A MODEL FOR MAGNETIC PLUCKING OF PIEZOELECTRIC BEAMS IN ENERGY HARVESTERS

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ABSTRACT

This paper introduces a calculation model for piezoelectric energy harvesters based on the frequency up-conversion method. Magnetic coupling is used to pluck a beam without physical impact. The piezo bimorph itself is modeled with a fully distributed parameter approach and then combined with a simple inverse square assumption for the magnetic forces. The results are verified experimentally and it is shown that the model is capable of reproducing the effects of various parameters, such as magnet orientation, initial gap between magnets and magnetic force.

KEYWORDS

Energy harvester; piezoelectric; oscillating proof mass; rotational; magnetic coupling

INTRODUCTION

Research interest in various energy harvesting methods has steadily grown over the last five to ten years. The idea of locally generating electrical energy from surrounding sources rather than using mains or battery power is appealing because of the potential reductions in maintenance requirements and ease of installation, e.g. for sensor networks in harsh or inaccessible locations. Developments in the area are being facilitated by the decreasing power consumption of modern electronics. For a thorough overview of the field the reader is referred to [1] and [2].

More specifically, for human body applications, e.g. medical sensors, wrist watches, etc., vibrational energy harvesting is widely believed to be the most promising. To counteract the challenges arising from the low frequency and randomness of human motion, a technique called plucking, frequency up-conversion or impulse excitation has gained a lot of consideration. The idea is to excite a large proof mass at low frequency and let it act upon a transducing element that then vibrates at its optimal frequency of operation. A popular method is to pluck a piezoelectric beam and let it ring down at its natural frequency, which improves the electromechanical coupling. Devices presented so far include a cylindrical harvester in the shape of a common battery using magnetic actuation [3]. A direct force knee joint energy harvester working with rotation is described in [4]. An impact device can be found in [5] and a number of designs are given in [6]. On the micro-scale, a piezoelectric device was introduced in [7].

Our inertial energy harvester (figure 1) has been presented previously in [8]. The basic principle is illustrated in figure 2. A rotor with an eccentric proof mass oscillates under external excitation. The tip of the fixed piezoelectric element and the rotor carry permanent magnets. When these two magnets pass each other the tip of the beam gets plucked and the piezo element freely vibrates. The advantage of this magnetic coupling is the absence of physical impact that could damage the brittle piezo material.



Figure 1: Piezoelectric rotational energy harvester

A design with magnetic coupling can be found in [9], operating under linear proof mass motion. In terms of rotional devices, a piezoelectric windmill was presented in [10]. For human motion, a rotating proof mass already proved to be advantageous in an electromagnetic device [11].



Figure 2: Basic arrangement and principle of the rotational beam plucking energy harvester

The aim of this paper is to present an experimentally verified model, to calculate the generated voltage and the tip displacement of a magnetically plucked piezoelectric beam, and to aid in design optimisation.

MAGNETIC PLUCKING MODEL

The arrangement that was considered in this study is depicted in figure 3. The piezoelectric bimorph beam carries a magnet that acts as a tip mass m_1 and the motion of the proof mass and driving magnet m_2 follows a prescribed motion with the vertical gap between the two being h. A simpler model, reducing the beam to a mass-spring-damper system, was presented in [8] and the

magnetic interaction force in *x*-direction, assuming an inverse square relationship with the distance between the magnets, was found to be:

$$F_{magx} = F_0 h^2 \frac{(x_2 - x_1)}{(h^2 + (x_2 - x_1)^2)^{3/2}}$$
(1)



Figure 3: Model of the magnetic coupling between the beam and driving rotor

The present version includes the distributed parameter solution for a bimorph piezoelectric beam described in [12] to calculate the generated voltage and tip displacement of the beam. This approach uses Euler-Bernoulli beam theory to determine the stress in the piezoelectric layers in the bimorph of figure 3 when it is bent. The vibration response relative to the base at each point along the beam can be represented by a convergent series with a spatial component $\phi_r(y)$, essentially describing the shape of each mode of vibration, and a time component $\eta_r(t)$:

$$w_{rel}(y,t) = \sum_{r=1}^{\infty} \phi_r(y) \eta_r(t)$$
(2)

Accordingly, the equivalent equation for the modal mechanical coordinate $\eta_r(t)$ is:

$$\frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r\omega_r\frac{d\eta_r(t)}{dt} + \omega_r\eta_r(t) - \widetilde{\theta}_r\nu(t) = F_r(3)$$

Given that a piezoelectric material reacts to an external stress by generating a voltage (and vice versa), a second equation for the voltage response v(t) is needed:

$$C_{\widetilde{p}}^{eq} \frac{dv(t)}{dt} + \frac{v(t)}{R_L} + \sum_{r=1}^{\infty} \widetilde{\theta}_r \frac{d\eta_r(t)}{dt} = \mathbf{0}$$
(4)

In these equations ζ_r is the modal mechanical damping, ω_r the modal resonance frequency, F_r the modal external force, $C_{\tilde{p}}^{eq}$ the equivalent capacitance of a bimorph piezoelectric beam and R_L the load resistance. The modal electromechanical coupling term $\tilde{\theta}_r$ is a constant that represents the coupling between the mechanical and the electrical domain in equations (3) and (4). It depends upon material and geometric parameters and the eigenfunctions $\phi_r(y)$.

The calculations and measurements are based on a bimorph piezoelectric beam with dimensions $60 \times 5 \times 0.5$ mm with an attached resistive load of $120 \text{ k}\Omega$ and a capacitance of 24.6 nF. It is important to note that this model also holds true for micro-scale devices.

The experiments were performed on a piezoelectric beam clamped between two layers of perspex and a pair of $5 \times 5 \times 1$ mm N52 type NdFeB magnets were used for the magnetic coupling. The driving magnet was swept past the tip magnet via a perspex rod mounted on a stepper motor at 2 rpm continuous rotation. This explains slight differences between the measurement results and the simulations, the latter being calculated with prescribed sinusoidal motion of 2 Hz and 10 mm amplitude. In preliminary investigations it proved sufficient to solve equations (3) and (4) up to the third vibration mode only. When plucked at the tip, the first mode of vibration of a beam is already by orders of magnitude more dominant than the second and third ones.

RESULTS

The sinusoidal motion of the driving magnet and the corresponding calculated tip displacement of the piezoelectric beam for attractive and repulsive arrangements of the magnets respectively are shown in figures 4 and 5. The initial gap is h = 2 mm. The repulsive arrangement results in a pronounced single plucking of the beam whereas the attractive set-up shows two separate actuations. In the latter case the tip of the beam gets accelerated towards the approaching magnet and oscillates at a higher frequency while travelling along with the driving magnet before finally getting released.



Figure 4: Simulated driving magnet and beam tip displacement at 2 Hz and 2 mm gap for an attractive magnet arrangement



Figure 5: Simulated driving magnet and beam tip displacement at 2 Hz and 2 mm gap for a repulsive magnet arrangement



Figure 6: Simulated voltage at 2 Hz and 2 mm gap for an attractive magnet arrangement



Figure 7: Simulated voltage at 2 Hz and 2 mm gap for a repulsive magnet arrangement

This becomes more obvious when looking at the corresponding voltage outputs in figures 6 and 7. The attractive arrangement shows a higher frequency oscillation after the catch phase and before release. This is due to a stiffening spring effect caused by the interaction between the magnets. Furthermore, repulsive magnets lead to a higher voltage due to a larger initial deflection of the beam tip and generally present a 'cleaner' plucking that could make impedance matching with the load, for maximum power output, easier.

The experimental equivalents of figures 6 and 7 are given in figures 8 and 9. The measured voltage across the piezo beam supports the predictions from the simulation in all qualitative aspects. The voltage is higher for a repulsive magnet arrangement and it presents a clean decaying oscillation compared to the distinct catch and release phases of the attractive arrangement. As stated above, the experimental set-up only allowed continuous rotation of 2 rpm, which is why the graphs only show two actuations instead of four. Also, the beam actuations in the simulated voltage curves clearly show an alternating pattern resulting from the sinusoidal input. With a high number of parameters influencing the calculations, such as material and piezoelectric constants, mechanical damping and initial magnetic force assumptions, the slight differences in absolute voltage compared to the measurements are not surprising.



Figure 8: Measured voltage at 2 rpm and 2 mm gap for attractive magnets



Figure 9: Measured voltage at 2 rpm and 2 mm gap for repulsive magnets

The model being established, it can now be used to investigate the influence of different parameters on the plucking of the beam and the resulting voltage. Figures 10 and 11 show the effect of an initial gap increased to h = 4 mm (double the previous value) for the attractive magnet arrangement. Consistent with the inverse square assumption for the magnetic coupling, the initial magnetic force was accordingly divided by a factor of 4. The influence of the gap size is quite clear - in comparison to figure 4, the beam only experiences a very gradual deflection and never vibrates at its natural frequency. This



Figure 10: Simulated driving magnet and beam tip displacement at 2 Hz and 4 mm gap for an attractive magnet arrangement

is unwanted as it will result in a lower power output of the device and negatively affect the operational frequency bandwidth. As can be seen in figure 11, the achieved voltage is also much lower.

Again, despite the measurement noise, the experimental voltage output in figure 12 shows good agreement with the predicted result from figure 11, taking the previously mentioned differences in the set-up into account.



Figure 11: Simulated voltage at 2 Hz and 4 mm gap for an attractive magnet arrangement



Figure 12: Measured voltage at 2 rpm and 4 mm gap for an attractive magnet arrangement

CONCLUSIONS

In this paper a calculation model for magnetic beam plucking in energy harvesters is presented. Magnetic actuation of piezoelectric beams has the advantage of being contactless and thus avoiding damage to the brittle piezo material. However, the parameters, most importantly initial gap and magnetic force, need to be well chosen to achieve good results. The model makes use of the distributed parameter solution for piezo bimorphs introduced in [12] and combines it with a simple expression for the magnetic interaction force following an inverse square law with the distance between magnets. Despite this slightly crude approximation, the comparison between experiment and calculation shows good agreement. The effects of magnet orientation (attractive or repulsive) and increased gap size were well predicted. This opens the opportunity for future investigations to further optimize and fully understand magnetic plucking of piezoelectric beams. For instance, the calculations could easily be adjusted to provide average power output into an electrical load under different excitations.

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