COMPARISON OF LONG-TERM STABILITY OF AM VERSUS FM GYROSCOPES

Burak Eminoglu[1], *Yu-Ching Yeh*[1], *Igor I. Izyumin*[1], *Isaac Nacita*[2], *Mark Wireman*[2], *Adam Reinelt*[2], *and Bernhard E. Boser*[1]
[1]University of California, Berkeley, California, USA
[2]746th Test Squadron, Holloman AFB, New Mexico, USA

ABSTRACT

A vibrating ring gyroscope is operated in AM and FM mode to demonstrate improved long-term stability of FM operation. Both bias and scale-factor (SF) accuracy were tested over a one-month period with devices power cycled between measurements. Bias and SF improve by more than two orders-of-magnitude from 335mdps to 2.86mdps and from 518ppm to 4.31ppm in the AM and FM modes, respectively. The temperature coefficients decrease from over 700dph/°C to less than 3dph/°C for the bias and 105ppm/°C to 1.25ppm/°C for the SF. The in-run bias improves from 19dph to less than 1.5dph. AM and FM tests use the same transducer with no additional trimming, demonstrating the fundamental advantage of FM over AM readout.

INTRODUCTION

All sensors measure their input relative to a reference. Conventional AM gyroscopes measure rate indirectly via the Coriolis force and synthesize the reference implicitly from a combination of transducer dynamics and readout circuit gain [1]. By contrast, FM gyroscopes measure rate directly as a frequency and use an external precision reference clock [2, 3]. This feature of FM gyroscopes provides good long-term stability and high performance without trimming and sophisticated calibration techniques.

FM gyroscopes can be operated in different modes to meet varying performance requirements. Quadrature FM operation (QFM) provides high dynamic range and wide bandwidth [2]. However, its bias is subject to the transducer resonant frequencies which exhibit a strong temperature dependence resulting in poor long-term stability.

Lissajous FM operation (LFM) eliminates this dependency by modulating the rate signal to a frequency that equals the mode-mismatch of the transducer. Better than 10ppm scale factor stability and 10dph bias stability over a 2-day period using this approach are described in [3].

This work introduces indexed FM operation (IFM) to further improve long-term stability thanks to reduced sensitivity of the oscillation amplitude to rate. In this operating mode the x- and y-modes of a z-axis gyroscope are operated with a phase shift that alternates between +90° and -90°, corresponding to circular clock and anti-clockwise orbits. Taking the difference between the rate measured in the two states rejects long-term variation of the transducer resonant frequencies. Additionally, the symmetry of the sensor and controller suppresses damping related errors and result in 2.81mdps ZRO and 4.31ppm SF repeatability over a 1-month period with power cycled between measurements, a more than two orders-of-magnitude improvement over the performance of the same sensor when operated in AM mode.

FM Gyroscope Operation

MEMS gyroscopes can be modeled as 2-DOF resonators coupled by Coriolis force. AM gyroscopes excite one of the modes, referred to as driven mode, at its resonant frequency at constant amplitude and infer rate from the amplitude of the orthogonal sense mode.

By contrast, in FM gyroscopes both modes are driven at equal amplitude at their respective frequencies and rate is inferred from the change of the free running oscillation frequency. In QFM operation, a controller uses an appropriate tuning means to adjust the phase difference between the x- and y-modes to a constant +90° or -90°. In presence of a rotation signal, the proof mass retains its angular momentum in the inertial frame resulting in a change of the oscillation frequency.

For an ideal Foucault pendulum, a rotation of 360°/s results in a 1Hz frequency shift in the oscillation frequency, corresponding to a unity SF. In practical transducers the scale factor is reduced by the angular gain *k*, which is set by geometry and exhibits very low sensitivity to environmental parameters, typically at the sub-ppm level, translating into excellent SF stability [4].

By contrast, the ZRO is set by the nominal resonant frequency which has a large temperature coefficient (approximately -30ppm/°C for Silicon). Alternating the orbit between clock and counter-clock wise oscillations as shown in Figure 1 up-converts the rate signal to the modulation frequency where it can be separated from long-term drift phenomena by electronic filtering.

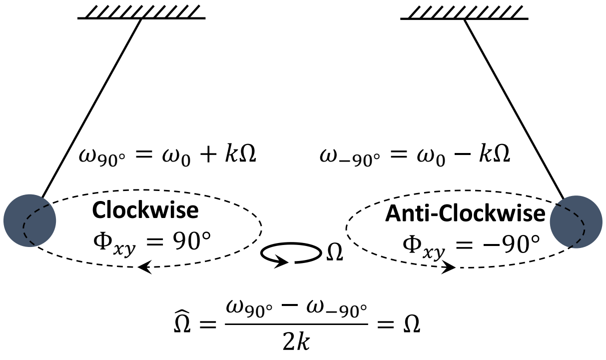


Figure 1: FM gyroscope with phase indexing.

Comparison of AM and FM Gyroscope Errors

Table 1 compares the outputs from AM and FM gyroscopes, using the terminology from Lynch's error model [1]. The quadrature error is omitted for brevity and cross-axis damping terms are assumed to be equal. Equations (2) and (3) for QFM and IFM operation differ

978-1-5090-1973-1/16/$31.00 ©2016 IEEE

primarily by the suppression of the resonant frequency ω in the latter.

The SF of the AM gyroscope is a complex function of several design parameters including proof mass velocity $\dot{x}$ and electro-mechanical transduction η. By contrast, the SF of the FM gyroscopes equals the angular gain k modified by a term with the normalized mismatch Δv of the x- and y-mode oscillation velocities squared. For 1ppm SF stability the velocities (and hence amplitudes) must exhibit less than 0.14% variation, a relatively modest requirement. By contrast, to meet the same requirement with an AM gyroscope, each parameter contributing to its SF must be controlled to a fraction of 1ppm absolute accuracy, a requirement that is nearly impossible to meet with practical implementations.

Table 1: Gyroscope error analysis for (1) force-rebalance AM (voltage readout), (2) QFM, and (3) IFM operation.

$\hat{V}_y = \underbrace{\frac{2mk\dot{x}}{\eta}}_{\text{scale factor}} \left\{\Omega + \underbrace{\frac{1}{2k}\Delta\left(\frac{1}{\tau}\right)\sin 2\theta_\tau}_{\text{bias}}\right\}$	(1)
$\omega_{\pm 90^o} = \pm k\left(1+\frac{\Delta v^2}{2}\right)\left\{\Omega + \frac{\Delta v}{2k}\Delta\left(\frac{1}{\tau}\right)\sin 2\theta_\tau\right\} + \omega$	(2)
$\frac{\Delta\omega_{\pm 90^o}}{2} = \underbrace{k\left(1+\frac{\Delta v^2}{2}\right)}_{\text{scale factor}}\left\{\Omega + \underbrace{\frac{\Delta v}{2k}\Delta\left(\frac{1}{\tau}\right)\sin 2\theta_\tau}_{\text{bias}}\right\}$	(3)

Similarly, the bias from aniso-damping is attenuated by the same factor Δv, translating into an at least two orders-of-magnitude advantage of FM over AM operation. The quadrature error is removed with synchronous demodulation. The symmetrical design of the FM readout facilitates more accurate phase matching compared to AM gyroscopes [5], translating into reduced quadrature leakage.

IFM Operation

Mode reversal has been proposed in AM gyroscopes as a means to reduce long-term drift [6]. In this operating mode, the roles of the drive and sense axes are periodically interchanged. Unfortunately, this solution is difficult to realize especially with high-Q transducers, as are generally required for high performance, limiting the reversal rate to a fraction of the transducer bandwidth.

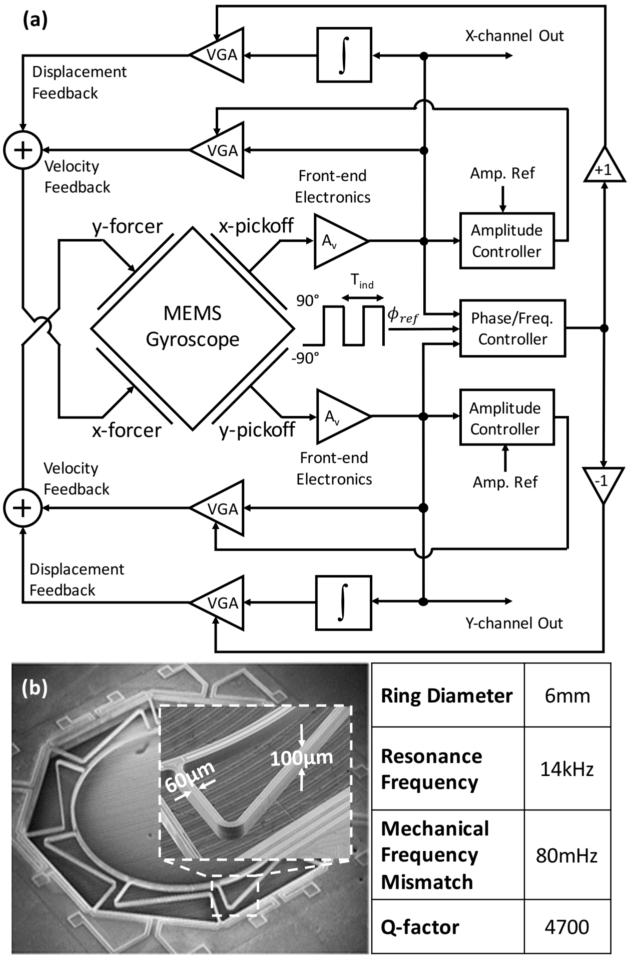
IFM operation avoids this tradeoff between resonator Q and modulation frequency. Since indexing only changes the relative phase of the x- and y-mode oscillations but does not alter the amplitude and hence energy in each mode, the modulation rate is set by the frequency tuning range Δf_{xy}, which is independent of resonator bandwidth and for example in transducers utilizing capacitive transduction can be very large. The transient duration in this case is the time required to change the relative phase between +90° and -90° and equals $1/2\Delta f_{xy}$ plus the settling time of the phase control loop which in practice equals a few oscillation periods.

Experimental Characterization

AM and FM performance was compared with a commercial gyroscope transducer, which is inherently mode-matched with a frequency split less than 0.1Hz [7]. The device was tested in the facilities of United States Air Force 746th Test Squadron.

Figure 2 shows the block diagram and transducer parameters, and Figure 3 shows the complete system. The front-end electronics and controllers are implemented with discrete components, while a custom frequency-to-digital converter ASIC is used to measure the IFM frequency [8]. A 10MHz temperature compensated crystal oscillator (TCXO) with a frequency stability of ±0.1ppm between 0°C to 70°C [9] supplies the reference to the frequency-to-digital converter (FDC), ensuring good scale factor accuracy.

The tested device uses magnetic transduction and lacks a means to tune the mechanical resonant frequencies. Phase shifts instead are generated by the controller with forces that are in-phase with displacement [10]. The ratio of these forces to the oscillation amplitude mimics a spring constant and can thus be used to tune the oscillators. Due to limitations of the maximum tuning force that can be realized by this method, the tuning range Δf_{xy} is limited to 1Hz.



Ring Diameter	6mm
Resonance Frequency	14kHz
Mechanical Frequency Mismatch	80mHz
Q-factor	4700

Figure 2: (a) Block diagram of IFM controller, (b) MEMS gyroscope with device parameters [11].

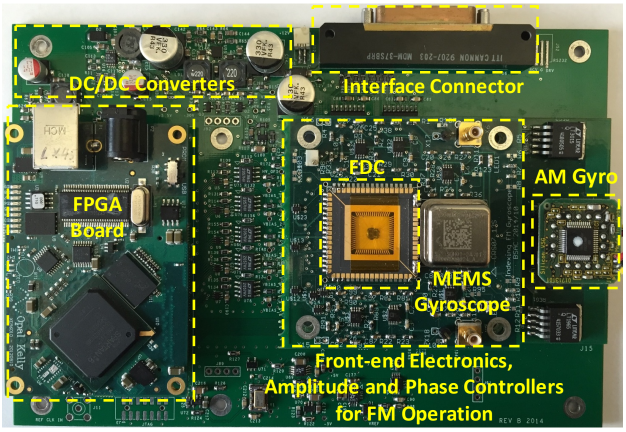


Figure 3: Board photo of the tested device.

Figure 4 shows the in-run Allan-Variance for the AM and FM operations. Compared to AM operation, FM operation reduces the bias as expected, but indexing results in elevated ARW. This is caused by rate noise above the 0.8Hz indexing rate folding into the signal-band. A transducer with higher tuning range f_{xy} or a combination of two or more sensors can be used to eliminate this penalty.

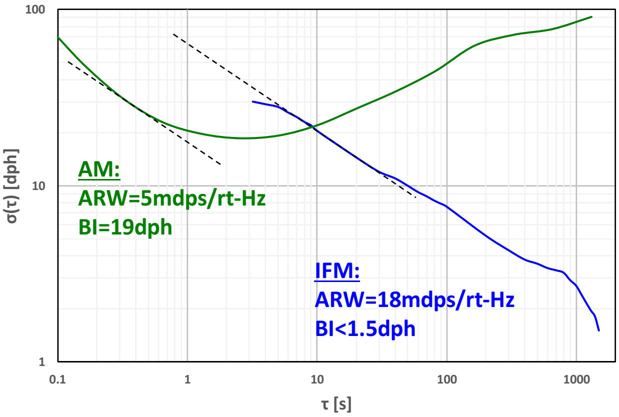


Figure 4: Allan variance plots for AM and IFM.

Figure 5 and 6 show the measured bias and SF error over a 30-day period during which power was switched off between measurements. Both results demonstrate a more than two orders-of-magnitude improvement of FM over AM operation of the same transducer. In SF measurements, FM gyroscope was tested up to ±600dps, which is the maximum rate of the rate table. FM gyroscopes have inherently much higher full-scale range. In the current system, the limitation for the full-scale SF is coming from the current FDC, which is able to measure ±1000dps.

Temperature sensitivity is lowered by a similar factor, as demonstrated by the measurement results shown in Figure 7 and 8. The TC of the SF in FM operating mode is as 1.25ppm/°C. This value is close to the TC of the angular gain of HRGs, which reported in [4] as 0.5ppm/°C. These results show that FM gyroscopes achieve similar long-term SF stability without sophisticated calibration or trimming and are thus promising for low-cost high performance gyroscopes.

Table 2 summarizes the results, indicating the performance advantage of FM versus AM operation.

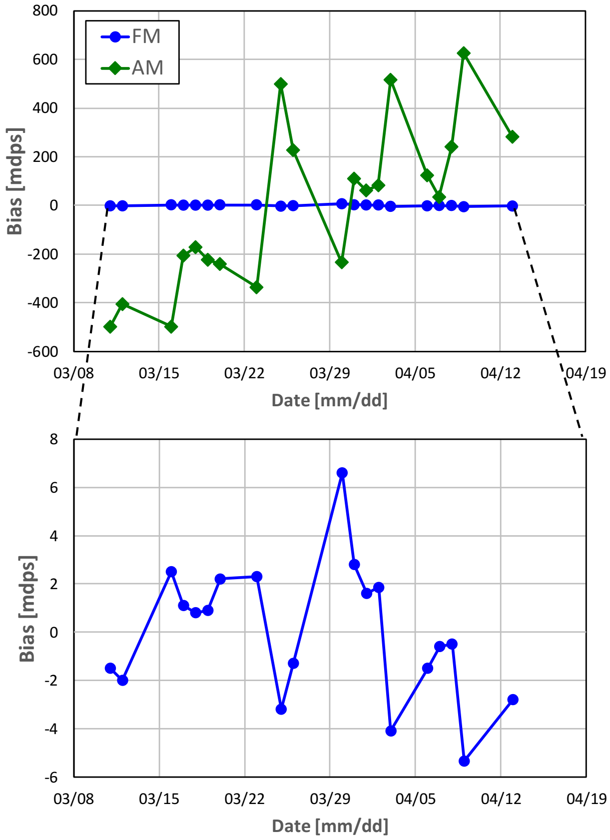


Figure 5: 1-month turn on bias repeatability results.

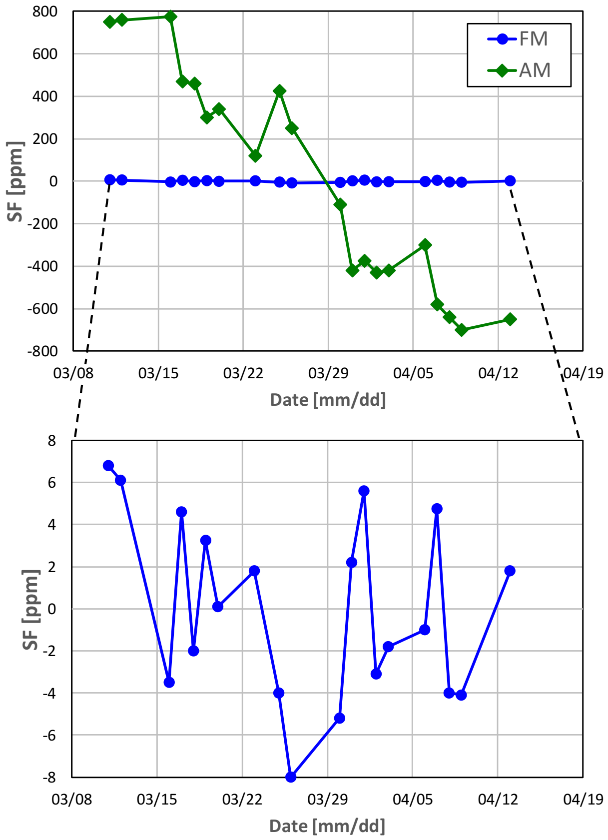


Figure 6: 1-month turn on SF repeatability results.

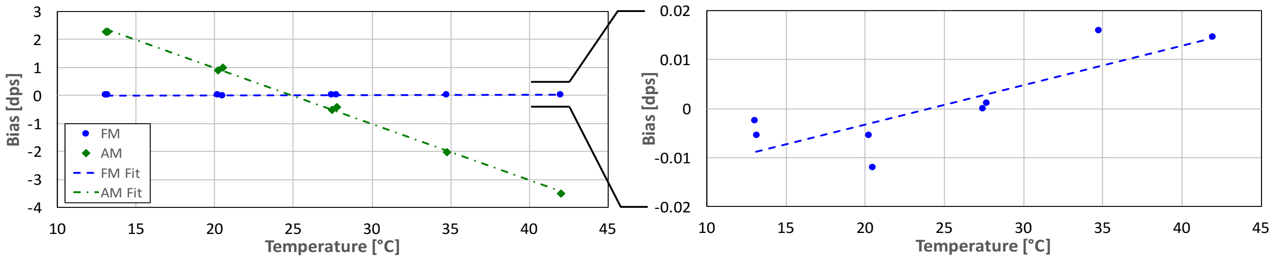


Figure 7: Temperature sensitivity of bias.

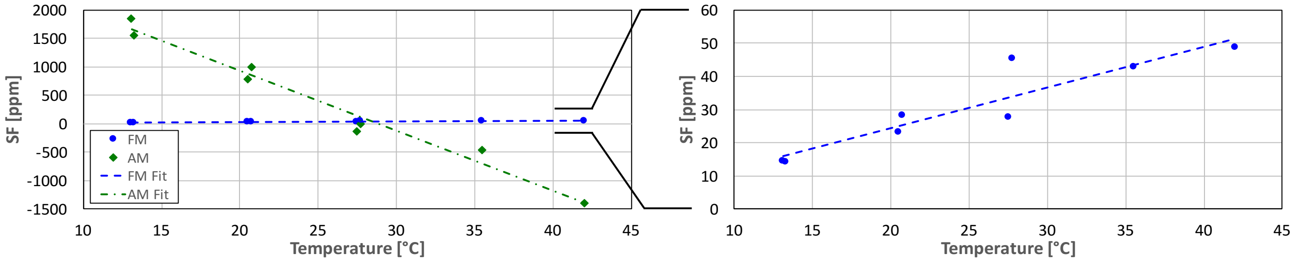


Figure 8: Temperature sensitivity of SF.

Table 2: Performance summary.

	AM	FM	Improvement
Bias(1σ)	335mdps	2.86mdps	117x
SF(1σ)	518ppm	4.31ppm	120x
TC Bias	711dph/°C	2.9dph/°C	245x
TC SF	105ppm/°C	1.25ppm/°C	84x
In-run Bias	19dph	<1.5dph	13x

CONCLUSIONS

Conventional AM gyroscopes measure rate indirectly via the amplitude of vibration of the sense channel. As a consequence, the scale factor depends on several fabrication parameters including electro-mechanical transduction and absolute velocity of the drive axis. Each of these parameters must be controlled to a fraction of the desired scale-factor accuracy, a significant barrier to achieving better than 100ppm stability.

FM gyroscopes, by contrast, detect rate in the form of a frequency change which can be measured with high accuracy by using a crystal or other accurate clock as reference, readily enabling sub 10ppm accuracy without trimming or calibration.

Indexed FM gyroscopes benefit from a similar reduction of zero-rate output. Unlike AM gyroscopes using mode-reversal to improve bias stability which requires a tradeoff between transducer Q and reversal rate, the indexing rate of FM gyroscopes only depends on the frequency tuning range and is independent of the transducer quality factor. FM operation can therefore take full advantage of improvements resulting in higher transducer Q and correspondingly higher performance.

ACKNOWLEDGEMENTS

The authors would like to thank Mitchell Kline for consultation on FM gyroscopes and Oleg Izyumin for assistance with board layout and fabrication. This material is based upon work supported by the Defense Advanced Research Projects Agency (DARPA) under Contract No. W31P4Q-12-1-0001.

REFERENCES

[1] D. D. Lynch, “Vibratory gyro analysis by the method of averaging,” *in 2nd Saint Petersburg Int. Conf. gyroscopic technol. and navigation I*, May 1995, pp.26-34.

[2] M. Kline, Y. Yeh, B. Eminoglu, H. Najar, M. Daneman, D. Horsley, and B. E. Boser, “Quadrature FM gyroscope,” *in IEEE MEMS 2013*, pp. 604–608.

[3] I. I. Izyumin, M. Kline, Y.-C. Yeh, B. Eminoglu, and B. E. Boser, “A 7ppm, 6deg/hr frequency output MEMS gyroscope,” *in IEEE MEMS 2015*, pp.33-36.

[4] D. M. Rozelle, “The hemispherical resonator gyro: From wineglass to the planets”, in *Proc. AAS/AIAA Space Flight Mechanics Meeting*, Feb. 2009, pp. 1157–1178.

[5] C. D. Ezekwe, W. Geiger, and T. Ohms, “A 3-Axis Open-Loop Gyroscope with Demodulation Phase Error Correction,” *in ISSCC 2015*, pp.478-479.

[6] S. F. Wyse and D. D. Lynch, “Vibrating MEMS Gyroscope and Method for Minimizing Bias Errors Therein,” U.S. Patent 7,188,525 B2, Mar. 13, 2007.

[7] Datasheet for CRS07: Angular Rate Sensor [Online]. Available: http://www.siliconsensing.com/media/30685/DocNo-CRS07-00-0100-132-Rev-1PDF.pdf

[8] I. Izyumin, M. Kline, Y.-C. Yeh, B. Eminoglu, and B. E. Boser, “A 50 µW, 2.1 mdeg/s/√ Hz frequency-to-digital converter for frequency-output MEMS gyroscopes,” *in ESSCIRC 2014*, pp. 399–402.

[9] Datasheet for M100: High Precision TCXO [Online]. Available: http://www.conwin.com/datasheets/tx/tx382.pdf

[10] B. Eminoglu, M. H. Kline, I. Izyumin, Y. Yeh, and B. E. Boser, “Ratiometric readout technique for MEMS gyroscopes with force feedback,” *in IEEE/ION PLANS 2014*, pp.1203-1208.

[11] S. Clarke, (2013, December 11). *Inertial technology for north finding* [Online]. Available: http://www.siliconsensing.com/media/310275/sssl-inertial-technology-for-north-finding-11-dec-2013.pdf

CONTACT

*B. Eminoglu; eminoglu@eecs.berkeley.edu