3D Transient Analysis of Ultrasound Propagation Using Finite Difference Time Domain Method and Its Experimental Verification

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Abstract—**A 3D transient analysis of ultrasound propagation using the finite difference time domain method is investigated. The resolution requirement on the discretization of space and time is examined using a simple first order model. The 3D transient analysis of the ultrasound propagation in the air surrounding a rectangular pipe is carried out taking into account the distribution of the displacement on the surface of the transmitter. The weighted integration of time-varying acoustic pressure on the surface of the receiver is compared with the measured voltage waveform. It is shown that the displacements of transducer are nonhomogeneous, and this should be taken into consideration for both transmitter and receiver in order to express the real phenomena of ultrasonic propagations.**

Keywords-ultrasound; propagation; 3D transient analysis; finite difference time domain method; transducer

I. INTRODUCTION

Numeric techniques are useful in the design of measurement systems which use ultrasonic sensors such as telemeters, thermometers, flowmeters, etc. Although simple methods [1] using only the propagation speed affected by the properties of the medium or the ray tracing modeling [2] of an ultrasound have been used, it is difficult to cope with wave-type phenomena, such as diffraction or interference in the frame of these methods. The finite element method [3] can consider the above physical wave-type phenomena. However, it is difficult due to huge memory requirements to apply this method to 3D analysis of ultrasound propagation because the wavelength is very short and high spatial resolution is required. The boundary element method [4] can also deal with these wave-type phenomena but it is difficult to consider variations of the medium properties in space. Given these reasons, the finite difference time domain (FDTD) simulation [5] seems to be attractive because it is an explicit one without solving large scale linear equations and the medium is divided into finite elements. Recently, the 3D simulation using the FDTD method became possible due to the progress of computer science.

In this paper, a 3D transient analysis of ultrasound propagation using the FDTD method is investigated. First, in order to clarify the required space and time resolutions, their effect on the accuracy is investigated using a simple first order model. Then, a 3D transient propagation analysis in the air surrounding a rectangular pipe is carried out taking into account the non-uniformity of the transducer displacement. The effect of the modeling of the transducer on the accuracy is investigated by comparison with the measured results.

II. METHOD OF ANALYSIS

A. Governing Equations and Discretization Method

The acoustic field is governed by the Euler's equation and the equation of continuity as follows:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho}\nabla p \tag{1}$$

$$\frac{\partial p}{\partial t} = -k\nabla \bullet \boldsymbol{v} \tag{2}$$

where $\boldsymbol{v}$, p, ρ, and k are the particle velocity, the acoustic pressure, the medium density, and the bulk modulus, respectively. The attenuation due to the losses is neglected in this paper.

Both governing equations are discretized by the FDTD method with the first order staggered mesh in this paper. In this method, the time and space differential terms are discretized by the central difference approximation and the unknowns $\boldsymbol{v}$ and p are solved in turn.

B. Effects of Grid Width and Time Interval on Accuracy

In this section, in order to clarify the resolution of space and time required for accurate analysis, the effects of the grid width and time interval on the accuracy of the propagation time and the amplitude of waveform are investigated.

Fig. 1 shows a simple first order model. The medium is the air (ρ = 1.205 kg/m^3, k = 142297 Pa). The transient propagation analysis giving the acoustic pressure p with the following time variations at point o is carried out:

$$p(t) = (0.5 - 0.5\cos 2\pi ft)\sin 2\pi ft \tag{3}$$

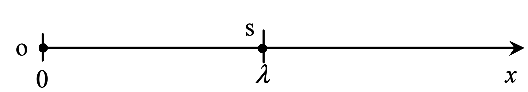

Figure 1. One dimensional analyzed model.

where f is the frequency and it is set to be 400 kHz. The propagation time T and the maximum value p_{max} of acoustic pressure at the point s, which is a wavelength λ (=0.859mm) apart from point o, obtained from the propagation analysis are compared with the real values. The real value of T is 2.50μs (=λ/C, acoustic speed C is 343.64m/s) and that of p_{max} neglecting the attenuation is 0.695 Pa.

Fig. 2 shows the relationship between the number N_s of subdivisions per wavelength λ and the errors ε_t, ε_p of T, p_{max} in the case when the time interval Δt of time step is small enough (Δt =0.1ns). When N_s is over 30, both errors become less than 1.5%. Fig. 3 shows the errors ε_t, ε_p due to the time interval Δt at N_s =30, in which the real values are assumed to be those at Δt=0.1ns. This figure shows that Δt should be chosen to be less than that in the stable condition ($\Delta x/C$=83.3ns; Δx is the width of mesh).

III. VERIFICATION

A. Model Description

The 3D verification model, in which two transducers facing each other are set in a rectangular pipe, is shown in Fig. 4. The diameter of transducer is 10mm and the driven frequency is 400 kHz. The distances between both transducers are set to be 10, 30, and 50 mm in turn. The square-wave voltage lasting 3.5 cycles, shown in Fig. 5, is applied to the transmitter, and the voltage at the receiver is measured.

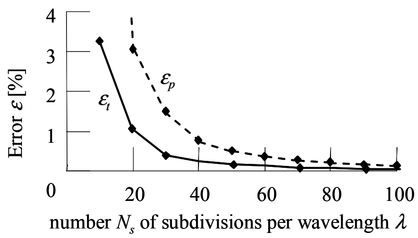

Figure 2. Effects of the grid size on the accuracy (Δt=0.1ns).

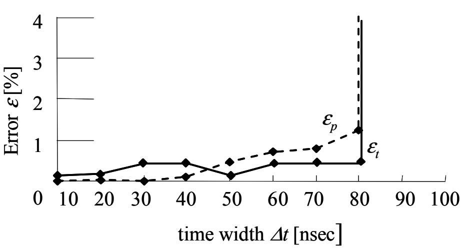

Figure 3. Effects of the time width Δt on the accuracy (N_s=30).

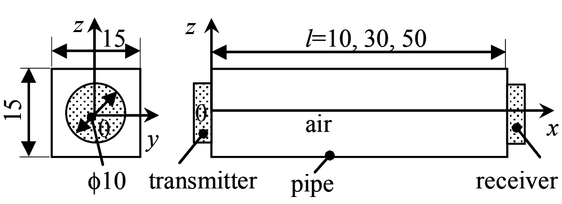

Figure 4. 3D verification model.

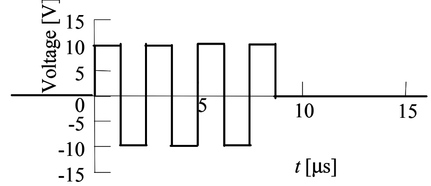

Figure 5. Waveform of the voltage applied to the transmitter.

B. Measurment Condition

Fig. 6 shows the experimental model. The transducers are set in the rectangular pipe made of acrylic resin. The resonance frequency of both transducers is 400 kHz and an adjustment layer on the surface of piezoelectric element is added in order to use in the air. The measured model is put in the room with constant temperature of 25 ° . The voltage shown in Fig. 2 is applied to transmitter using the function generator (NF Corp.: WF1943B, 14bit, 120MS/s), and the voltage of the receiver is measured by the memory storage (Iwatsu Electric Co., Ltd: DS9121, 10bit, 20MS/s).

C. Analyzed Condition

The wave propagation in the air only surrounding the pipe is analyzed. Only 1/4 of whole region is analyzed due to the symmetry. On the inner surface of the pipe and the surface of receiver, the Dirichlet boundary condition $\boldsymbol{v}=\boldsymbol{0}$ is imposed. The widths Δx and Δt of grid and time step are chosen to be 25μm and 5ns, respectively, which are in accord with the conditions mentioned in Section II. B.

On the surface of the transmitter, the distribution of x-component v_x of the actual velocity of the surface movement, by applying the voltage shown in Fig. 5, is given taking into account the time variation. The actual velocities at the grid point with interval of 0.5mm are obtained from the x-component Dx of the actual displacement measured by the laser interferometer (Technar: TWM-5000). The examples of the time variations of Dx at some points are shown in Fig. 7. Dx is normalized by the maximum value in time at the center point. The amplitude of the displacement Dx of the transducer is largest at the center and it suddenly damps at the other points. The phase is also different at each point. For comparison, the analysis giving the uniform v_x in space with the waveform shown in Fig. 5 on the surface is also carried out.

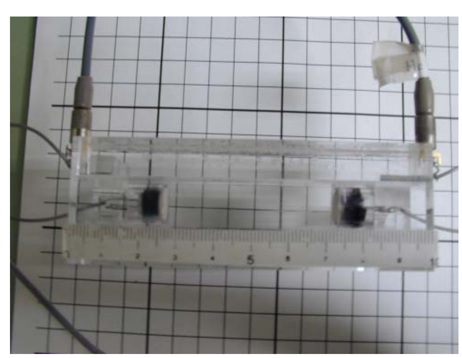

Figure 6. Experimental model.

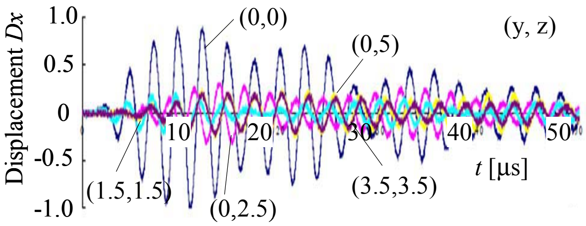

Figure 7. Time variation of the transducer displacement.

For the receiver, the time-varying integration of the acoustic pressure p is calculated instead of the voltage waveform. The integration is weighted by the maximum displacements in time at each point and it is carried out only within ϕ6 mm which is the size of the Piezoelectric element. For comparison, the integration without weight in the whole region of the surface of the receiver is also carried out.

D. Results and Discussion

Fig. 8 shows the voltage waveforms of the receiver obtained from the measurement. The shape of the voltage waveform of l=10mm is similar to the time variation of the displacement Dx at the center of transducer. In the cases of l=30, 50 mm, the maximum value of the voltage is smaller and the voltage after 7 periods becomes much smaller compared with l=10mm.

Fig. 9 shows the time variation of the acoustic pressure p distribution of l=50mm at z=0mm obtained from propagation analysis. When the uniform v_x is given on the surface of the transmitter, the shape of waveform is not changed and the attenuation observed in Fig. 8 of the measured results cannot be expressed as shown in Fig. 9(i). On the other hand, when the actual velocity v_x is imposed, the acoustic pressure p at the center axis (x axis) becomes smaller with time due to the diffusion, as shown in Fig. 9(ii). Therefore, the actual condition should be imposed on transmitter.

Fig. 10 shows the time variation of the integration of the acoustic pressure p at the receiver obtained from the

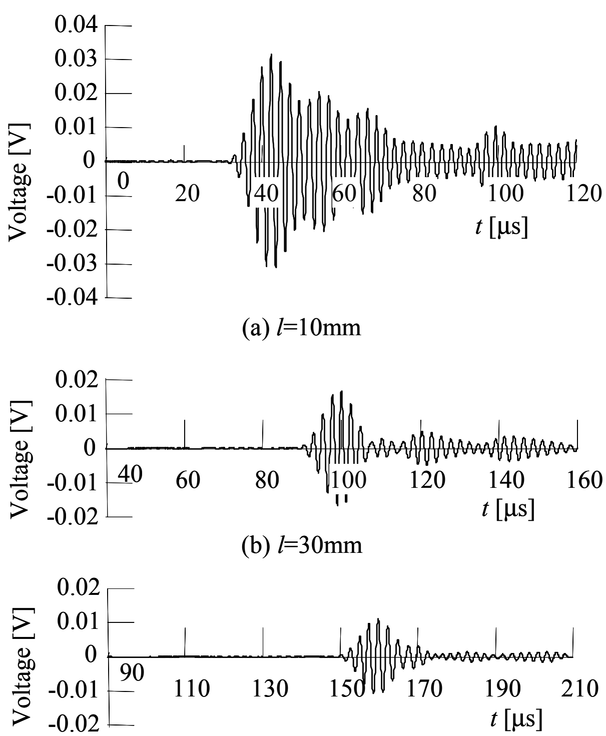

Figure 8. Voltage waveform at the receiver (measurement).

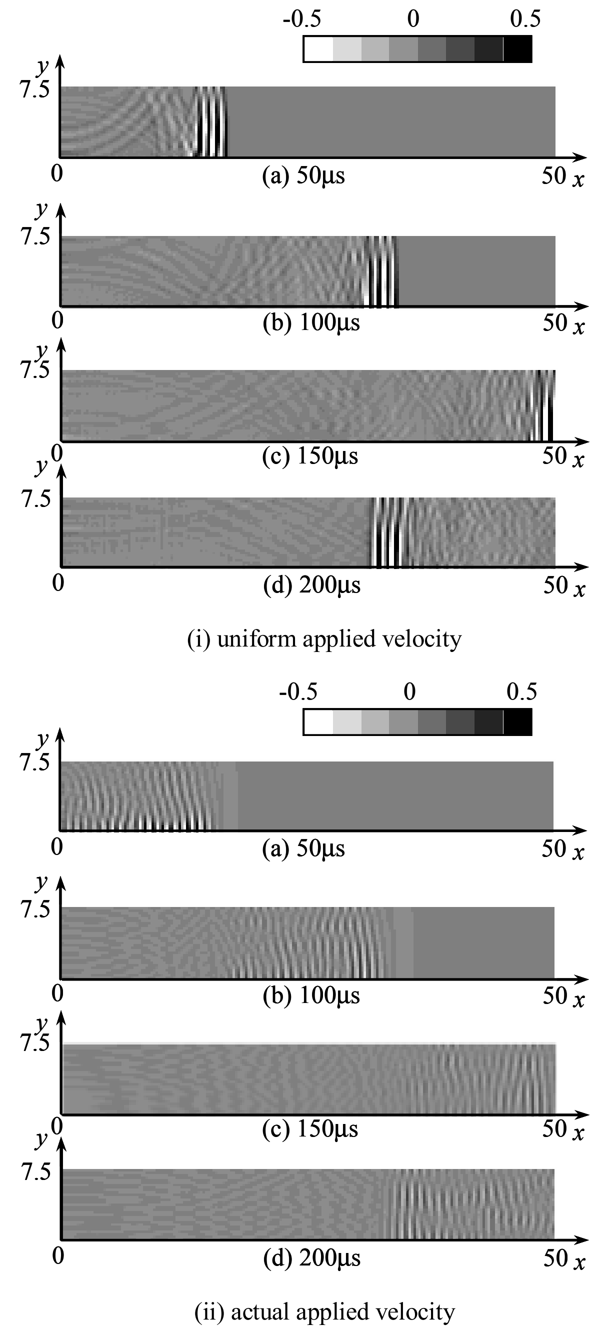

Figure 9. Time variation of distributions of acoustic pressure (z=0).

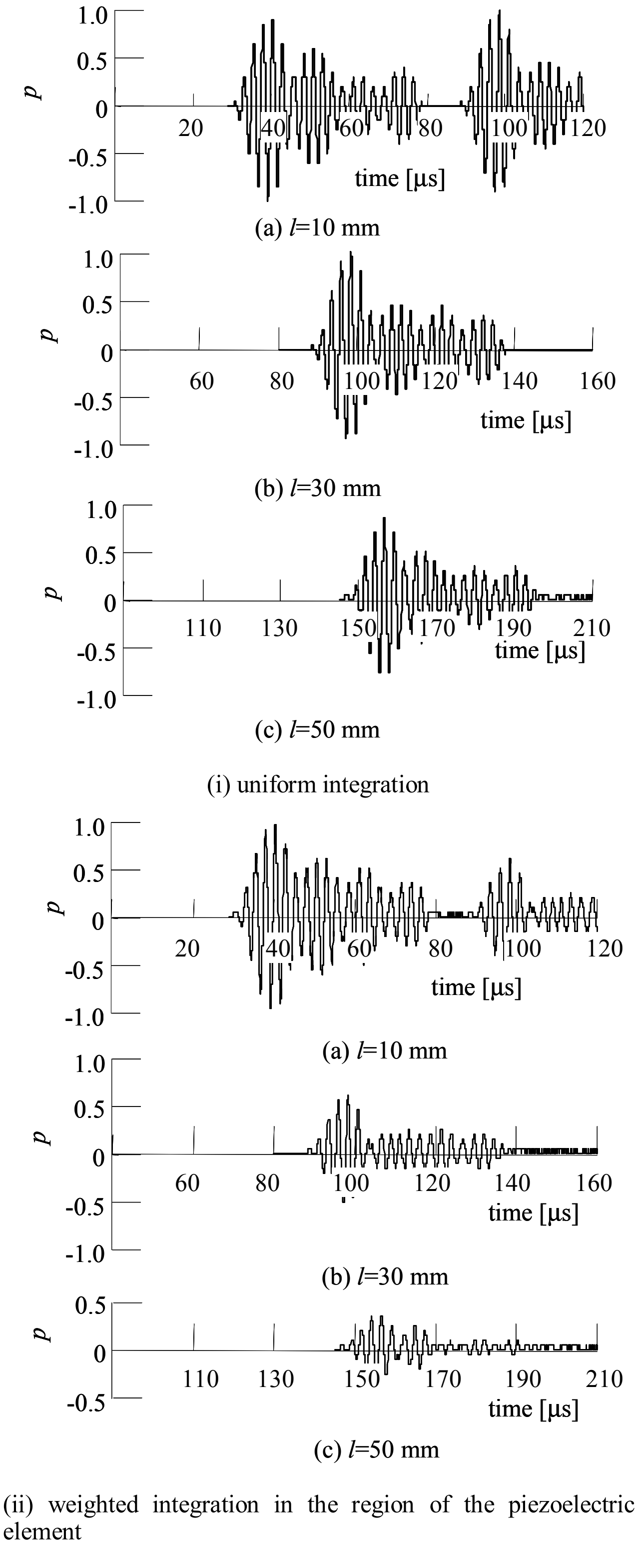

Figure 10. Time variation of integration of acoustic pressure at the receiver (analysis).

propagation analysis when the actual velocity v_x is given on the surface of the transmitter. When the uniform integration is applied, the effect of attenuation of the acoustic pressure p at the x-axis shown in Fig. 9 (ii) cannot appear, as shown in Fig. 10 (i). On the other hand, when weighted integration is carried out only in the region of the piezoelectric element, the tendencies of not only the attenuation of the maximum value, but also the variation of shape of waveform with the distance l coincide with the measurement.

IV. Conclusions

A 3D transient analysis of ultrasound propagation using the FDTD method is investigated and verified by experiment. The results obtained can be summarized as follows:

1) In order to carry out ultrasound propagation using the FDTD method accurately, the width Δx of the mesh element should be less than 1/30 of the wavelength. And the time interval Δt should be chosen to be less than that of stable condition.

2) In order to express the real phenomena of the propagation by the numerical analysis, the non-uniformity of transducer displacement should be considered.

3) Finally, the voltage waveform at the receiver can be expressed by imposing the real velocity distribution obtained from the measurement on the transmitter and modeling the output voltage by integration of acoustic pressure, which is weighted by the displacement of the transducer and limited to the region of the piezoelectric element.

A simulation of the piezoelectric element will be introduced in this propagation analysis in the future.

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