

# A NOVEL RESONANT ACCELEROMETER BASED ON MODE LOCALIZATION OF WEAKLY COUPLED RESONATORS

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## ABSTRACT

This paper describes a novel MEMS resonant accelerometer based on two weakly coupled resonators (WCRs) using the phenomenon of mode localization. To the best of the authors' knowledge, it is the first time that this principle is experimentally demonstrated for an accelerometer. When acceleration acts on the two proof masses, there will be a differential electrostatic stiffness perturbation introduced on the WCRs, which leads to mode localization and mode shape change. Therefore, the acceleration can be sensed by measuring the amplitude ratio shift. The measured relative shift in amplitude ratio ( $\sim 312162$  ppm/g) is 302 times higher than the shift in resonance frequency ( $\sim 1035$  ppm/g).

## KEYWORDS

Resonant accelerometer, mode localization, high sensitivity, weakly coupled resonators

## INTRODUCTION

Over the past years, a new transduction scheme based on mode localization of weakly coupled resonators (WCRs) has become an effective way to enhance the sensitivity of MEMS resonant sensors. The mode localization phenomenon can be observed in a nearly symmetric WCRs system, the presence of a small symmetry-breaking perturbation on the structure will lead to a vibration energy confinement [1]. Based on mode localization of the WCRs, ultrahigh sensitive mass sensors [2-3], displacement sensor [4], and stiffness sensors [5-6] have already been demonstrated. Different from the traditional resonant sensors, for which the resonance frequency is chosen as the output metrics, in this work eigenstates or amplitude ratios are chosen as the output metrics. Using mode localization, the sensitivity of WCRs based sensors could be enhanced by orders of magnitude compared to traditional resonant sensors.

How to improve sensitivity of MEMS resonant accelerometers [7] remains as one of the biggest challenges in the MEMS field. So far, research to increase sensitivity focused mainly on the mechanical design, e.g. by adding a leverage mechanism [8] and increasing the length-to-width ratio [9]. The emergence of a transduction scheme based on the phenomenon of mode localization has the potential to provide a novel solution to enhance sensitivity of resonant accelerometers by several orders of magnitude. In this paper,

such a resonant accelerometer is experimentally demonstrated for the first time.

## THEORY AND ANALYSIS

The schematic diagram of the mode localization based resonant accelerometer is shown in Figure 1. Two proof masses (Mass 1 and Mass 2) are suspended by folding beams. Two double ended tuning fork (DETf) resonators are coupled by two bridge-type coupling beams, which forms an ideal symmetric WCRs. The resonators are driven and sensed by comb-fingers based capacitive transduction.

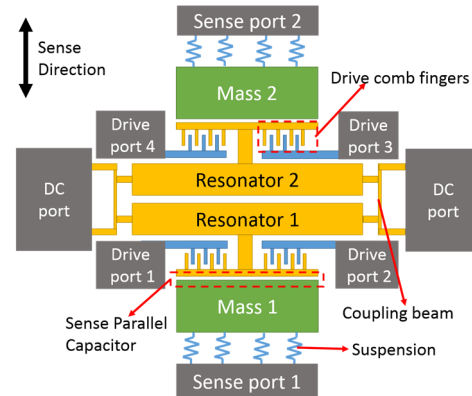


Figure 1: Schematic of the novel resonant accelerometer based on weakly coupled resonators.

The gaps between the proof masses and WCRs are  $g_0$ , across which a potential difference between the proof masses and WCRs is applied; this introduces an electrostatic negative stiffness to the DETf beams. When acceleration acts on the proof mass,  $g_0$  changes and causes the stiffness of the DETf changes which, in turn, causes mode localization, and thus the amplitude ratio of the WCRs changes. The acceleration can be sensed by measuring the variation of the oscillation amplitude ratio. Perturbations acting on the two resonators are isometric and opposite. The block diagram of the WCRs is illustrated in Figure 2.

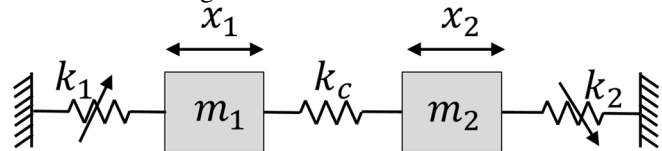


Figure 2: Block diagram of the WCRs.

The mass and stiffness of the two resonators are assumed ideally symmetric:  $m_1 = m_2 = m$ ,  $k_1 = k_2 = k$ , the coupling stiffness is  $k_c$  and the perturbation on resonator 1 is  $\Delta k$ . The dynamic equations of the system without considering damping are given as:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k + k_c + \Delta k & -k_c \\ -k_c & k + k_c - \Delta k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (1)$$

where  $x_1$  and  $x_2$  are the vibration displacements of the two resonators. The eigenvalues (i.e. the resonance frequency) and eigenstates (i.e. the amplitude ratio) can be calculated as:

$$\omega_i^2 = \frac{k + k_c \mp \sqrt{\Delta k^2 + k_c^2}}{m} \quad (i=1,2) \quad (2)$$

$$u_i = \frac{\Delta k \pm \sqrt{\Delta k^2 + k_c^2}}{k_c}$$

The relative shift (sensitivity) based on eigenvalue and eigenstates when there is a stiffness perturbation to any resonator is given as:

$$\left| \frac{\omega_i - \omega_i^0}{\omega_i^0} \right| \cong \frac{k_c \mp \sqrt{\Delta k^2 + k_c^2}}{2k} \quad (3)$$

$$\left| \frac{u_i - u_i^0}{u_i^0} \right| \cong \frac{\Delta k}{k_c} \left( 1 \pm \frac{\Delta k}{2k_c} \right)$$

The relative shift of the resonance frequency and the dimensionless amplitude ratio at the first resonance mode as a function of the stiffness perturbation obtained from numerical simulations are shown in Figure 3. It can be seen that the smaller the coupling factor  $\kappa = k/k_c$  is, the bigger the curve slope is, which means the sensitivity is higher. The sensitivity based on the amplitude ratio is  $1/2\kappa$  times higher than the sensitivity based on the resonance frequency. When perturbation is 0, the amplitude ratio is 1 (or -1); with an increase of the perturbation, the curve splits with a higher slope. The sensitivity is nonlinear over the entire perturbation range, especially for  $\Delta k < k_c$ . Therefore, to assure the linearity of the accelerometer, the working point of the WCRs should be selected to keep the amplitude ratio as far as possible from a value of 1. Due to fabrication tolerances, the WCRs are not ideally symmetric, which leads to an amplitude working point other than 1 in the unperturbed case without special measures.

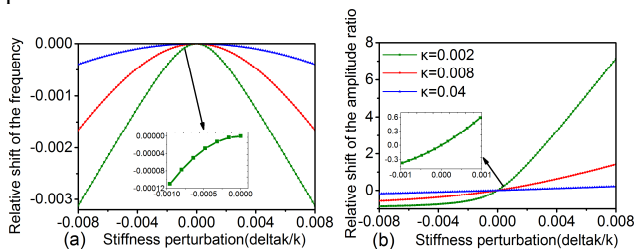


Figure 3: Numerical simulations of the relative shift of the frequency (a) and the dimensionless amplitude ratio (b) as a function of the stiffness perturbation.

The perturbation stiffness caused by the acceleration can be expressed as:

$$\Delta k_a(a) = -\varepsilon AV^2 / \left( g_0 - \frac{m_s a + \varepsilon AV^2 / 2g_0^2}{k_s - \varepsilon AV^2 / g_0^3} \right)^3 \quad (4)$$

Where  $\varepsilon$  is the permittivity,  $A$  is the area of the sense capacitor,  $m_s$  is the mass of the proof mass,  $V$  is the potential difference between the proof mass and WCRs,  $k_s$  is the stiffness suspension, and  $a$  is the acceleration. Therefore, the amplitude ratio as a function of the acceleration can be derived as:

$$u_i = \frac{\Delta k_a(a) \pm \sqrt{\Delta k_a(a)^2 + k_c^2}}{k_c} \quad (5)$$

## DEVICE FABRICATION AND EXPERIMENTAL SETUP

The accelerometer is fabricated in silicon-on-insulator (SOI) technology [10] and packaged in a vacuum environment ( $\sim 20$  mTorr). The fabrication process of the SOI technology, as shown in Figure 4, comprises the following main steps: (a) photoresist deposition and patterning; (b) DRIE etching; (c) removing the photoresist; (d) dicing; and (e) vapor HF release structures.

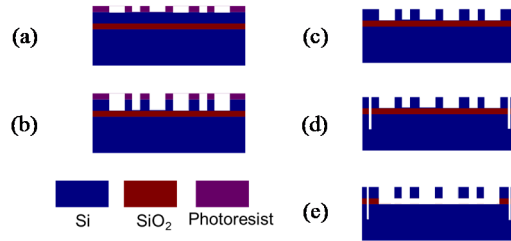


Figure 4: The fabrication process of the SOI technology.

An SEM image of the accelerometer is shown in Figure 5.

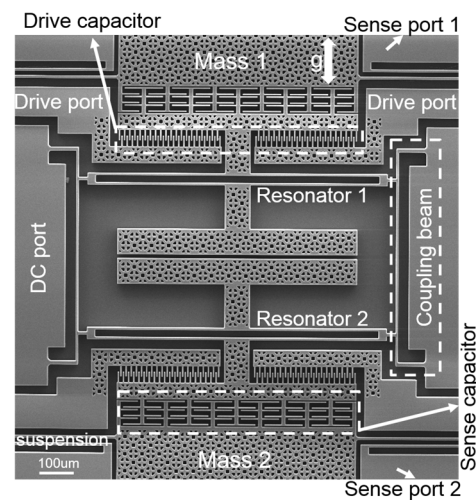


Figure 5: SEM image of the novel accelerometer.

The accelerometer was evaluated using an open loop interface circuit as shown in Figure 6. The proof masses were maintained at 0 V, while the DC voltage applied on the resonators was variable. A 30 mV AC sweep signal was applied on the drive ports to excite the resonators. The signal was amplified by a transimpedance amplifier (with a gain of 1 MOhm) and observed on a dynamic signal analyzer.

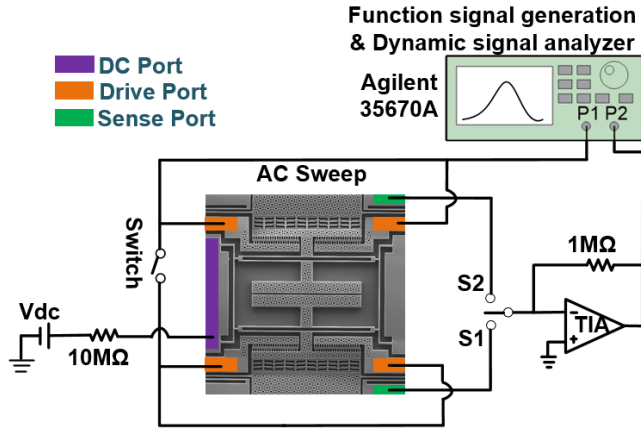


Figure 6: Open loop measurement setup.

## RESULTS AND DISCUSSIONS

The magnitude and phase frequency responses of the WCRs are shown in Figure 7, when DC voltage applied on the WCRs was 10 V. The signal was obtained after feed-through capacitance cancellation [11]. There are two vibration modes (in-phase and out-of-phase) for the WCRs, and the out-of-phase mode is at a lower frequency than the in-phase mode. The resonance frequency of the 1<sup>st</sup> mode is about 27132 Hz, and the 2<sup>nd</sup> mode about 27273 Hz. The frequency difference of the two modes is 141 Hz which means that the coupling factor  $\kappa = k_c/k \approx 0.0026$ .

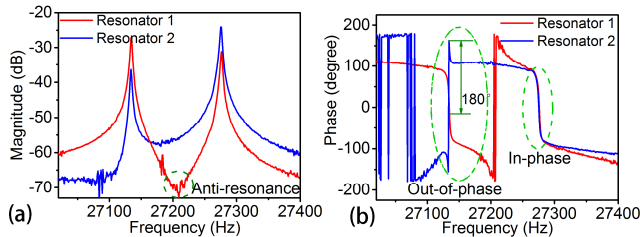


Figure 7: Magnitude (a) and phase frequency (b) responses (log scale) of the two resonators for zero acceleration. ( $Q$ -factor was approximately 9000).

Different from traditional resonant sensors, the output of mode localization based sensors were the eigenstates or amplitude ratio. In this work, the amplitude ratio was chosen as the output metric. The linear magnitude responses of the two resonators as a function of acceleration (Figure 8) was obtained by observing the variation of the amplitude of the

two resonators. It is obvious that the vibration amplitude and frequency of the WCRs are strong functions of acceleration.

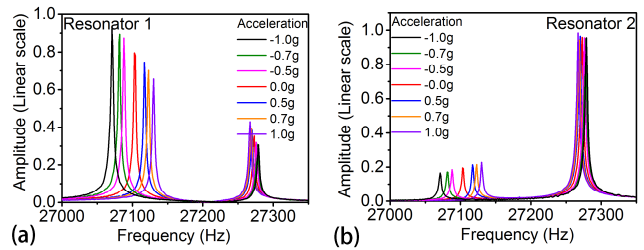


Figure 8: Frequency responses (linear scale) of the two resonators as a function of acceleration.

The amplitude ratio was calculated by dividing the peak value of the frequency response of the two resonators. Ideally, the vibration amplitudes of the two resonators should be identical and amplitude ratio should be 1 for zero acceleration. However, the amplitude ratio at the 1<sup>st</sup> mode was about 4.04 for zero acceleration, i.e. the working point without perturbation was 4.04. The mismatch is attributed to mass and stiffness asymmetry of the two resonators caused by the fabrication tolerances.

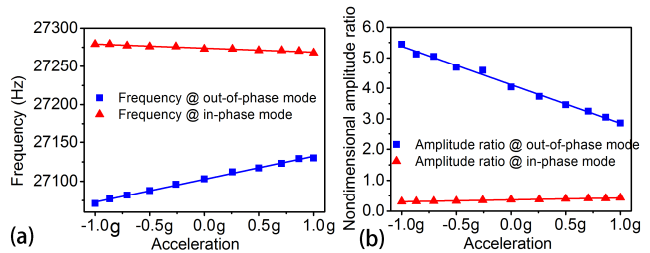


Figure 9: Frequency (a) and amplitude ratio (b) (amplitude of resonator 1 divided by amplitude of resonator 2) variations at the in-phase mode and the out-of-phase mode as a function of acceleration.

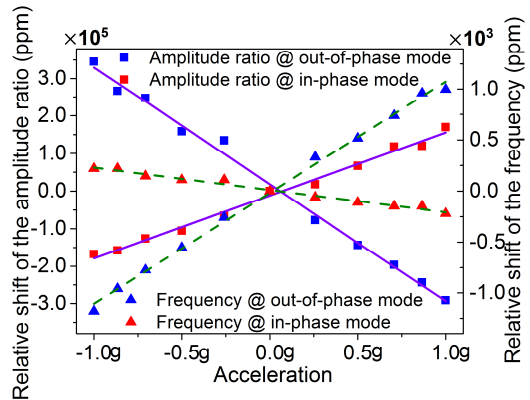


Figure 10: Relative shift of the resonance frequency and amplitude ratio as a function of acceleration. Sensitivity in amplitude ratio was 312162 ppm/g (fitted) while the sensitivity in frequency was 1035 ppm/g for the 1<sup>st</sup> mode.

The dependency of the resonant frequency and the amplitude ratio with acceleration are shown in Figure 9. It can be seen from Figure 9(a) that the sensitivity of the resonance frequency variation is 28 Hz/g for the 1<sup>st</sup> mode, whereas the sensitivity based on the amplitude ratio is 1.26/g (Figure 9(b), the amplitude ratio is dimensionless).

To compare the sensitivity of the amplitude ratio and resonance frequency on the same measurement scale, a relative sensitivity need to be used. As shown in Figure 10, the relative sensitivity of the amplitude ratio is ~312162 ppm/g while that of the resonance frequency is ~1035 ppm/g. This indicates that the sensitivity of the accelerometer based on the amplitude ratio is 302 times higher than the typical frequency sensing principle.

Furthermore, when the voltage applied on the resonators, the stiffness of the WCRs is softened, thus changing the coupling factor, therefore the sensitivity of the accelerometer can be made tunable variable. The sensitivity of the amplitude ratio, when the voltage applied on the WCRs changes from 8 V to 10 V, is shown in Figure 11.

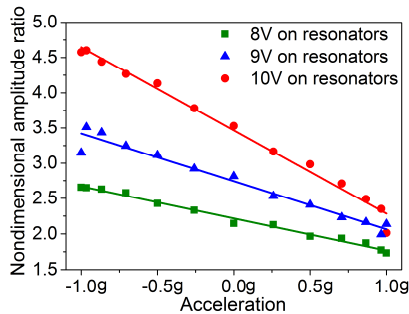


Figure 11: Sensitivity variation at out-of-phase mode as a function of acceleration with different DC voltage applied on the resonators.

## CONCLUSIONS

In this paper, a novel resonant accelerometer based on mode localization of weakly coupled resonators is experimentally demonstrated. The experimental results show that the sensitivity based on the amplitude ratio is 302 times higher than that of the resonance frequency. This is a considerable improvement for resonant inertial sensors. In future work, more detailed specifications of the accelerometer such as the resolution, linear range, linearity and stability will be investigated.

## ACKNOWLEDGEMENTS

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