New insights and extensions of split-spectrum algorithms from an optimum distributed detection perspective

Ignacio Bosch Roig
Dept. Communications
Polytechnic University of Valencia
Valencia, Spain
igbosroi@dcom.upv.es

Luis Vergara Domínguez
Dept. Communications
Polytechnic University of Valencia
Valencia, Spain
lvergara@dcom.upv.es

***Abstract*—In this paper we approach Split-spectrum (SS) automatic detection from a distributed detection perspective. The basic idea is to implement an energy detector at the output of every band-pass filter, and then to fuse all the detections to generate a final decision for every time index. The individual detectors are optimally designed by using the subspace matched filter theory (briefly, every band-pass filter is equivalent to the projection into a tuned subspace). The fusion is also optimized by using the well established theory of distributed detection. The proposed detection algorithms are applied to the analysis of the ultrasonic NDE of the inner layer structure of the vault of a Spanish basilica. The results show the interest of the proposed new detection schemes**

Keywords- Automatic, distributed, detection, monosensor, data fusion, decision, ultrasonic, NDE, consolidation, restoration, split-spectrum

I. INTRODUCTION

Detection of ultrasonic echo pulses embedded in a grain noise background is a classical problem in the area of non-destructive testing (NDT) of materials. The simplest model of grain noise considers many randomly superimposed echoes due to the complex microstructure of materials. Under very general conditions, neglecting multiple scattering, this superposition model implies that the grain noise is a non stationary stochastic process, having a Gaussian [1] or a non-Gaussian [2], distribution. Non stationarity is mainly due to the attenuation dependence on frequency [1], [3], meanwhile Gaussianity or non-Gaussianity depends on the effective number of scatters contributing to the recorded signal in a given instant [2], or to the possible presence of regularities in the spatial distribution of the scatters.

Let us stay the problem in the following terms. We want to determine the presence of a possible ultrasonic echo pulse $p(n)$ in a segment of the recorded and sampled ultrasonic signal $r(n)$.

$$\begin{aligned} H_1 \quad & r(n) = p(n) + g_1(n) \\ H_2 \quad & r(n) = g_2(n) \end{aligned} \tag{1}$$

With $n=n_s,...,n_s+N-1$, where n_s, n_s+N-1 are respectively the starting and the final sample numbers delimiting the segment, and $g_i(n)$ corresponds to the grain noise samples under hypothesis i.

Determining the presence of $p(n)$ implies some processing $f[\cdot]$ on the segment $z(n_s)= f[\mathbf{r}]$, with $\mathbf{r}=[r(n_s),\ldots,r(n_s+N-1)]^T$. If we move the value n_s along the recorded signal we obtain the sequence $z(n_s)$, which is the output sequence obtained after processing the input sequence $r(n)$.

Depending on the application we can be required to detect the presence of $p(n)$ in non-automatic manner, but, very often, automatic detection is a requirement. In the first case the usual goal is to maximize the signal to noise ratio enhancement (*SNRE*) factor. In the second case the output $z(n_s)$ is to be compared with a threshold t to make a sample by sample automatic decision on the two hypotheses of equation (1)

$$\begin{cases} if \quad z(n_s) > t \quad decide \ H_1 \\ if \quad z(n_s) < t \quad decide \ H_2 \end{cases} \tag{2}$$

Maximizing the probability of detection (*PD*) for a given probability of false alarm (*PFA*), or minimizing the probability of error (*PE*) needs a proper selection of the processing function $f[\cdot]$, which is not necessarily the same which maximizes *SNRE*. Both goals can be joined by gating the input sequence, thus generating a new output sequence $r_{out}(n_s)=r(n_s)$ if decide H_1, and $r_{out}(n_s)=0$ if decide H_2. It can be easily shown (see for example [4], page 111) that the *SNRE* corresponding to $r_{out}(n_s)$ when compared with $r(n_s)$ is $SNRE=PD/PFA^{0.5}$.

In this paper we assume that automatic detection (2) and/or gating post-processor are required, and, where possible, the Neyman-Pearson (N-P) criterion should be satisfied. Hence the basic problem faced in this paper is the optimum design of the processing function $f[\cdot]$. This will be constrained by the a priori knowledge we could assume about the echo pulse and about the grain noise. Thus, the most classical situation (largely worked in [4]) is that of assuming: perfect knowledge of vector $\mathbf{p}$; both $\{g_1(n)\}$ and $\{g_2(n)\}$ are locally stationary Gaussian inside every interval [n_s, n_s+N-1] having the same (and known) power spectrum $S_g(w)$. Then, the matched filter detector is the optimum solution [5], where $\mathbf{C}g$ is the grain noise local covariance matrix of $S_g(w)$.

$$z(n_s) = f(\mathbf{r}) = \mathbf{r}^T \mathbf{C}_g^{-1} \mathbf{p} \quad (3)$$

However, in most practical applications of ultrasonic NDT, the assumed hypotheses for (3) to be optimum are far from being accomplished. The vector **p** depends on the pulse arriving to the possible reflector, hence knowledge of **p** can not be assumed in general. The spectrum of the grain noise $S_g(w)$ are depth dependent [3]. However, the presence of a reflector produces variations in the vicinity of it, thus reducing the number of scatters. Finally, Gaussianity is not an adequate hypothesis for coarse grained materials [2], producing "spiky" grain noise records, or for materials exhibiting regular spreading of the scatters.

Split-spectrum algorithms offer a practical alternative to matched filter detectors for those cases where the mentioned hypotheses fail. They take advantage of the tuning frequency sensitivity of grain noise, when it is filtered by band-pass filters tuned along the grain noise bandwidth. It is assumed that presence of an isolated echo **p** due to a given reflector is not sensitive to the tuning frequency, and a constant amplitude shifting is induced in all the filter output levels, the general expression for the split-spectrum processing is

$$z(n_s) = f(\mathbf{r}) = f_{nl}(\mathbf{Hr}) = f_{nl}(\mathbf{v}) \quad (4)$$

where the rows $\mathbf{h}_i$, i=1…N, of matrix **H** (LxN) correspond to the finite impulse responses of the band-pass filters. $f_{nl}(\cdot)$ is a nonlinear function. Typical alternatives for $f_{nl}(\cdot)$ are

- $z(n_s)=f_{nl}(\cdot)$=min[v_i], minimization algorithm [6]
- $z(n_s)=f_{nl}(\mathbf{v})$=order statistic ($v_i$), [7]
- $z(n_s)=f_{nl}(\mathbf{v})$=number of positive values in real part of **v**, polarity thresholding algorithm [4], [8]

Split-spectrum algorithms based upon (4) may produce significant improvements in *PD*, and so in *SNRE*, when compared with just a gating of the original signals. They do not need knowledge of **p** or of $S_g(\omega)$. However, fitting the threshold t in (2), requires knowledge of the probability density function (pdf) of $z(n_s)$. On the other hand, the achieved *PD* will depend on the joint pdf of the elements of **v** under H_1.

Some *PD-PFA* analysis of minimization [6] and polarity thresholding [8], algorithms assume uncorrelated multivariate Gaussian distribution under both hypothesis

$$\begin{aligned} \mathbf{v} &: N(\mathbf{m}, \sigma_{Hg}^2 \mathbf{I}) \quad H_1 \\ \mathbf{v} &: N(\mathbf{0}, \sigma_{Hg}^2 \mathbf{I}) \quad H_2 \end{aligned} \quad (5)$$

where **m** is a mean vector due to the (assumed) shifting m produced at every filter output by the presence of **p** and σ^2_{Hg} is the variance due to the noise component **Hg** in vector **v=Hr=Hp+Hg**.

Assuming the model (5) for *PD-PFA* analysis of split-spectrum algorithms is controversial. If model (5) is adequate, then the optimum detector which maximizes *PD* is a matched (to **m)** filter: $z(n_s)=1^T\mathbf{Hr}=r(n_s)$, (a simple gating of the original signal). In practice when using split-spectrum algorithms, we have to question model (5): the vector mean shift **m** is a too simple model; grain noise statistics, (the variance), should be different for every hypothesis, reducing the grains contribution under H_1.

We propose in the next section a general split-spectrum scheme based on the use of distributed detection. They include, as particular cases, the minimization and order statistics algorithms, thus an indirect result of the work is a better understanding of why and when split-spectrum algorithms can improve trivial detectors. Experiments with simulated and real ultrasonic signals are considered in section 3.

II. SPLIT-SPECTRUM DISTRIBUTED DETECTION ALGORITHMS

A. Proposed scheme

From the foregoing discussions we are going to consider a different model than (5), namely:

$$\begin{aligned} \mathbf{r} &: N(\mathbf{p}, \mathbf{C}_{g1}) \quad H_1 \quad \mathbf{C}_{g1} = \sigma_{g1}^2 \mathbf{D}_g \\ \mathbf{r} &: N(\mathbf{0}, \mathbf{C}_{g2}) \quad H_2 \quad \mathbf{C}_{g2} = \sigma_{g2}^2 \mathbf{D}_g \end{aligned} \quad (6)$$

with $\sigma^2_{Hg1} << \sigma^2_{Hg2}$ where grain noise variance reduces in a significant manner when the pulse is present, where $\mathbf{D}_g$ is a matrix having its main diagonal normalised.

The optimum detector corresponding to model (6) is much dependent on the knowledge we have about **p**. Let us assume that $\mathbf{C}_{g2}^{-1/2}\mathbf{p}$ is inside a given subspace having a projection matrix $\mathbf{P}_i = \mathbf{h}_i \mathbf{h}_i^H / \mathbf{h}_i^H \mathbf{h}_i$. We could estimate **p** by projecting the pre-whitened observation vector $\mathbf{C}_{g2}^{-1/2}\mathbf{r}$ on that subspace.

$$z_i(n_s) = \mathbf{r}^H \mathbf{C}_{g2}^{-1/2} \frac{\mathbf{h}_i \mathbf{h}_i^H}{\mathbf{h}_i^H \mathbf{h}_i} \mathbf{C}_{g2}^{-1/2} \mathbf{r} \quad (7)$$

We propose the scheme of figure 1. Instead of making only one decision, as in equation (2), distributed decisions are made at the output of every band-pass filter (u_i(0 or 1)), which are fused to make the final decision (u_0 (0 or 1)), using an optimum fusion rule rule($u_{1,\ldots,}$ u_L) that maximizes the final probability of detection PD_{final}.

$$r_{out}(n_s) = \begin{cases} r(n_s) & if \quad rule(\mathbf{u}) = 1 \\ 0 & if \quad rule(\mathbf{u}) = 0 \end{cases} \quad (8)$$

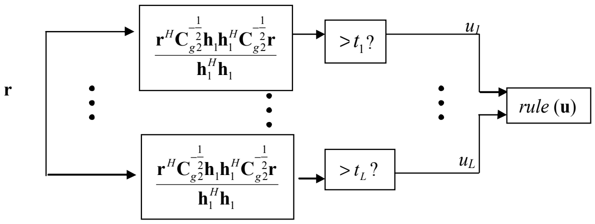

Figura 1. Example of Scheme of split-spectrum distributed detection algorithm (*SSDD*)

When DFT is used to implement the band-pass filters, twice the subspace energy follows a central chi-squared distribution with two degrees of freedom. Hence we have an easy way to finding the adequate threshold t_i at every filter output to obtain a desired probability of false alarm PFA_i, this in turn implies an easy way for fitting the final probability of false alarm PFA_{final} after fusion is done.

The *SSDD* algorithm assumes that the presence of the pulse "injects" energy along a given number of subspaces, but a minimum amount of energy is required in every subspace to guarantee detection. This suggests the convenience of some type of normalization in *SSDD* to actually make detection only dependent on the energy subspace distribution property of the pulse, and not on the pulse level. Thus we arrive to the normalised *SSDD* (*NSSDD*), where every filter output value is divided by the maximum of them. Then we could make a decision at every filter output.

$$z_i(n_s) = \frac{\mathbf{r}^T \mathbf{C}_{g2}^{-\frac{1}{2}} \mathbf{h}_i \mathbf{h}_i^H \mathbf{C}_{g2}^{-\frac{1}{2}} \mathbf{r}^T}{\max_j \mathbf{r}^T \mathbf{C}_{g2}^{-\frac{1}{2}} \mathbf{h}_j \mathbf{h}_j^H \mathbf{C}_{g2}^{-\frac{1}{2}} \mathbf{r}^T} \quad (9)$$

We have to consider the optimum design of the fusion rule. The optimum fusion rule for general models of correlated decisions has been derived in [9]. Calling m to the number of zeros in vector $\mathbf{u}$, we must consider the particular case of uncorrelated decisions under H_2 and pair wise correlations under H_1. Particularizing the general expression given in the equation (79) of [9] ($\rho^{H1}=\rho$ and $\rho^{H2}=0$). It can be shown, if $\Delta(m)$ is a non-increasing function of m, we can write the N-P optimum fusion rule [9]

$$rule(\mathbf{u}) = rule(m) = \begin{cases} 1, & if \quad m \le m_0 \\ 0, & if \quad m > m_0 \end{cases} \quad (10)$$

III. EXPERIMENTS WITH SIMULATED AND REAL DATA

A. Simulated detection analysis

We generate a number of records of simulated grain noise where some randomly distributed pulses to be detected have been inserted. Every simulated record is obtained by convolving zero-mean white noise records with an impulse response having the same waveform than the pulses to be detected.

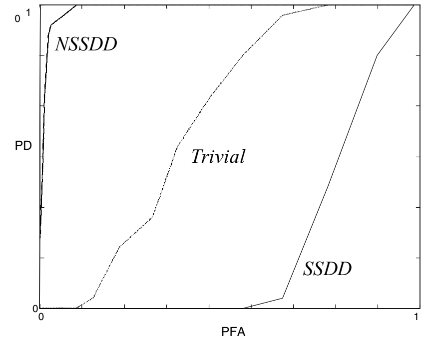

Figure 2. Raw ROC curves for the three alternatives

We know a priori the locations of the pulses to be detected, so we can count the number of detected and missed pulses, as well as the number of false pulse detections. A comparison among three detectors is done: *SSDD*, *NSSDD* and what we call trivial detector, which is just a gating of the prewhitened recorded signal. In figure 2, we show raw ROC curves computed from the number of true and false pulse detections. We conclude the superiority of *NSSDD* with respect to the trivial detector and *SSDD*. In particular *SSDD* seems to be very sensitive to the presence of some amount of grain noise in the time interval where the pulse appears. The results of the detection analysis indicate that conventional split-spectrum algorithms are severely limited for use in an automatic detection framework.

B. Real data detection analysis

To complete the experimental part of the paper we present some real data results corresponding to the ultrasonic analysis of the first layer profile of a basilica´s cupola 1:1 scale model. The ultrasonic analysis was made with the aim of showing the viability of ultrasonic non-destructive testing techniques to help in the process of restoration of heritage historical buildings.

The first layer of the cupola is a 0.3 cm stratum of mortar, and the second one is a 1.2 cm width stratum of plaster. The objective is to trace the interface profile between the first and second layers. This tracing gives valuable information to the restores. Thus, we have made non-destructive ultrasonic analysis using echo-pulse inspection mode with ultrasonic pulse generation with 5MHz transducer.

We have collected 75 A-scan of 100 μs along a vertical array of locations. On the other hand we obtained in laboratory an estimate of the expected delay between echoes form the interface (1.92 μs). That means that a possible first reflection from the interface should arrive at 3.84 μs, a second one at 2x3.84=7.68 μs, a third one at 11.52 μs and so on.

We have applied the three algorithms: trivial detector, *SSDD* and *NSSDD*, to the original signals considering the interval 4-20 μs, thus avoiding the initial idle time interval of the receiver (figure 3). Then we look for possible interface profiles outlined by the reflections of order greater than one (second, third and so on).

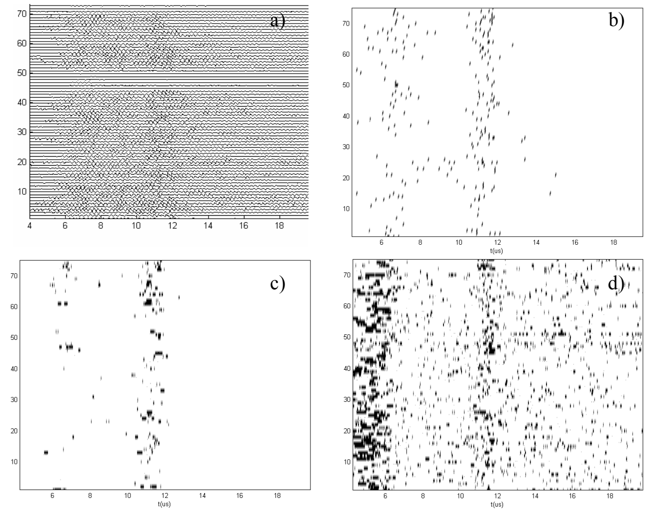

Figure 3. a) Original B-scan (75x4000) b) Trivial c) *SSDD* d) *NSSDD*

In figure 3b, 3c and 3d we show respectively the results for the trivial, *SSDD* and *NSSDD*. We show 2-D grey representations with black indicating detection. In the three cases the profile corresponding to the third reflection is "uncovered" by the processing. The second reflection is too much corrupted by multiple surface and inner reflections to allow reconstruction of the profile. A possible fourth reflection seems to be too attenuated to appear. However the third reflection is outlined by the three methods.

Apparently, the best results are obtained with *SSDD* algorithm, followed in this order by the trivial detector and *NSSDD*. Surprisingly, *NSSDD* seems to exhibit the largest number of false alarms, which is in contradiction with the superior performance predicted by theory and observed in simulations. But it should be considered that mortar is a material composed by sand and paste of cement. Two essential parts of its microstructure are air pores (sizes may vary from 10^{-10} to 10^{-4} *m*) and sand grains (10^{-4} to 10^{-3} *m*). The wavelength corresponding at 5 MHz, $\lambda=0,312\cdot10^{-3}$m, which of the order of the sand grains. Thus, in this case, together with the microstructure grain noise we have also isolated echoes from sand grains. Moreover, at 5 MHz we are also sensitive to surface irregularities, which add more isolated echoes to the recorded signal. As *NSSDD* is amplitude independent, it is able to detect not only echoes from the interface but also from the sand grains and the surface irregularities. On the other hand *SSDD* and the trivial detector are amplitude sensitive methods so that they filter out many of the (low level) isolated echoes. In this sense we can say that in *NSSDD*, we do not have more false alarms, but "too many detections".

IV. CONCLUSIONS

We have proposed new *SSP* algorithms for automatic detection of ultrasonic echoes in a grain noise background. The algorithms are derived in the framework of optimum distributed detection, so that control of *PFA* and maximization of *PD* are achieved. They give us insights into the *SSP* algorithms limitations, as the new ones are extensions of previous algorithms based on order statistics. In general terms we can say that *SSP* based algorithms are adequate to detect "isolated" echoes preceded and followed by important levels of grain noise.

We have shown that incorporation of a prewhitening (another novelty with respect to previous *SSP* algorithms) is inherent to the desired optimality. Some good results may be obtaining even from what we have called "trivial detector". We have also shown that approaching the algorithms in the framework of algebraic subspaces, thus exploiting the formal equivalence between a "tuned" filter and a band-pass "tuned" subspace is a powerful way to obtain optimal detectors at every filter output.

Finally, the simulated and real data experiments have demonstrated the interest of the proposed scheme. In particular, the application to the detection of the echoes due to the first layer of a cupola wall, had afforded good results.

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