PARAMETRIC AMPLIFICATION/DAMPING IN MEMS GYROSCOPES

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ABSTRACT

The attainable resolution of inertial sensors is ultimately limited by the cumulated noise level generated in both the mechanical domain (mechano-thermal noise) and the frontend of the electrical readout circuit, provided that deterministic errors, such as quadrature errors in the case of gyroscopes, are kept under control. Improving the resolution performance of MEMS structures mounts to being able to either increase the minimum detectable signal through an increased sensitivity, or to improve the signal-to-noise ratio (SNR). This paper reports on parametric amplification and damping employed in a MEMS gyroscope. Experiments confirm that parametric modulation through electro-mechanical coupling leads to both an increase spectral selectivity and a reduction of the equivalent input noise angular rate (from $0.046\,\deg/\left(\sec\sqrt{Hz}\right)$ to $0.0026\,\deg\big/\left(\sec\cdot\sqrt{Hz}\right)$ for a parametric gain of 5). In a more general analysis of a MEMS resonant structure, electro-mechanical parametric amplification decreases the mechano-thermal noise associated with the mode motion - the equivalent input noise acceleration was diminished from $0.033 m\cdot s^{-2}$ to $0.022 m\cdot s^{-2}$ for a parametric gain of 5. Both signal amplification and an attenuation of undesired signal components can be achieved by tuning the phase difference between the driving force and the parametric coupling. Therefore, the technique can be applied to reduce the quadrature error signal, which strongly constrains the maximum gain of the sensing circuit.

INTRODUCTION

Whilst there are several theoretical research papers describing the phenomenon of parametric resonance, there is limited reporting on its application to MEMS/NEMS devices. The seminal work reported in [1] demonstrated parametric amplification with a linear gain of at least 25 in micro-cantilevers by using electrostatic excitation and optical detection. Parametric amplification techniques, applied to a MEMS magnetometer, led to a magnetic field sensitivity increase of 51 times [2]. Honeywell patented the idea of implementing parametric amplification in MEMS gyroscope in 2002, where the external AC signal pumps energy at twice the resonant frequency into the sense-electrodes [3]. The feasibility of the concept was further analyzed in [4] for the case of MEMS tuning gyroscopes

Parametric amplification in ring MEMS gyroscopes is reported in [5]. Recent experiments [6] show that by using a parametric resonance based actuator, the driven mode mechanical response signal has a rich dynamic behaviour, with an amplified response within a large bandwidth (1kHz). This is advantageous for resonant MEMS gyroscopes, where the Coriolis force couples two orthogonal resonant modes that might have distinct but close resonant frequencies, influenced as well by the applied bias voltages.

The general structure of a MEMS vibratory gyroscope uses area-varying interdigitated capacitances for the voltage actuation of the driven mode. This configuration enables linear forces and large displacements (typically around 10µm), at the expense of higher driving voltages. The Coriolis-induced displacements in the sense mode are orders of magnitude smaller, and it is therefore critical to ensure that they are above the sensitivity threshold of the readout circuit and the mechano-thermal noise level. The displacement detection in the sense mode uses gap-varying capacitances, in order to achieve maximum sensitivity.

Present paper focuses on the analysis of parametric pumping in the sense mode, using a common mode voltage applied to gap-varying comb drives.

MEMS GYROSCOPE STRUCTURE

A proof mass suspended by crab leg fixtures ensures the two orthogonal degrees of freedom (driven/sense) of the MEMS gyroscope. The structure is implemented in SOIMUMPS technology, with a minimum gap size of 2µm and a thickness of 25µm for the structural layer.

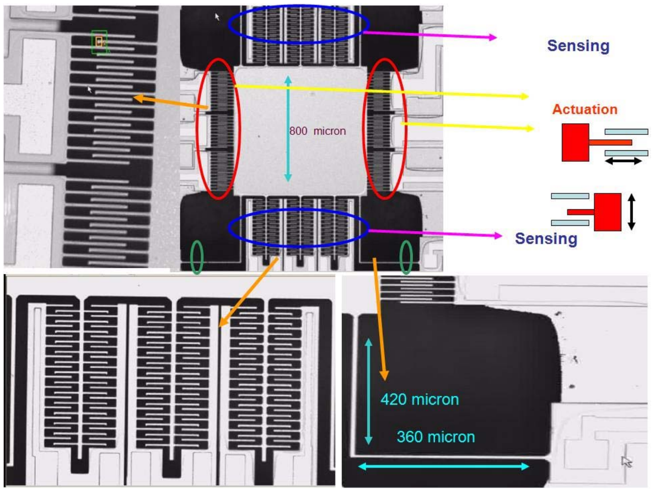

Figure 1: Gyroscope fabricated in SOIMUMPS (25µm) technology. Red/Blue markers-Actuation/Sensing combs.

Figure 1 shows distinct sets of actuating and sensing comb fingers. The horizontal actuation (0x) is achieved by applying AC voltages between the moving and the fixed fingers of the actuators (with a 5µm gap) and a DC voltage to the proof mass. When an external angular rate Ω_z rotates the sensor about 0z axis, the resultant Coriolis force will induce a proportional motion in the sense mode (0y). The 0y displacement is measured using asymmetric comb drives, with gaps of 6µm and 9µm, intentionally left large to facilitate the parametric amplification experiments. The dynamic behavior of the structure obeys the differential equations (1) and (2)

$$m\ddot{x}+c_x\dot{x}+(k_{yx}+k_x)x=F_{actuation}+F_{noise,x}(c_x,t)+2m\dot{y}\Omega_z \tag{1}$$

$$m\ddot{y}+c_y\dot{y}+(k_{xy}+k_y)y=-2m\dot{x}\Omega_z+F_{noise,y}(c_y,t) \tag{2}$$

Here k_x and k_y are the equivalent stiffness coefficients in x and y direction, c_x and c_y are the damping coefficient in x and y direction, and $m=m_x=m_y$ is the equivalent mass of the movable part. k_{xy} and k_{yx} are the cross coupling coefficient, which occurs due to imperfections in fabrication. $F_{actuation}$ is the electrostatic force that puts the gyroscope in motion in the x direction and is directly proportional to the square of voltage applied. The Coriolis force $F_{c,y}=-2m\dot{x}\cdot\Omega_z$ induces the motion in the sense (0y) direction – the equivalent effect in the drive mode is negligible relative to the magnitude of the electrostatic actuation. The noise forces in equations (1) and (2) are correlated with the respective damping coefficients, to establish a thermodynamic equilibrium between the microstructure and the surrounding fluid.

Noise in MEMS gyroscope

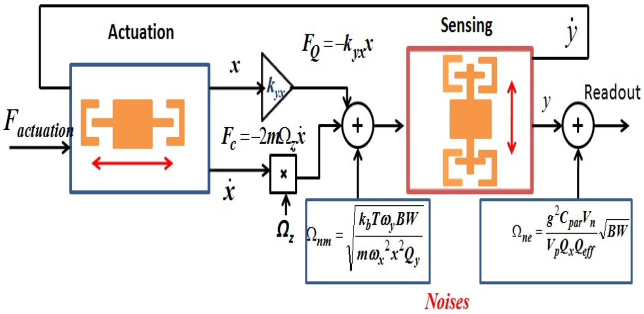

Figure 2: System level block diagram. Coriolis and quadrature forces induce motion in the sense mode. Mechano-thermal (Ω_{nm}) and electronic noise (Ω_{ne}) components are illustrated.

Mechanical sensitivity of the gyroscope can be expressed as sense mode displacement per unit angular rate, $Y(\omega)/\Omega_z$. The global noise limits the achievable sensitivity – the total noise equivalent rotation (Ω_{TNE}) is the sum of two uncorrelated components: the mechanical (or Brownian) noise equivalent rotation (Ω_{nm}) and the electronic noise equivalent rotation (Ω_{ne}): $\Omega_{TNE}^2=\Omega_{nm}^2+\Omega_{ne}^2$. Individual noise components are given by eqns. 3.a, 3.b:

$$\Omega_{nm}=\sqrt{\frac{k_BT\omega_yBW}{m\omega_x^2x^2Q_y}} \tag{3a}$$

$$\Omega_{ne}=\frac{d^2C_{par}V_n}{V_pQ_xQ_{eff}}\sqrt{BW} \tag{3b}$$

Here, k_B is the Boltzmann constant, d is the gap of comb fingers at rest, V_n is the white noise voltage, $C_{parasitic}$ is the net capacitance of bond pads, wires etc. V_p is the voltage applied to the substrate and Q_{eff} is the effective quality factor. One assumes that the structure is driven at the resonance frequency in 0x (driven mode). Technology and layout imperfections result in a supplementary, undesired quadrature component of the 0y – the net effect is modeled as an equivalent quadrature force $F_Q=-k_{yx}x$. The typical quadrature forces are much larger than the Coriolis forces ($F_c=-2m\Omega_z\dot{x}$), and with a 90^0 phase shift (essential for being able to reduce them through parametric damping). Errors due to quadrature coupling are deterministic, unlike the electronic and mechano-thermal noises. The proposed parametric electro-mechanical technique can be used for amplifying the Coriolis coupling and damping the undesired quadrature coupling. This enhances the mechanical signal and increases the SNR.

Parametric amplification in gyroscope

Parametric amplification is a nonlinear effect, due to the modulation of the equivalent spring constants (e.g., through common-mode voltages), and it can be applied in principle to both the driven and sensing modes (with dedicated capacitive combs). If one assumes harmonic actuation at resonance in 0x mode and consider the nonlinear form given by *Mathieu's equation* instead of (1), one has:

$$\ddot{x}+\frac{c_x}{m}\dot{x}+\frac{k_x+k_{yx}}{m}(1+\frac{\Delta k_x}{k_x}\sin(\omega_pt+\phi_p))x=\frac{F_{actuation}}{m}\sin(\omega_xt+\phi_{0x}) \tag{4}$$

Here the pumping frequency $\omega_p=2\omega_x$, while ϕ_x,ϕ_p are the phases of driving force and of the parametric pump, respectively. Δk_x is the amplitude of the modulation factor for the spring constant. To attain maximum gain, the phase of the parametric pump should be orthogonal to the drive phase [7]. A similar parametric pumping scheme is required in the sensing mode, where the phase of the electro-mechanical pump is tuned such that the energy is pumped into the resonator synchronously with the Coriolis force (a lag of 90 degree relative to the driving force).

$$\ddot{y}+\frac{c_y}{m_y}\dot{y}+\frac{k_y+k_{xy}}{m_y}(1+\frac{\Delta k_y}{k_y}\sin(\omega_pt+\varphi_p-90))y=2\dot{x}\Omega_z\sin(\omega_xt+\varphi_{0x}-90) \tag{5}$$

The sign of the Coriolis force has been incorporated as a phase change in eq. 5. Assuming the system is actuated at its resonance, the steady state solution of (5) is given by:

$$y_{ss}=\frac{F_{Coriolis}}{k_y}(1-\frac{Q_y\Delta k_y}{2k_y})\sin(\omega_xt+\phi_x-90) \tag{6}$$

EXPERIMENTAL SETUP

To experimentally prove the technique, the Coriolis force term is mimicked by a voltage controlled electrostatic force in the structure shown in Figure 3. As mentioned before, there are three independent bond pads for actuation and sensing combs. One set of the comb drives is used to mimic a Coriolis force (blue connections in Figure 3).

Other two sets of the comb drives (red connections in Figure 3) are used for parametric pumping, using common mode voltages. Existing defects like missing fingers in the fabricated devices (Figure 4) generate the equivalent of quadrature signal components. A common mode AC voltage is applied on the differential gap-varying capacitances and used to pump energy from electrical to mechanical domain. The resulting electrostatic force results in:

$$F_{electrostatic} = \frac{C_0}{2}\left[\frac{1}{(1-\frac{y(t)}{d})^2} - \frac{1}{(1+\frac{y(t)}{d})^2}\right]u_{CM}^2(t) \qquad (7)$$

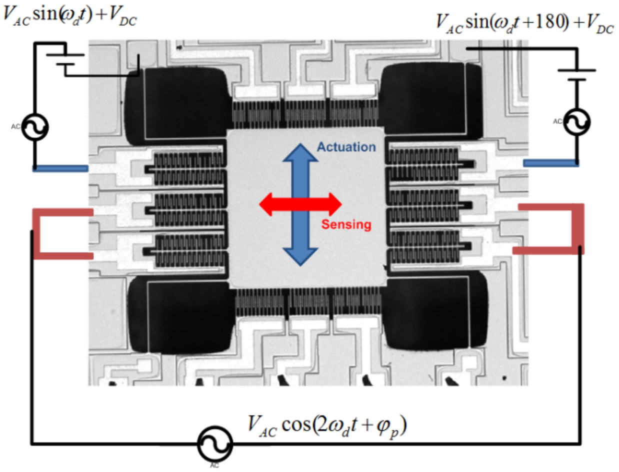

Figure 3: Device concept and working principle.

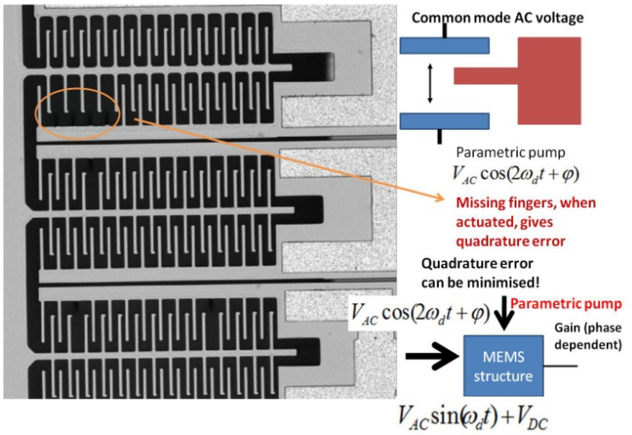

Figure 4: Sense comb drives with gap varying fingers.

Here, C_o is the capacitance at rest position and u_{CM} is the common mode voltage. For $y\langle d_0$, where d_0 is the zero-voltage gap, the net effect of the electrostatic force is a modulation by u_{CM} of the equivalent spring constant k_y:

$$m_y\ddot{y} + c_y\dot{y} + \left(k_y - \Delta k_y\left(\frac{u_{CM}}{V_o}\right)^2\right)y = F_{Coriolis} + F_{Quadrature} \qquad (8)$$

For an harmonic common mode voltage of the same frequency as the driving mode actuation voltage, $u_{CM}(t) = V_0\cos(\omega_y t + \varphi)$, the **relative phase φ** is the essential parameter for the electro-mechanical parametric coupling.

Experimental results

The experimental validation measured optically the induced displacements in the proof mass, for various parametric pumping conditions. Polytec PMA-500 equipment was used for the test and characterization of the gyroscope structures. Both the scanning laser-Doppler vibrometry and the video-stroboscopic planar motion analyzer modules were used to extract the parameters related to the driven and sensing resonant modes. The extracted stiffness coefficient k_y=132.23N/m, for a sense mode resonant frequency of 8.57kHz. The equivalent of a Coriolis force generated by $\Omega_z = 1\%_{\sec}$ is generated electrically for inducing the 0y motion of the proof mass, while a common mode voltage of adjustable phase delay and amplitude is used for parametric pumping.

Figure 5 illustrates the dependence of the net parametric gain (amplification of F_c effect) on the u_{CM} amplitude. A maximum gain factor of 25 was experimentally measured before reaching dynamic instability in 0y motion.

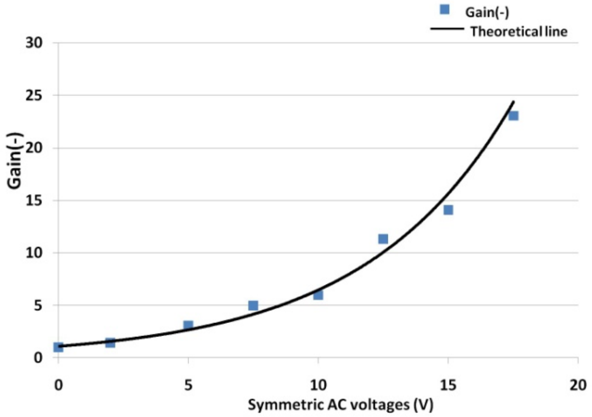

Figure 5: Experimental (points) and theoretical (line) parametric gain vs. pump voltage amplitude

Parametric coupling increases the selectivity of the resonant mechanical structure, leading to a larger equivalent quality factor and a lower noise level, as illustrated in Figs. 6 and 7. Noise estimation was done for both the equivalent 0y resonator and for the MEMS gyroscope. The mechano-thermal noise depends on the damping coefficient and the operating bandwidth. Figure 6 shows the measurements indicating the reduction of the noise force ($F_{n,y} = \sqrt{4k_B T c_y \cdot BW}$) with a higher parametric gain, for a constant measurement bandwidth (BW). For a gain of 5, the equivalent input acceleration noise diminishes from 0.033ms^{-2} to 0.022ms^{-2}. The increased spectral selectivity will reduce as well, in the case of the gyroscope, the equivalent noise input angular rate (given by eqn. 3a) – for a gain of 5, $\Omega_{z,n}$ reduces from $0.0046°/(s\sqrt{Hz})$ to

$0.0026^\circ/\left(s\sqrt{Hz}\right)$. Figure 8 shows the dependence of the parametric pumping on the relative phase between the driving force and the common mode AC voltage. A reduction of the normalized gain up to 0.6 was measured (relative to the case of no parametric pumping). According to theory, the phase dependence of the gain is given by [7]:

$$G=\left[\left(\frac{\cos\phi}{1+Q_y\dfrac{\Delta k}{2k_y}}\right)^2+\left(\frac{\sin\phi}{1-Q_y\dfrac{\Delta k}{2k_y}}\right)^2\right]^{1/2} \qquad (9)$$

Consequently, the parametric gain is maximum at $\phi = 90^\circ$ and reaches a minimum at 180 degrees, facts validated experimentally. The parameters extracted from the experimental measurements were used in complementary Simulink simulations, which have shown a good matching with the data. Numerical simulations indicate an attenuation of 0.8 of the motion component when the phase difference is set to $\phi = 180^\circ$. This parametric damping mode is extremely useful for the attenuation of quadrature error component.

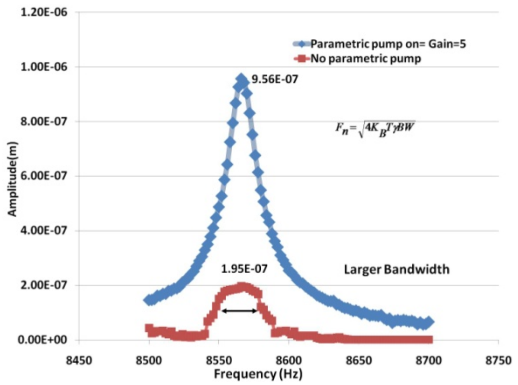

Figure 6: Influence of the parametric amplification on the equivalent bandwidth.

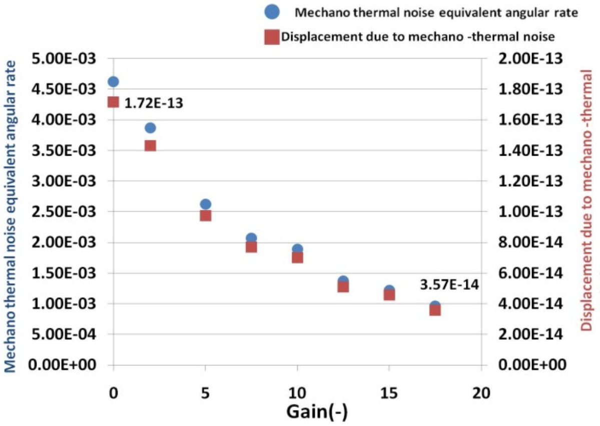

Figure 7: Comparison of mechano-thermal noise equivalent angular rate (blue) and displacement due to mechano thermal noise (red) vs. Gain.

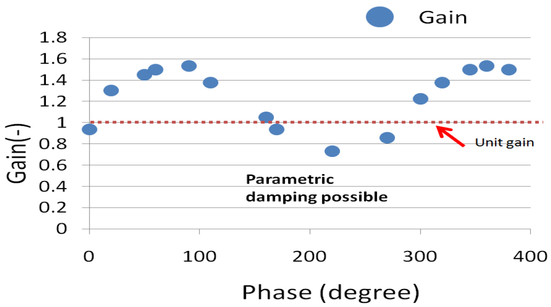

Figure 8: Phase dependence of the normalized gain

CONCLUSION

The minute capacitive variations in MEMS inertial sensing, coupled with the effects of both random (noise) and deterministic (quadrature errors) signal components require an amplification of the mechanical vibration signal before the conversion into an electrical one. We have shown, validated by measurements, that mechanical motion in the sense mode can be amplified or attenuated, depending on specific phase synchronization conditions between the mechanical motion and the common-mode AC actuation. Parametric gains in the range 5-25 have been experimentally measured on the test structures. The technique can be therefore applied either for the amplification of the Coriolis-induced motion, or for a reduction of the quadrature error.

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REFERENCES

[1] D. Rugar and P. Grutter, "Mechanical Parametric Amplification and Thermo Mechanical Noise Squeezing", *Phys. Rev. Lett.*, vol67, pp. 699-702, 1991.

[2] M.J. Thompson, D.A. Horsley, "Parametrically Amplified MEMS Magnetometer", *in Proc. Transducers '09*, Denver, Co, USA, June 21-25, 2009, pp-1194- 97.

[3] Honeywell , "MEMS Gyroscope with Parametric Gain" *US Patent* 06715353, 2002.

[4] L. Yongsik, "A study of parametric excitation applied to a MEMS tuning fork gyroscope", PhD dissertation, University of Missouri Columbia, 2007.

[5] B.J, Gallacher, J S Burdess and K M Harish, "A Control Scheme for A MEMS Electrostatic Resonant Gyroscope excited using Combined Parametric Excitation and Harmonic Forcing", *J.Micromech.Microeng.*, vol. 16, pp. 320-331, 2006.

[6] A. Laura Oropeza-Ramos, Christopher B. Burgner, and Kimberly L.Turner,"Robust Micro-rate Sensor Actuated by Parametric Resonance", *Sens. Actuators A:Phys.*, vol.152, pp. 80-87, 2009.

[7] T. Ono, H. Wakamatsu, M.Esahi, "Parametrically Amplified Thermal Resonant Sensor with Pseudo- Cooling Effect", *J.Micromech.Microeng.*, vol.15, pp. 2282-2288, 2005.