good electrical equivalent of the piezoelectric resonator. Fig. 5 below shows the model, its resistance and reactances when the circuit is harmonically excited, and its typical admittance curve (bottom)

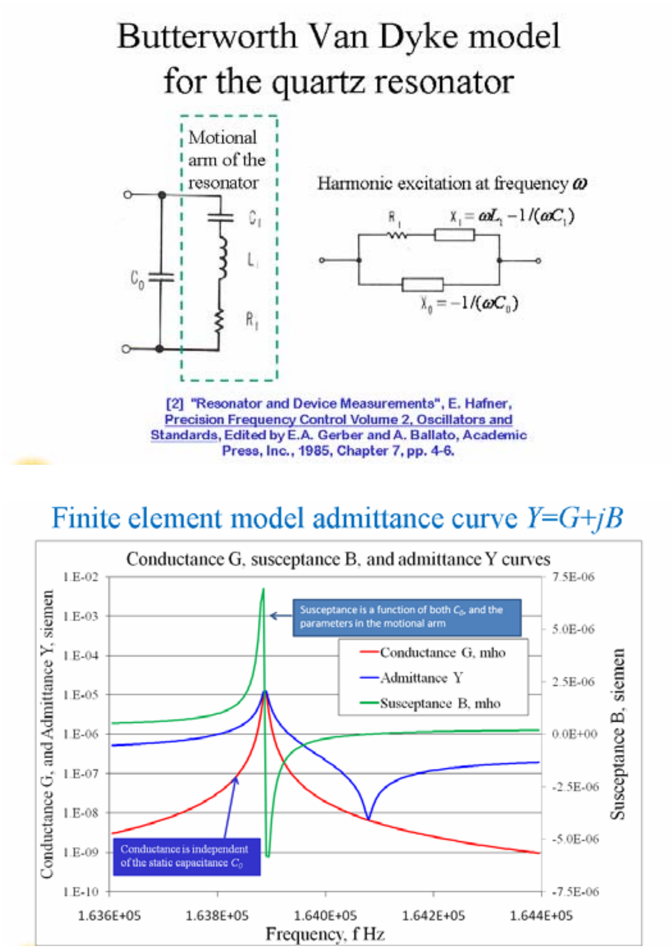

Figure 5. Butterworth Van Dyke model for the quartz resonator, and its typical admittance curve (bottom)

We observe that the static capacitance C_0 is not part of the motional arm of the resonator. C_0 can be obtained from the susceptance at a very low frequency such as 1 Hz. At this low frequency, the resonator is essentially stationary, and it acts essentially like a capacitor. Hence at 1 Hz:

$$C_0 = \frac{B}{2\pi} \qquad \text{Eq(12)}$$

When C_0 is known, the parameters of the motional arm could be determined from the following derived relations:

$$B_1 = B - \omega C_0 \qquad \text{Eq(13)}$$

$$R_1 = \frac{G}{G^2 + B_1^2} \qquad \text{Eq(14)}$$

$$X_1 = \frac{B_1}{G^2 + B_1^2} \qquad \text{Eq(15)}$$

The reactance X_1 could be differentiated with respect to the frequency using a central difference method:

$$\left[\frac{dX_1}{df}\right]_i = \frac{\left.X_1\right|_{i+1} - \left.X_1\right|_{i-1}}{f_{i+1} - f_{i-1}} \qquad \text{Eq(16)}$$

At the series resonance frequency, $\omega = \frac{1}{\sqrt{L_1 C_1}}$, it can be further shown that

$$L_1 = \frac{1}{4\pi}\frac{dX_1}{df}\bigg|_{f = f_s}$$

$$C_1 = \frac{1}{4\pi f_s^2 L_1}\bigg|_{f = f_s} \qquad \text{Eq(17)}$$

$$Q = \frac{1}{2\pi f_s C_1 R_1}\bigg|_{f = f_s}$$

2) Results for a 2-degree Z-cut quartz tuning fork resonator

A 164 KHz, 2-degree Z-cut quartz tuning fork was studied. The main mode of this tuning fork is shown below in Fig. 6

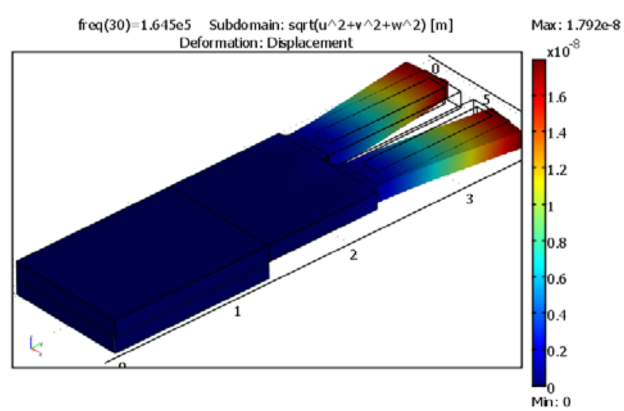

Figure 6. Main mode of a 164 KHz, 2-degree Z-cut quartz tuning fork resonator. C_0 = 0.3 pF.

The static capacitance for this tuning fork was calculated using Eq(12), and C_0 was determined to be 0.3 pF. We can plot Eqs(14) and (17) as a function of the excitation frequency, and this was shown in a plot of the admittance curve in Fig. 7.

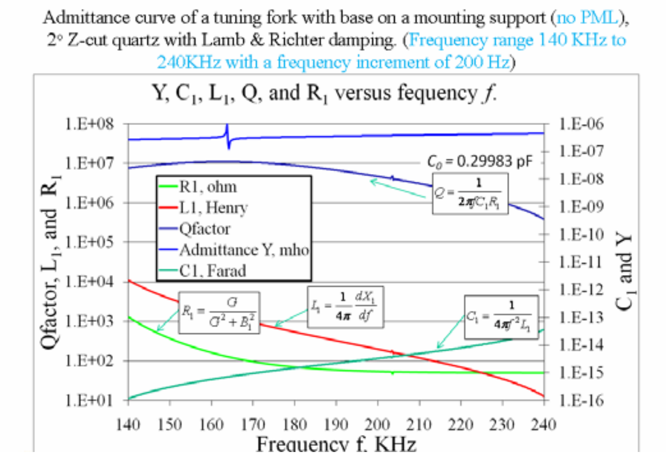

Figure 7. Admittance curve of the 163 KHz tuning fork along with the curves obtained from Eqs(14) and (17).

We observe the curves from Eqs(14) and (17) are smooth lines, hence they are not sensitive to the frequency increments of the admittance curve. When a vertical line is drawn on the graph at the series resonance frequency we could obtain values for R_1, L_1, C_1, and Q (see Fig. 8). These values were used to calculate a predicted admittance curve which was compared with the finite element admittance curve, and this was shown in Fig. 9. Good comparison was observed which validated our algorithm for extracting the Butterworth Van Dyke model parameters from the admittance curve.

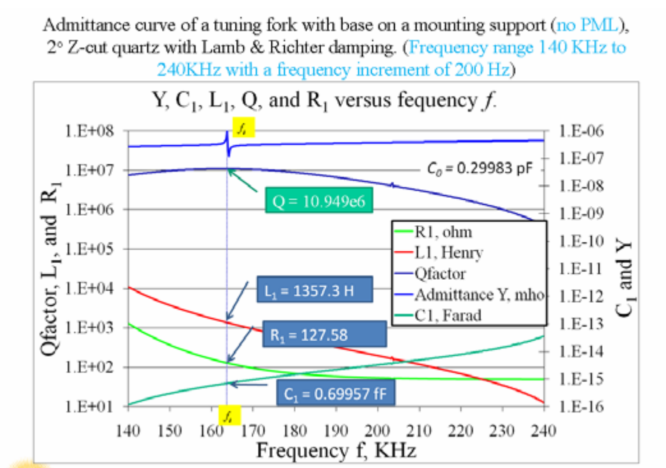

Figure 8. Extraction of the Butterworth Van Dyke model parameters using Eqs(14) and (17).

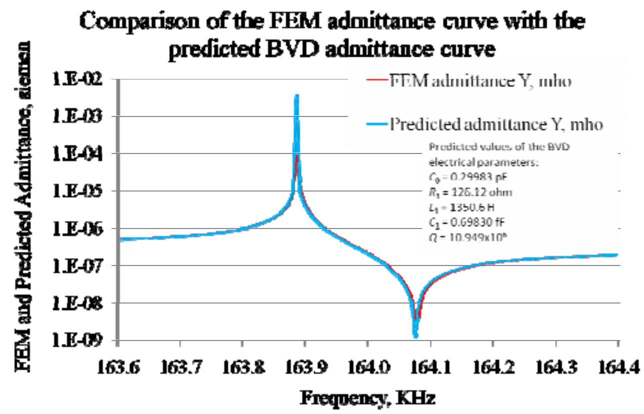

Figure 9. Extraction of the Butterworth Van Dyke model parameters using Eqs(14) and (17).

V. Effects of the Mounting Base and Perfectly Matched Layer Model (PML) on the Motional Resistance R_1 and Q of Quartz Resonators

The motional resistance R_1 and Q of the quartz resonator were of particular interest when the objective was the development of frequency devices with a low phase noise and a high frequency stability based on a high Q factor.

The effect of a perfectly matched layer (PML) base on the Butterworth Van Dyke model parameters of the 164 KHz tuning fork (Fig. 6) was studied. The results showed that PML base changed the values of R_1 and Q of the tuning fork dramatically while the parameters C_0, L_1, and C_1 remained unchanged.

Table I: Effects of the PML base on the Butterworth Van Dyke parameters of the 164 KHz, 2-degree Z-cut quartz tuning fork resonator.

BVD parameters	No PML in base	PML in the base
C_0	0.3000 pF	0.300 pF
R_1	126.1 ohm	69480 ohm
L_1	1351 H	1351 H
C_1	0.6983 fF	0.6983 fF
Q	10.95×10^6	20020

The effect of the base could further be demonstrated by the effects of packaging on the frequency response of the quartz resonator. This is shown below in Fig. 10 for a 13.3 MHz AT-cut quartz plate where its frequency response was affected by its packaging, and the reduction in R_1 and Q of the resonator was observed.

We drew the conclusion that an optimal design for a high Q miniaturized resonator required the inclusion of the base support to form a resonator-base structure. The base formed an essential component of the frequency device.

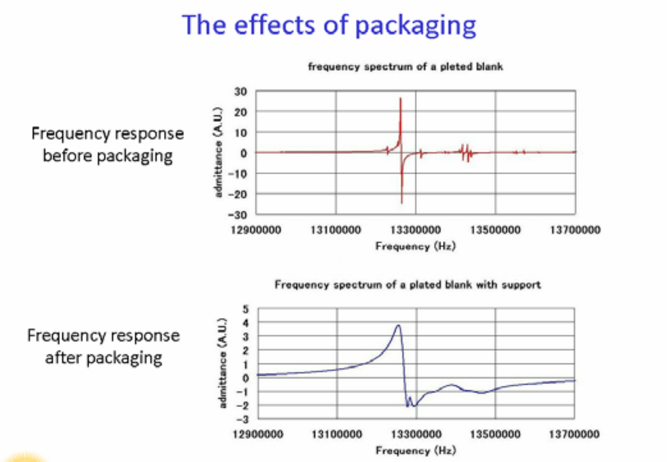

Figure 10. The effects of packaging on the frequency response of a 13.3 MHz AT-cut quartz resonator.

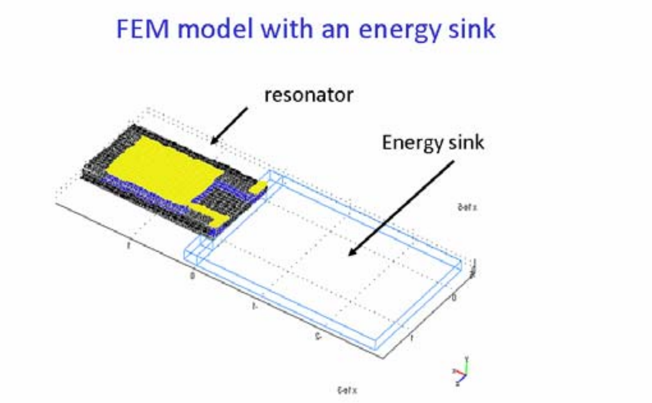

Figure 11. An AT-cut resonator mounted as a cantilever fixed onto a mounting base.

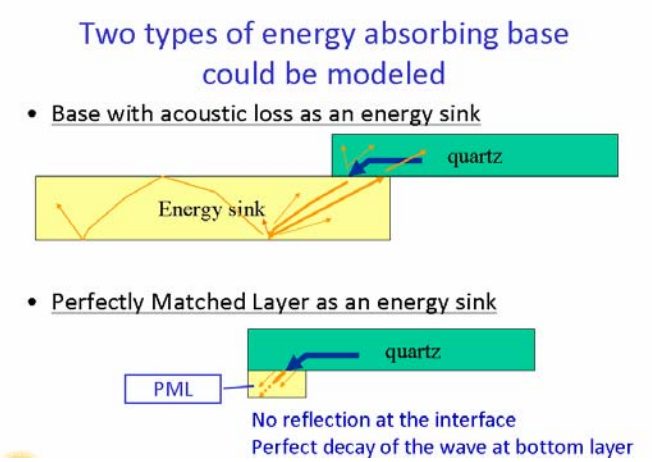

Figure 12. Energy absorption in the base can be thought as either an energy sink (top), or PML base (bottom).

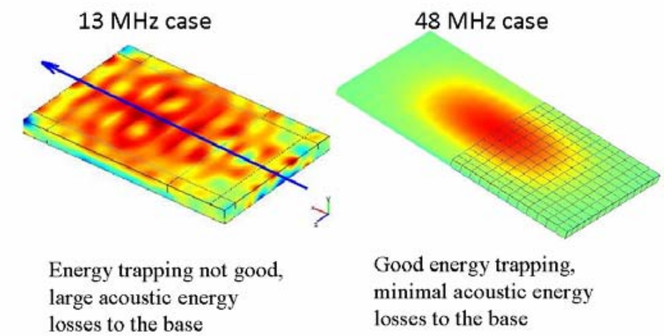

Figure 13. Two cases of AT-plate resonator. Left: 13 MHz resonator with poor energy trapping. Right: 48 MHz resonator with good energy trapping.

a. Models for the mounting base of the quartz resonator

The model of the mounting base as an energy sink or PML base was studied. Figure 11 shows the AT-cut resonator mounted as a cantilever fixed onto a base, and Fig. 12 shows the base could be thought of as an energy sink which absorbed acoustic energy radiated from the AT-cut resonator into the base. The PML is also an energy sink in that acoustic waves and their energy are propagated into the base without reflections. Two cases of AT-cut plate resonator were studied; the first case was that of a 13 MHz resonator with poor energy trapping, while the second case was that of a 48 MHz resonator with good energy trapping. Figure 13 below shows the two resonator cases.

Frequency response analyses were carried out on the two cases. The finite element had Lamb and Richter's[2] viscosity constants for quartz while the conductivity was neglected since it was negligible compared to the viscosity losses. For each case, the frequency response analyses were carried without PML in the base support and then with PML in the base support. In both cases of the resonators, better comparisons with the experimental results were found from finite element models that include energy losses in the base such as that provided by the PML base. Figure 14 and 15 showed the better comparisons of experimental results with the finite element models that had both viscosity losses and PML base. Tables II and III provided the numerical values of the Q and R_1 obtained from the finite element models and compared with the measured values. In both cases of the poor energy trapped 13 MHz resonator, and good energy trapped 48 MHz resonator, the numerical results showed better comparisons with their measured values when the base allows for energy dissipation such as with the PML base.

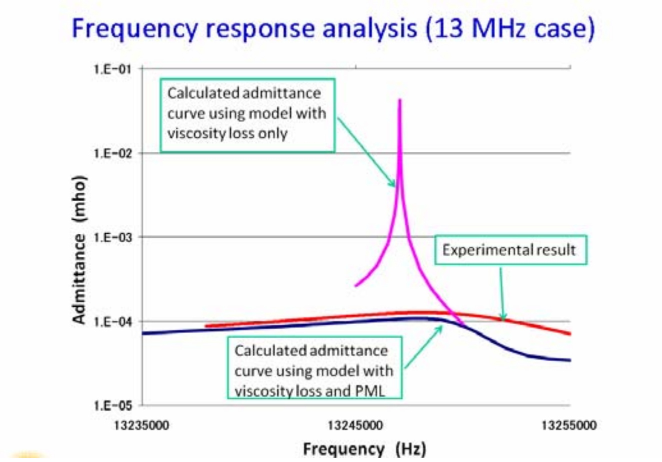

Figure 14. A 13. MHz AT-cut resonator with poor energy trapping mounted as a cantilever fixed onto a mounting base without PML and with PML.

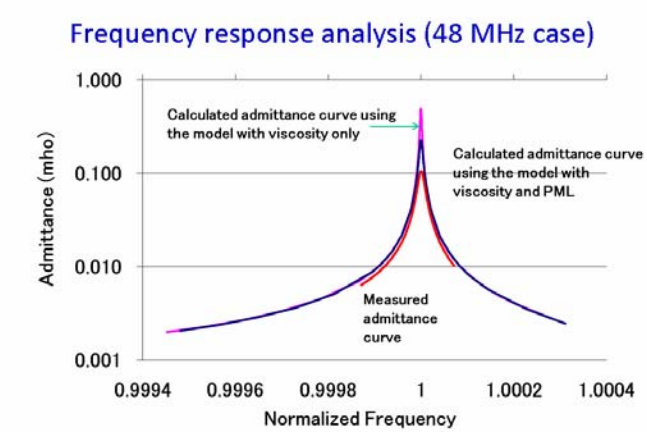

Figure 15. A 48 MHz AT-cut resonator with good energy trapping mounted as a cantilever fixed onto a mounting base without PML and with PML.

TABLE II: COMPARISON OF MEASURED RESULTS OF THE 13 MHZ AT-CUT QUARTZ RESONATOR WITH THE FINITE ELEMENT MODELS

Sample	F_R, MHz	Q factor	R_1, ohm
Measured values	13.253	1067	9256
Finite element model with viscosity, and base with PML	13.247	982	9256
Finite element model with viscosity and base without PML	13.247	630000	24

TABLE III: COMPARISON OF MEASURED RESULTS OF THE 48 MHZ AT-CUT QUARTZ RESONATOR WITH THE FINITE ELEMENT MODELS

Sample	F_R, MHz	Q factor	R_1, ohm
Measured values	47,997	69110	9.41
Finite element model with viscosity, and base with PML	48.165	199423	4.40
Finite element model with viscosity and base without PML	48.165	427159	2.03

Warner[3] had shown that the Q of AT-cut resonators at various frequencies were shown to decrease with frequency. We compare the Q obtained from the finite element models of AT-cut resonators with his equation and measured values of Q. This was shown in Fig. 16, and the graph showed that the Q obtained from the finite element models with Lamb and Richter's viscosity constants and a base without PML follow Warner's[3] equation reasonably well. The measured Q at various frequencies however were below that of Warner's. The finite element models with both the viscosity constants and a base with PML brought the Q lower and closer to the measured Q.

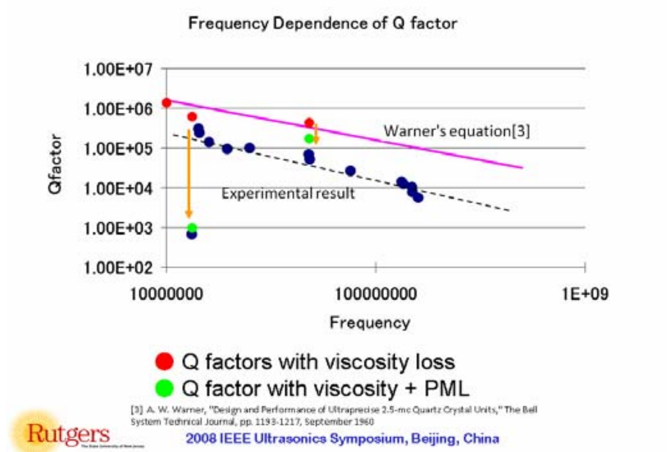

Figure 16. Comparison of the Q factors obtained from finite element models of AT-cut resonators with Warner's[3] equation and measured Q factors. The finite element models with Lamb and Richter's quartz viscosity constants and PML base bring the Q factors closer to the measured Q factors.

VI. CONCLUSIONS

Piezoelectric models with dissipation in viscosity and conductivity were implemented for quartz resonators. Lamb and Richter's [2] viscosity constants, and conductivity constants by Lee, Liu, and Ballato[1] were used. The conductivity losses in quartz were negligible when compared with the losses due to viscosity, and were neglected. Equations for determining the Butterworth Van Dyke model electrical parameters C_0, R_1, L_1, C_1,and Q from the finite element admittance curve were presented and the parametric values obtained were found to compare well with the finite element admittance curve. Studies of a 164 KHz, 2-degree Z-cut quartz tuning fork and AT-cut quartz plate resonators of various frequencies were performed, and it was found that the energy losses at the mounting base had an important role in the determination of the motional resistance R_1, and Q of the frequency devices. On the other hand the energy loss at the base had little effects on the other electrical parameters C_0, L_1, and C_1. The energy loss at the base lowered the Q of resonators and brought their values closer to the measure Q values. The Q from the finite element models with viscosity losses only fitted well with Warner's[3] equation but were higher than the measured Q values.

The finite element models with viscosity and base energy dissipation would be useful for designing new quartz resonators. An optimal design for a high Q miniaturized resonator required the inclusion of the base support to form a resonator-base structure. The base formed an essential component of the frequency device

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