Chirplet Transform for Ultrasonic Signal Analysis and NDE Applications

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Abstract— In this investigation, the chirplet transform is introduced as a means to obtain not only time-frequency representation, but also to estimate the echo amplitude, time of arrival, center frequency, bandwidth, phase, and chirp rate of multiple interfering ultrasonic echoes. This transformation can be used for signal decomposition and successive parameter estimation of multiple interfering echoes. It has been shown that by using both simulated chirp signals and the ultrasonic experimental data, the chirplet signal decomposition algorithm performs robustly, yields accurate echo estimation and results in SNR enhancements. Numerical and analytical results show that the algorithm is efficient and successful in precise signal representation. This type of study addresses a broad range of applications including flaw detection, deconvolution, object classification, velocity measurement, and ranging systems.

Keywords - chirplet ; time-frequency; parameter estimation

I. INTRODUCTION

The chirp signal is a type of signal often encountered in ultrasonic testing of dispersive and/or inhomogeneous materials consisting of complex microstructures. The chirp signal parameters correlate to the physical properties, and the estimation of these parameters leads to the characterization of materials. In this study, we present a successive parameter estimation algorithm that relies on the assumption that any ultrasonic signal, no matter how complex it is, can be decomposed into the superposition of multiple single chirplet echoes. The goal is to efficiently estimate the parameters of the individual echoes. Most importantly, with ultrasonic signal parameters we can establish the analytical relationship between the signal model and the physical properties of materials. The chirplet-based parameter estimation algorithm developed in this study is an extension of our early work for signal decomposition and parameter estimation [1-3]. It is an effective tool for the estimation of narrow-band, broad-band, dispersive or nondispersive ultrasonic echoes. Once the signal is decomposed by a family of chirplet echoes, these echoes individually or collectively can be used to describe the nonstationary behavior of the signal and also can be utilized for time-frequency analysis.

The chirplet transformation has been developed in [4-5] as a useful and practical method for time-frequency analysis which has been applied to radar signals and image processing. Further implementations and applications of the adaptive chirplet transform for sonar and speech analysis have been presented in [6-8]. In this paper, we introduce a successive and efficient chirplet decomposition algorithm that employs an adaptive Gaussian chirplet kernel as the general model for the estimation of the ultrasonic echo parameters. This algorithm adaptively tracks and locates the individual echoes for efficient and precise estimation of the all the parameters of these individual echoes. Hence, these estimated parameters can be used for achieving high data compression rate while keeping high fidelity signal reconstruction. To demonstrate the superior time-frequency performance of the chirplet transform, ultrasonic flaw echoes embedded in grain scattering have been analyzed. In this study it has been shown that the chirplet transform outperforms conventional time-frequency transformations such as short-time Fourier transform, Wigner-Ville transform.

II. SUCCESSIVE PARAMETER ESTIMATION ALGORITHM

The purpose of the successive parameter estimation algorithm is to efficiently estimate the parameters of the individual ultrasonic echoes. The parameters can be used not only for data compression but also as a quantitative procedure for examining the physical properties of the materials. In pulse-echo ultrasonic testing the backscattered echo from a single reflector can be modeled as

$$f_\Theta(t) = \beta \exp\left(-\alpha_1 (t-\tau)^2 + i2\pi f_c (t-\tau) + i\phi + i\alpha_2 (t-\tau)^2\right) \quad (1)$$

Where $\Theta = [\alpha_1, \alpha_2, \beta, f_c, \phi, \tau]$ denotes the parameter vector, α_1 is the bandwidth factor, α_2 is the chirp-rate, β is the amplitude, f_c is the center frequency, ϕ is the phase, and τ is the time-of-arrival of the ultrasonic echo. These parameters can be estimated successively using the chirplet transform (CT). The successive parameter estimation algorithm is a recursive method that starts with a time-frequency (TF) representation of the ultrasonic signal based on the CT. The CT of $f_\Theta(t)$ with respect to a chirplet kernel $\Psi_{\hat{\Theta}}$ is defined as

$$CT\left(\hat{\Theta}\right)=\int_{-\infty}^{+\infty} f_{\Theta}(t)\Psi_{\hat{\Theta}}^{*}(t)dt \tag{2}$$

Where $\hat{\Theta}=\left[\gamma_1, \gamma_2, \eta, \frac{\omega_0}{2\pi a}, \theta, b\right]$ denotes the vector of estimated parameters and

$$\Psi_{\hat{\Theta}}^{*}(t)=\left(\frac{2r_1}{\pi}\right)^{1/4}\exp\left[-\gamma_1(t-b)^2-i\omega_0\left(\frac{t-b}{a}\right)-i\theta-i\gamma_2(t-b)^2\right] \tag{3}$$

represents the chirplet kernel and $\eta=\left(\frac{2r_1}{\pi}\right)^{1/4}$ in order to normalize the energy of the chirplet kernel to unity. Hence, the CT of a single ultrasonic echo can be expressed as

$$CT\left(\hat{\Theta}\right)=\beta(2\pi\gamma_1)^{1/4}\frac{1}{\sqrt{\alpha_1+\gamma_1-i\alpha_2+i\gamma_2}}\exp\left[-\frac{\left(\omega_c-\frac{\omega_0}{a}\right)^2}{4[\alpha_1+\gamma_1-i\alpha_2+i\gamma_2]}-\frac{(\alpha_1-i\alpha_2)(\gamma_1+i\gamma_2)(b-\tau)^2}{[\alpha_1+\gamma_1-i\alpha_2+i\gamma_2]}+(\phi-\theta)+\frac{\frac{\omega_0}{a}(\alpha_1-i\alpha_2)+i\omega_c(\gamma_1+i\gamma_2)(b-\tau)}{[\alpha_1+\gamma_1-i\alpha_2+i\gamma_2]}\right] \tag{4}$$

where $\omega_c=2\pi f_c$. The maximum similarity between the input signal, $f_{\Theta}(t)$, and the chirplet kernel, $\Psi_{\hat{\Theta}}$, leads to correct estimation of echo parameters, $\hat{\in}$. It can be shown that the peaks of TF representation of the ultrasonic echo can be used to estimate the center frequency, f_c, and time-of-arrival, τ. To accomplish this goal, the magnitude of the $CT\left(\hat{\Theta}\right)$ is used for the TF representation of the signal, which is given by

$$\left|CT\left(\hat{\Theta}\right)\right|=(2\pi\gamma_1)^{1/4}\beta\left[(\alpha_1+\gamma_1)^2+(\alpha_2-\gamma_2)^2\right]^{-1/4}\exp\left[-\frac{\left(\omega_c-\frac{\omega_0}{a}\right)^2(\alpha_1+\gamma_1)^2}{4[(\alpha_1+\gamma_1)^2+(\alpha_2-\gamma_2)^2]}-\frac{(\alpha_1^2\gamma_1+\alpha_2^2\gamma_1+\gamma_1^2\alpha_1+\gamma_2^2\alpha_1)(b-\tau)^2}{4[(\alpha_1+\gamma_1)^2+(\alpha_2-\gamma_2)^2]}\right] \tag{5}$$

The maximum of the above equation can be obtained by taking partial derivatives of $\left|CT\left(\hat{\Theta}\right)\right|$ in respect to a (which corresponds to the center frequency) and b (which corresponds to the time-of-arrival). The solutions of $\frac{\partial\left|CT\left(\hat{\Theta}\right)\right|}{\partial a}=0$ and $\frac{\partial\left|CT\left(\hat{\Theta}\right)\right|}{\partial b}=0$ are

$$b=\tau \quad , \quad \frac{\omega_0}{a}=\omega_c \tag{6}$$

It is important to point out that under the condition of (6), the estimation of the peak position in TF domain is not a function of the bandwidth factor, γ_1, chirp-rate, γ_2, and phase, θ, of the ultrasonic echo. Furthermore, the peak value of $\left|CT\left(\hat{\Theta}\right)\right|$ is proportional to the amplitude of the actual echo and leads to the estimation of β.

Based on the above estimation of a and b, the estimation of the chirp-rate γ_2 becomes a one-dimensional estimation problem. Hence, the maximum of $\left|CT\left(\hat{\Theta}\right)\right|$ yields the optimal solution for γ_2. This can be achieved by taking the derivative of $\left|CT\left(\hat{\Theta}\right)\right|$ in respect to γ_2 and setting it to 0, the solution of $\frac{\partial\left|CT\left(\hat{\Theta}\right)\right|}{\partial\gamma_2}=0$ is

$$\gamma_2=\alpha_2 \tag{7}$$

Similarly, the estimation of the bandwidth factor is carried out by taking the partial derivative of $\left|CT\left(\hat{\Theta}\right)\right|$ in respect to the bandwidth factor, γ_1, and setting it to 0, The solution of $\frac{\partial\left|CT\left(\hat{\Theta}\right)\right|}{\partial\gamma_1}=0$ is

$$\gamma_1=\alpha_1 \tag{8}$$

Since there is no information about signal phase in the magnitude representation of the CT, the real part of the CT is used to estimate the phase of the echo, θ. The maximum of $\mathrm{Re}\left(CT\left(\hat{\Theta}\right)\right)$ yields the estimation of θ. This can be obtained by taking the partial derivative of $\mathrm{Re}\left(CT\left(\hat{\Theta}\right)\right)$ in respect to θ and setting it to 0, which results in

$$\theta=\phi\ \pm 2k\pi \tag{9}$$

where $k=1,2,3,...$ These results show that the chirplet transform leads to an exact estimation of the time-of-arrival, center frequency, phase, bandwidth factor, and chirp-rate of the ultrasonic echo.

III. Performance Evaluation Using Simulated And Experimental Echoes

The successive parameter estimation technique can be applied to decompose ultrasonic signals into a linear combination of chirp components, which is

$$s(t) = \sum_{j=0}^{N-1} f_{\Theta_j}(t) \qquad (10)$$

where $f_{\Theta_j}(t)$ is chirplet model and Θ_j is the parameter vector of $f_{\Theta_j}(t)$ [refer to (1)].

The decomposition is performed as follows. First, based on the CT TF representation of ultrasonic signal, the most dominant chirp echo is windowed and estimated using the successive parameter estimation algorithm. Then, the estimated echo is subtracted from the original signal. Next, the second echo is estimated from the remaining signal. This process is repeated until the reconstruction error, E_r, is below an acceptable value $E_{\min}$. The value of $E_{\min}$ is determined based on the requirements of the reconstruction quality of the signal.

The procedure used to design the window (see Fig. 1) is based on the determination of the peaks and valleys of the CT TF representation of the signal (Fig. 1a). Upon detection of a peak, an automatic window uses the projection of the CT in the frequency domain (Fig. 1b) and in the time domain (Fig. 1c) to localize the peaks and valleys in the CT TF representation. The windowing algorithm automatically traces a reference line that intercepts the uncompleted echo which has the maximum peak. The boundaries of the window are determined by the valleys around the peak.

The performance of the chirplet decomposition algorithm is expected to degrade in the case of heavily overlapping echoes. If more than two peaks are found, the window for each echo is constructed by the intersection of such boundaries based on peaks and valleys. In noisy environments, it is important to constrain the size of the window in order to limit the amount of noise into the signal estimation step. But it also is important to keep as much of the signal information as possible. Hence, the boundaries in the TF representation may not truthfully characterize the location of the echoes. For this reason, the width of the window can be set to a predetermined value that may depend on the noise level and energy of the echo. These are the tradeoffs one must take into consideration when designing the TF windowing algorithm.

The estimated echoes are superimposed onto the actual signal in Fig 2. From these results, one can conclude that the successive parameter estimation algorithm is successful even in a situation where multiple echoes interfere in both time and frequency.

The signal decomposition algorithm is also evaluated using an ultrasonic experimental signal consisting of multiple interfering echoes. Fig 3 shows an experimental signal acquired from a steel sample block with a flat-bottom hole (i.e., target) using a 5 MHz transducer and sampling rate of 100 MHz.

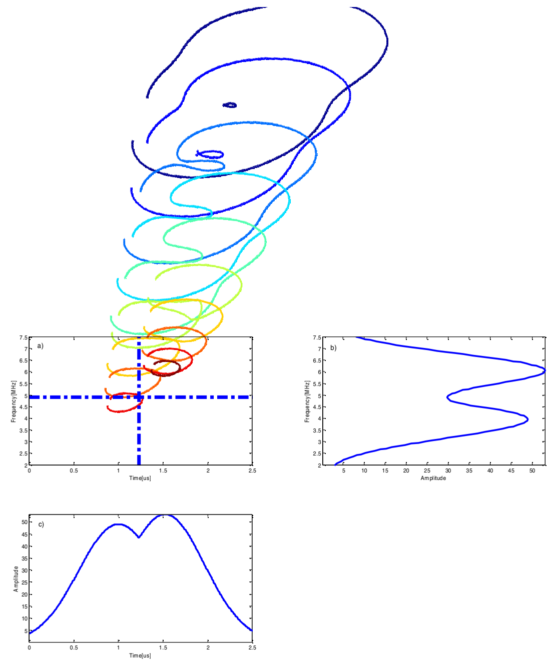

Figure 1. a) TF representation of two interfering echoes, b) Projection in the frequency domain, c) Projection in the time domain.

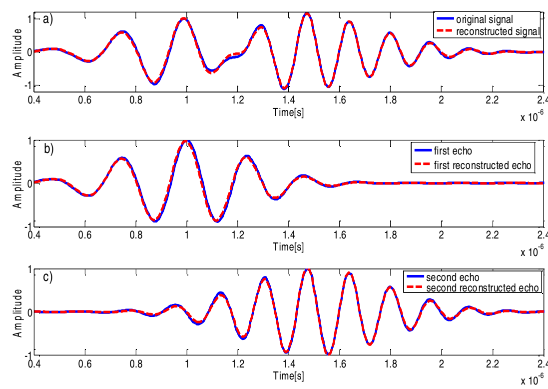

Figure 2. Actual (solid line) and estimated (dashed line) signals, a) Two interfering echoes, b) First echo, and c) Second echo.

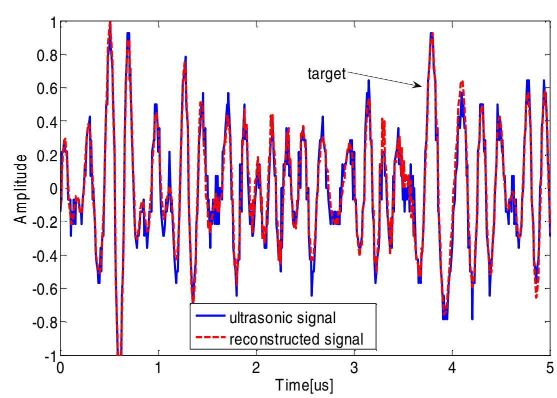

Figure 3. Experimental signal superimposed with the reconstructed signal (dash line) .

It can be seen that the experimental signal has a poor SNR and the target echo shows interference from microstructure scattering and measurement noise. The reconstructed signal of 15 estimated chirplets is shown in Fig 3 (dash line). Fig 4 shows different TF representation of the experimental signal and the reconstructed signal. Fig 4a, Fig 4b and Fig 4c show the CT, WVD and STFT of the ultrasonic backscattered signal. It can be seen that the WVD of the experimental signal [see Fig 4b] smears out the actual information of microstructures. High fidelity TF representation can be attained by using an adaptive chirplet composition and parameter estimation algorithm based on CT representation of the signal. The superimposed WVD of the estimated chirplets is shown in Fig 4d. These results clearly demonstrate that the chirplet decomposition has been successful in estimating echoes and filtering out the noise.

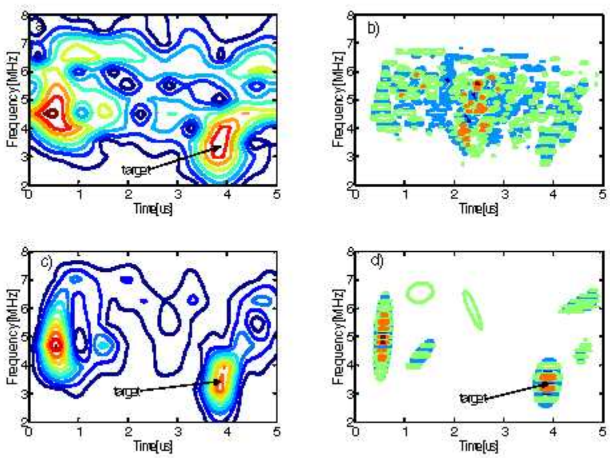

Figure 4. a) CT of ultrasonic backscattered signal. b) WVD of the ultrasonic backscattered signal c) STFT of the reconstructed signal d) superimposed WVD of the estimated chirplets.

IV. CONCLUSION

In this study we have analyzed a signal modeling and successive parameter estimation technique based on the chirplet decomposition to compress and denoise ultrasonic signals. The chirplet transform has been introduced to estimate the time-of-arrival, the center frequency, the phase, the bandwidth factor, the chirp-rate, and the amplitude of ultrasonic echoes. It has been shown through analytical derivations and computer simulations that the parameter estimation algorithm based on the CT can efficiently estimate all the echo parameters. Overall, the signal modeling and parameter estimation algorithm presented in this paper not only offers data compression capabilities, but also provides parameters that can be used for signal analysis, pattern recognition, target sizing and material characterization.

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