Spatio-Temporal Deconvolution of Pulsed Ultrasonic Fields Received by a Transducer of Linear Aperture : A Simulation Study

Wahiba Djerir(1*), Tarek Boutkedjirt(1♣), Med Ouali Si-Chaib(2) and Hakim Djelouah(1)

(1) Faculté de Physique, Université des Sciences et de la Technologie Houari Boumédiène, BP 32, El-Alia, DZ-16111 Algiers, Algeria
(2) Faculté des Sciences de l'Ingénieur, Université de Boumerdes, DZ-35000 Boumerdes, Algeria

Abstract – **In ultrasonic field measurements, the signal delivered by the receiving transducer is affected by the receiver spatio-temporal transmission properties. The pressure is spatially averaged over its finite-size aperture. Furthermore, frequency variations of its transfer function may distort the output signal. The efficiency of the deconvolution of these effects is shown by means of numerical simulations. The pulsed pressure field radiated by a wideband 10.3mm-radius planar transducer, with a central frequency f_c = 2.25 MHz (λ_c: corresponding ultrasonic wavelength in water) is considered. The receiver is a linear aperture 25 µm- thick PVDF membrane hydrophone. The study shows that the quality of the reconstructed signal depends strongly upon the signal-to-noise ratio (SNR), the aperture dimensions and the distance from the source. For an aperture of length $L = 2.6$ mm $\approx 3.9\lambda_c$, placed on axis at $z = 3$ mm, correlation coefficients between the reconstructed pressure and the original one, $r_{p\hat{p}}$, of 0.999, 0.988 and 0.655 have been obtained, for SNRs of 60, 40 and 20 dB respectively. For a greater aperture ($L = 4.6$ mm $\approx 6.9\lambda_c$) and SNR = 40 dB, no satisfactory results could be obtained at this distance ($r_{p\hat{p}}$ = 0.912). At a greater axial distance (z = 20 mm), better deconvolution results could be achieved ($r_{p\hat{p}}$ = 0.928).**

I. Introduction

For the measurement of pulsed ultrasonic fields, different techniques exist. However, the most recommended technique remains the use of piezoelectric PVDF hydrophones as receivers. This is due to their advantageous acoustic properties and commercial availability [1]. Nevertheless, the electric signal delivered by these devices may be affected by their spatio-temporal transmission properties. Because of the high working frequencies, their aperture dimensions become greater than the wavelengths of the ultrasonic wave. In this case, they furnish a spatially averaged value of the measured acoustic quantity. Furthermore, the variations of their sensitivity in this frequency range have to be considered. The possibility of deconvolving the spatial effects has been shown for harmonic ultrasonic fields [2]. The present contribution is a step towards the generalization of the study to pulsed ultrasonic fields.

II. Direct problem

A. Linear, one-dimensional model

The pulsed ultrasonic field radiated in a liquid is considered. This field is received by a hydrophone of linear aperture with a finite length L and a relatively negligible width (linear aperture). Thereby, a radial scan of the field, in the hydrophone length direction (x-direction), at a distance z_0 from the transmitter, is performed. The spatial domain studied is thus restricted to one dimension. For an incident acoustic pressure $p(x, z_0, t)$, the hydrophone considered as a space and time invariant linear system, characterized by the spatio-temporal impulse response $h(x,t)$, delivers an electric voltage :

$$v(x, z_0, t) = p(x, z_0, t) \otimes_{xt} h(x,t) + n(x, z_0, t) . \quad (1)$$

$\otimes_{xt}$ is the spatio-temporal convolution operator and $n(x, z_0, t)$ a non-correlated to the signal noise component.

For a synchronous vibration of all aperture points to an incident plane wave, the impulse response $h(x,t)$ can be written: $h(x,t) = h_1(x) h_2(t)$, where $h_1(x)$ and $h_2(t)$ are the spatial and temporal impulse responses of the hydrophone respectively. Thus, the hydrophone output voltage becomes:

$$v(x, z_0, t) = \langle p(x, z_0, t) \rangle \otimes_t h_2(t) + n(x, z_0, t) . \quad (2)$$

$\langle p(x, z_0, t) \rangle = p(x, z_0, t) \otimes_x h_1(x)$ is the spatially averaged pressure and, $\otimes_x$ and $\otimes_t$, are the spatial and temporal convolution operators respectively.

B. Hydrophone spatio-temporal transmission effects

As a basis for the numerical simulation, the pulsed field of a planar circular piston (radius: 10.3 mm) set in a rigid baffle is considered. The temporal variation of the pressure generated at the transmitter surface is that of a Gaussian modulated sinusoid

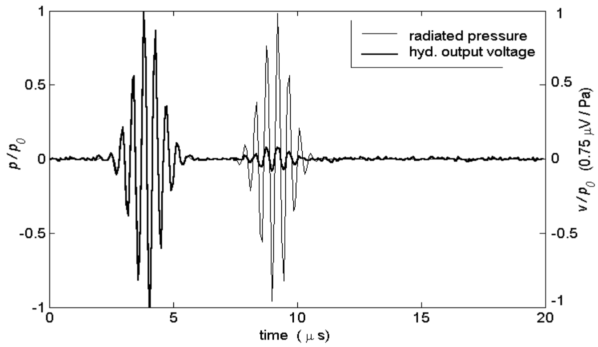

Figure 1. Radiated pressure and output voltage of the hydrophone of linear aperture (L= 2.6mm= 3.9 λ_c , λ_c = 0.67mm), on axis at z_0 =3mm (SNR = 40dB).

of frequency f_c = 2.25 MHz, which corresponds to a central wavelength of the ultrasonic pulse in water λ_c =0.67mm. At the axial distance z_0 and for different radial positions x , the ultrasonic pressure shows the well known contributions of plane and edge waves. On axis, the two contributions have the same amplitude and opposite polarity (Fig. 1).

The same figure shows the open-circuit output voltage of the hydrophone, when it is placed on axis, at z_0 =3mm. This signal has been calculated according to (1), by assuming a hydrophone aperture of length (L =2.6mm=3.9 λ_c) on which the receiving sensitivity is constant (ideal aperture). The hydrophone considered is constituted of a 25µm thick PVDF membrane with 0.1µm thick spot poled gold electrodes. Furthermore, the hydrophone output signal is supposed to be corrupted by a stationary, non-correlated, white and Gaussian noise, with a signal-to-noise-ratio SNR=40dB, the reference being the response to the maximal pressure amplitude of the incident plane wave [3].

The figure shows that, the influence of the hydrophone temporal impulse response on the "measured" pressure field is of less importance (same waveform for the plane wave). This is due to the "slow" variations of the hydrophone temporal transfer function in the frequency band of the incident pressure pulse [3]. On the contrary, spatial averaging considerably affects the edge wave.

III. INVERSE PROBLEM

In order to retrieve the radiated pressure field from the "measured" data, a method based on the deconvolution of the hydrophone spatio-temporal effects is proposed.

A. Temporal deconvolution

The aim of the method is to obtain an estimated value of the averaged pressure from the hydrophone output voltage. The acquisition system being supposed linear, the estimated value $<\hat{p}(x,z_0,t)>$ can be obtained by using a temporal filter with the impulse response $h_{F2}(t)$ .That is:

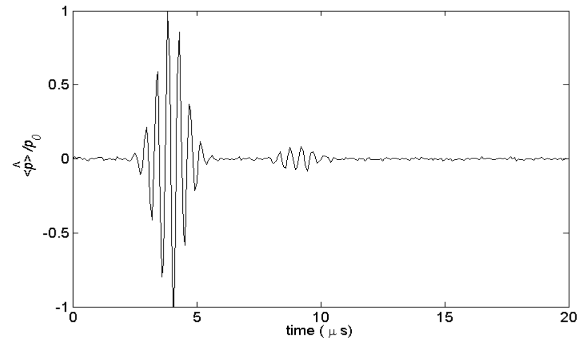

Figure 2. Reconstructed averaged pressure by means of a temporal inverse filter. Aperture: L=2.6mm, SNR=40dB.

$$<\hat{p}(x, z_0,t)>=h_{F2}(t)\otimes_t v(x, z_0,t) . \qquad (3)$$

For the deconvolution, a temporal inverse filter is used. Its transfer function is simply the inverse of that of the hydrophone ($H_{F2}(f)=1/H_2(f)$). Fig. 2 shows that this operation leads to excellent results.

B. Spatial deconvolution

The objective is to obtain an estimated value, $\hat{p}(x,z_0,t)$, of the "original" incident pressure, $p(x,z_0,t)$, from the estimated value of the spatially averaged one, $<\hat{p}(x,z_0,t)>$. This can be achieved by using a reconstruction filter of spatial impulse response $h_{F1}(x)$; that is :

$$\hat{p}(x, z_0,t)=h_{F1}(x)\otimes_x<\hat{p}(x, z_0,t)> . \qquad (4)$$

As a criterion for the evaluation of the quality of the deconvolution procedure, the correlation coefficient, $r_{\hat{p}p}$, between the reconstructed pressure $\hat{p}(x,z_0,t)$ and that radiated, $p(x,z_0,t)$, is used. On axis, this coefficient is given by:

$$r_{\hat{p}p}=\sum_{k=1}^{N} p^*(i_0,k)\hat{p}(i_0,k) \Big/ \sqrt{\sum_{k=1}^{N}|p(i_0,k)|^2\cdot\sum_{k=1}^{N}|\hat{p}(i_0,k)|^2} . \qquad (5)$$

i_0 and k are the indices related to the position x=0 and the time t respectively and N is the number of temporal samples.

1) Spatial inverse filter

The spatial transfer function of this filter is the inverse of the hydrophone transfer function ($H_{F1}(f_x)=1/H_1(f_x)$). When using this filter, the deconvolution result is shown in Fig. 3. In this case, a correlation coefficient for the ultrasonic pressure reconstructed on axis $r_{\hat{p}p}$ =0.855 is obtained. In contrast to the temporal deconvolution, the spatial inverse filter doesn't permit the reconstruction of the pressure field because of the zeros of

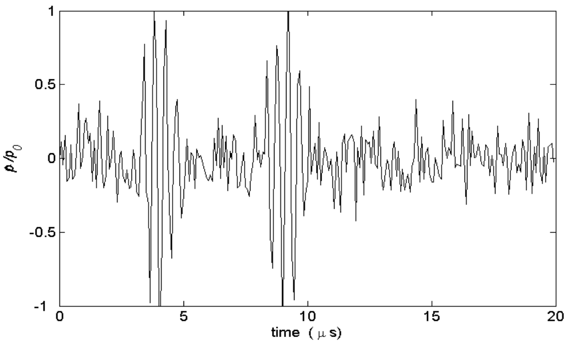

Figure 3. Reconstructed pressure, on axis at z_0 =3mm, by using a spatial inverse filter (L=2.6mm, SNR=40dB).

the spatial transfer function of the hydrophone. This highlights the ill-posedness of this inverse problem. Even if these zeros are numerically avoided, the problem remains ill-conditioned because of the drastic amplification of the noise (Fig. 3).

2) Spatial Wiener filter

To overcome the ill-posedness of the problem, a regularization procedure based on a Wiener reconstruction filter is used. Its spatial transfer function is given by [3,4]:

$$H_{F1}(f_x) = \frac{H_1^*(f_x)\Phi_{pp}(f_x,z_0)}{\left|H_1(f_x)\right|^2\Phi_{pp}(f_x,z_0)+\Phi_{nn}(f_x,z_0)} \quad (5)$$

where * denotes the complex conjugation operation. $\Phi_{pp}(f_x,z_0)$ and $\Phi_{nn}(f_x,z_0)$ are respectively the spatial power spectrum densities (PSD) of the pressure to be reconstructed and of the noise. This filter has been implemented under two different conditions :

Wiener filter with a-priori known PSDs

In this case, a-priori knowledge of the PSDs in (5) is supposed. The comparison of the reconstructed pressure (Fig. 4a) when using this "ideal" Wiener filter with the radiated pressure (Fig. 1) shows that, under these ideal conditions, excellent reconstruction results can be achieved ($r_{\hat{p}p}$ =0.995).

Wiener Filter with estimated PSDs

Generally, there is no sufficient information on the ultrasonic field to be investigated. In this case of no a-priori knowledge of the PSDs, Φ_{pp} and Φ_{nn}, these quantities are replaced by their estimated values, $\Phi_{\hat{p}\hat{p}}$ and $\Phi_{\hat{n}\hat{n}}$, respectively. The latter can be obtained from the power spectrum of the hydrophone output signal, Φ_{vv} [3]. The variations of the pressure so reconstructed are shown in Fig. 4b. Though the use of Wiener filter under these "real" conditions leads to results of less quality ($r_{\hat{p}p}$ =0.988) compared to those obtained in the "ideal" case, there is still good agreement between the reconstructed pressure and the radiated one.

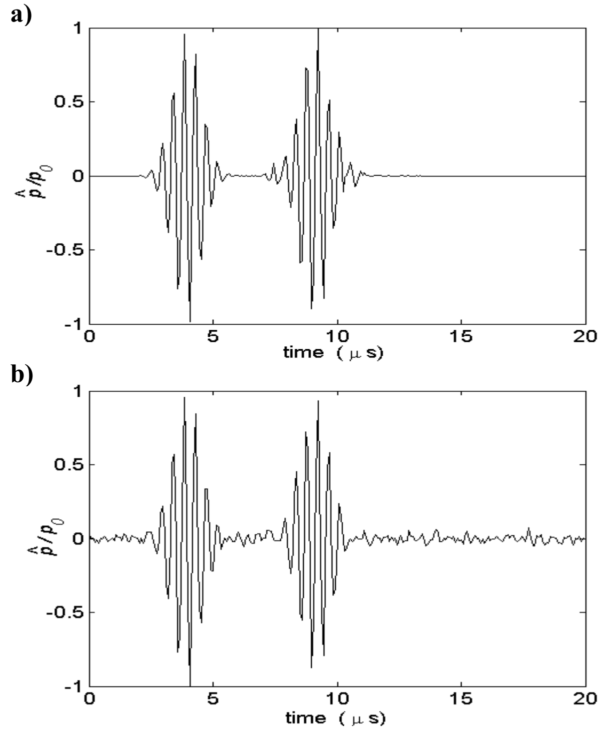

Figure 4. Reconstructed pressure, on axis at z_0 =3mm, by using a spatial Wiener filter (L=2.6mm, SNR=40dB) with a) a- priori known PSDs, b) estimated PSDs.

3) Effect of SNR

In order to study the influence of the noise on the reconstruction quality, the deconvolution has been performed on going from signals furnished by the hydrophone with different noise levels. Fig. 5a is obtained for SNR= 60dB. It shows that the "real" Wiener filter can furnish good reconstruction results ($r_{\hat{p}p}$ =0.999) when the "acquired" signals are not too noisy. On the contrary, for a low SNR (20dB), the reconstruction results become poor (Fig. 5b, $r_{\hat{p}p}$ =0.655).

4) Effect of hydrophone dimensions

The less advantageous reconstruction procedure (Wiener filter with estimated PSDs) has been tested for a greater aperture (L =4.6mm=6.9 λ_c) with an SNR= 40 dB. When the reconstruction results obtained by using this filter (Fig. 6a) are compared with those obtained at the same distance (z_0 =3mm) for an aperture of 2.6mm length (Fig. 4b), it can be noticed that the quality of the reconstruction diminishes with increasing hydrophone dimensions ($r_{\hat{p}p}$ =0.912 instead of 0.988). In Fig. 6a, although the edge wave has a greater amplitude than that spatially averaged (Fig. 2), the reconstructed pressure field differs from the original one. This shows that for this noise level (40 dB), the limits of the reconstruction procedure with a "real" Wiener filter at this distance and for these dimensions are already attained.

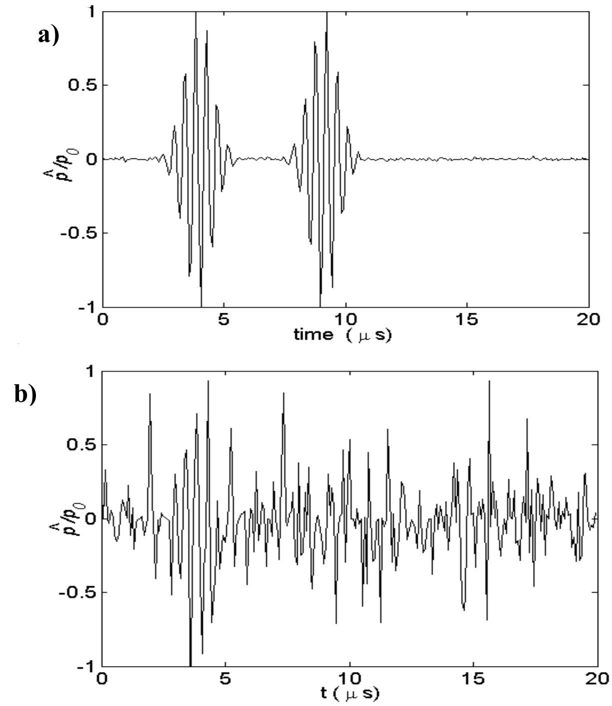

Figure 5. Reconstructed pressure, on axis at z_0 =3mm, by using a spatial "real" Wiener filter (L=2.6mm). a) SNR=60dB, and b) SNR=20dB.

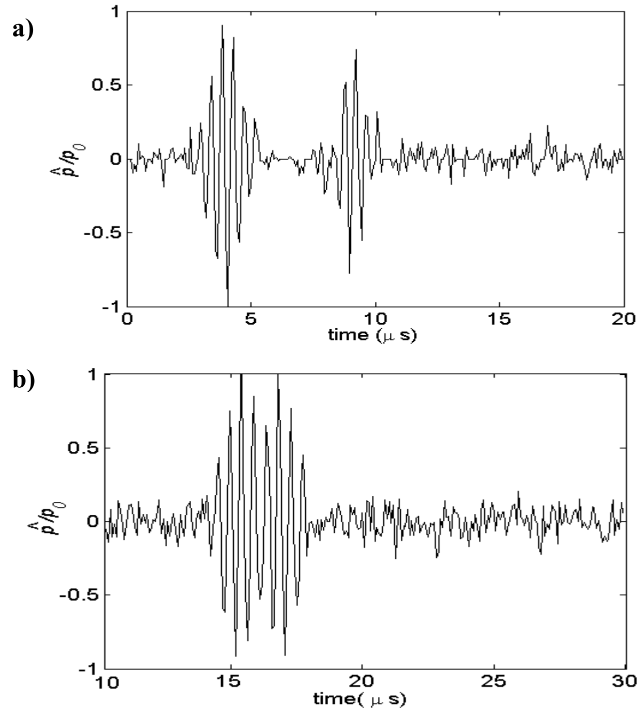

Figure 6. Reconstructed pressure on axis, by using a spatial Wiener filter with estimated PSDs. L=4.6mm, SNR=40dB. a) z_0 =3mm, b) z_0 =20mm.

5) Effect of axial distance to the source

The spatial frequencies bandwidth of the investigated ultrasonic field decreases, at a given time-frequency with the axial distance to the source [2]. The influence of this parameter for pulsed fields, has been studied by performing the reconstruction at a greater distance (z_0 =20mm).

Fig. 6b shows the reconstruction results, at this distance, obtained by using a spatial Wiener filter with estimated PSDs. Here, the sound field has been "acquired" by a hydrophone of 4.6 mm aperture length and SNR=40dB. In this figure, the edge wave, though interfering with the plane wave because of the distance considered and the pulse width, is properly reconstructed ($r_{\hat{p}p}$ =0.928). This improvement of the reconstruction quality allows using greater aperture dimensions at this distance before the limits of the procedures are attained.

IV. Conclusion

In this study, it has been shown that it is possible to deconvolve the effects of the hydrophone spatio-temporal transmission properties. On the contrary to the temporal deconvolution, the spatial deconvolution is ill-posed. This has been overcome by using regularization procedures such spatial Wiener filtering. This permitted the pulsed pressure field to be reconstructed from "measurement" data with a better resolution.

The study has been achieved for the field of a circular planar transducer and a receiver of linear aperture. It can, however, be generalized to any kind of ultrasonic field and any aperture geometry (circular, rectangular). Though the showed results concerned the on-axis region, the deconvolution allows the reconstruction of the original pressure at any region of the ultrasonic field, provided SNR is sufficient.

The reconstruction results have shown the strong dependency of the reconstruction quality upon SNR, receiver aperture dimensions and axial distance to the source. In all cases, the greater is the SNR, the better are results. In addition, the farther is the scanned field region, the greater the aperture dimensions can be, before the limits of the reconstruction procedure are reached.

References

[1] G.R. Harris, R.C. Preston, A.S. DeReggi, "The impact of piezoelectric PVDF on medical ultrasound exposure measurements, standards, and regulation," *IEEE Trans. UFFC,* vol. 47 (6), pp. 1321-1335, 2000.

[2] T. Boutkedjirt and R. Reibold, "Reconstruction of ultrasonic fields by deconvolving the hydrophone aperture effects. I. Theory and simulation, II. Experiment" *Ultrasonics,* vol. 39 (9) pp. 631-649, 2002.

[3] W. Djerir and T. Boutkedjirt, "Reconstruction of pulsed ultrasonic fields by deconvolution of the spatio-temporal effects of the receiving hydrophone: Theory and simulation," *Forum Acusticum 2005, Budapest (Hungary) 29 Aug.- 02 Sept. 2005,* pp. 2833-8, 2005.

[4] W.K. Pratt, "Generalized Wiener filtering computation techniques," *IEEE Trans. C.*, vol. 21, pp. 636-41, 1972.

* E-mail address: wahi_2004@yahoo.fr
♣ E-mail address: tarek_boutkedjirt@hotmail.com