Frequency Stabilization of Lasers by Locking to an Atomic Isoclinic Point

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Abstract—**To overcome the temperature sensitivity of lasers locked to Doppler-broadened transitions, we propose a scheme in which the laser is locked to an *isoclinic point* of the atom's spectrum: a frequency at which the first derivative of the absorption spectrum does not change upon a physical change of the sample. Here, we demonstrate that an isoclinic point exists in the Rb^{87} D$_1$ spectrum at 795 nm, and that it is located midway between the F$_g$=2 → F$_e$=1 and F$_g$=2 → F$_e$=2 transitions. Locking a VCSEL diode laser to this isoclinic point, we have been able to demonstrate a very low sensitivity of the locked laser's frequency to temperature variations of the atomic vapor.**

I. INTRODUCTION

As is well known, the optical frequency of a laser can be stabilized by locking it to an atomic resonance [1], and though sub-Doppler spectroscopy [2] yields very narrow resonant features for this purpose, linear absorption spectroscopy (LAS) has clear advantages for ultraminiature atomic physics (e.g., atomic clocks and magnetometers). Specifically, LAS allows for simplicity and compactness of design, and without the need for overlapped beams there is less sensitivity to microphonics. Unfortunately, the Doppler-broadened absorption spectra of LAS often consist of overlapping atomic transitions, which, in the case of the stable alkalies, are due to unresolved (or partially resolved) excited-state hyperfine structure. In such cases, the peak frequencies of the LAS features depend on the degree to which the various partially resolved transitions overlap, and this in turn depends on temperature via the transitions' Doppler width [3]. The issue of spectral overlap and its temperature dependence is especially problematic for LAS in Rb and Cs, the "workhorse" elements for vapor-phase atomic clocks [4-7], since it can give rise to a temperature sensitivity of the locked laser's frequency that is on the order of 10^{-11}/°C.

Here, we discuss a scheme for laser stabilization based on an atomic *isoclinic point*, which by definition is a "wavelength, wavenumber, or frequency at which the first derivative of an absorption spectrum of a sample does not change upon a chemical reaction or physical change of the sample" [8]. Specifically, as will be shown subsequently, there is an isoclinic point for Rb^{87} at a frequency midway between the Doppler-broadened F$_g$=2 → F$_e$=1 and F$_g$=2 → F$_e$=2 D$_1$ transitions. To demonstrate the efficacy of isoclinic point stabilization, we have locked a VCSEL diode laser to the Rb^{87} isoclinic point, and have observed a very low sensitivity of the locked laser's frequency to temperature variations of the atomic vapor.

II. THEORY

Doppler broadening is routinely problematic in linear, vapor-phase, absorption spectroscopy, especially when it leads to overlapped optical transitions. The issue is not always the loss of resolution (which can often be mitigated, to a degree, through improvements in the signal-to-noise ratio), but the fact that overlapping transitions have peak absorption frequencies that shift with vapor temperature. For example, as illustrated in Fig. 1 for the D$_1$ transition of Rb^{87}, the Doppler-broadened absorption lines corresponding to transitions from the $5^2S_{1/2}$ ground-state hyperfine levels to the $5^2P_{1/2}$ excited-state hyperfine levels overlap. Consequently, the peaks of the absorption lines are "pulled" relative to the transitions' true resonant frequencies by the wings of the neighboring lines. Change the degree of spectral overlap, and the absorption lines' peak frequencies will shift. Since the Doppler-broadened widths of absorption lines change with temperature, so too does the degree of spectral overlap among the transitions; and this in turn gives the peak frequencies of a linear vapor-phase absorption spectrum a sensitivity to vapor temperature.

To illustrate the problem more quantitatively, consider two neighboring Doppler-broadened transitions, A and B, as illustrated in Fig. 1, where w_D is the Doppler-broadened full-width half-maximum (FWHM). For a laser frequency ω_L tuned near these absorption lines, the irradiance transmitted by a vapor of length L will be given by

$$I(L) = I_o \exp[-NL[\sigma_A(\Delta_A) + \sigma_B(\Delta_B)]]. \tag{1}$$

Here, $\sigma_J(\Delta_J)$ is the cross section of the Jth resonance and Δ_J is the detuning from the true resonant frequency of the transition: $\Delta_J = \omega_L - \omega_J$. Taking the derivative of Eq. (1) with respect to laser frequency we have

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$$\frac{dI(L)}{d\omega_L} = -NLe^{-N[\sigma_A(\Delta_A)+\sigma_B(\Delta_B)]L}\left(\frac{d\sigma_A}{d\omega_L}+\frac{d\sigma_B}{d\omega_L}\right), \quad (2a)$$

which provides a necessary condition for local extrema in the atoms' absorption spectrum:

$$\left(\frac{d\sigma_A}{d\omega_L}+\frac{d\sigma_B}{d\omega_L}\right) = 0 \cdot \quad (2b)$$

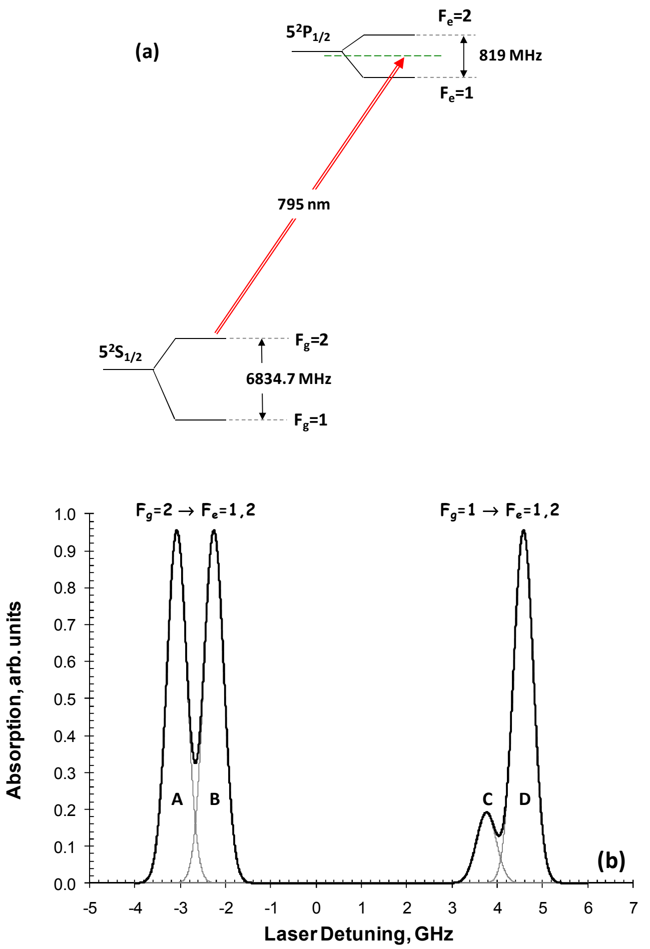

Figure 1. (a) Energy level diagram for the Rb^{87} D_1 transition at 795 nm. (b) Theoretical absorption spectrum of the Rb D_1 transition near room temperature.

For the case of the extremum located near absorption line A, this procedure yields the peak frequency of the A transition:

$$\omega_{pA} = \omega_A - \Delta_B\left(\frac{\sigma_{pB}}{\sigma_{pA}}\right)e^{-4\ln(2)\left(\Delta_B/w_D\right)^2}, \quad (3a)$$

or

$$\omega_{pA} \cong \omega_A - (\omega_A - \omega_B)\left(\frac{\sigma_{pB}}{\sigma_{pA}}\right)e^{-4\ln(2)\left((\omega_A-\omega_B)/w_D\right)^2}, \quad (3b)$$

where σ_{pJ} is the peak absorption cross section of the J^{th} transition, and where ω_{pA} is seen to have a temperature-dependent shift due to the temperature sensitivity of the Doppler width. (To be clear, ω_{pA} is the peak frequency of the absorption line, while ω_A is the transition's intrinsic resonant frequency.) In particular, for small changes about some reference temperature T_o, and defining Δ_{AB} as $\omega_A-\omega_B$, the peak frequency of the transition will vary like

$$\frac{\delta\omega_{pA}}{\delta T} = -4\ln(2)\frac{\Delta_{AB}}{T_o}\left(\frac{\sigma_B(\Delta_{AB})}{\sigma_{pA}}\right)\left(\frac{\Delta_{AB}}{w_D(T_o)}\right)^2 \cdot \quad (4)$$

For absorption line A in Fig. 1 near room temperature, this yields $\delta\omega_{aP}/\delta T \cong 16$ kHz/°C, or in fractional frequency y 4.2×10^{-11}/°C. This is a relatively large temperature sensitivity for the peak frequency of a transition, and demonstrates the significant role that a vapor's temperature variations can play in precision spectroscopy.

Reflecting on the above discussion, it may be noted that near resonance the absorption cross-section of one overlapped transition is essentially independent of temperature, while the other cross-section retains a relatively strong temperature dependence. It is this asymmetry in the temperature sensitivity of the two cross sections near resonance that causes the peak frequency to vary with temperature. Conversely, near the midpoint between the two resonances, both cross-section derivatives in Eq. (2b) are temperature dependent. In particular, if we define ω_m as the frequency corresponding to the local minimum near the midpoint, then for reasonably resolved, Doppler-broadened absorption lines (and assuming $\sigma_{pA} \cong \sigma_{pB}$) it is straightforward to show that

$$\omega_m \cong \left(\frac{\omega_A+\omega_B}{2}\right) - \frac{(\sigma_{pA}-\sigma_{pB})}{(\sigma_{pA}+\sigma_{pB})}\frac{\Delta_{AB}w_D^2}{4\ln(2)\Delta_{AB}^2-2w_D^2} \cdot \quad (5)$$

In this case, the temperature dependence of the minimum only arises when the absorption cross-sections of the two transitions are unequal. When σ_{pA} equals σ_{pB} the second term on the right-hand-side of Eq. (5) is identically zero, and the frequency of the local extremum equals the intrinsic midpoint frequency of the two transitions independent of temperature: the midpoint frequency is an isoclinic point.

III. EXPERIMENT

As suggested by Fig. 1, and as can be verified by detailed calculation, the F_g=2 → F_e=1 and F_g=2 → F_e=2 D_1 transitions (i.e., A and B in the figure) have identical cross-sections. Thus, the midpoint between these two transitions should provide an experimental realization of an isoclinic point. To test this idea, Fig. 2 shows a block diagram of our experimental arrangement. A VCSEL diode laser (vertical-cavity surface-emitting laser) with a nominal output power of a few hundred microwatts had its wavelength modulated at a few tens of kilohertz, and was then split into two beams. These two beams passed through two identical, cylindrical, Pyrex absorption cells with a radius of roughly 1 cm and a length of about 3 cm. The absorption cells contained isotopically enriched Rb^{87} (i.e., $N_{85}/(N_{85}+N_{87}) \cong 0.01$), and the transmitted light intensity was detected with identical Si photodiodes and sent to two identical lock-in amplifiers (LIAs).

The output of LIA–B was first passed to a PID controller (proportional-gain/integrator/differentiator) and then added to the modulation voltage sent to the VCSEL injection-current controller. The VCSEL wavelength was thereby locked to some LAS feature of the D_1 Rb absorption spectrum as manifested in absorption cell B. We could choose either the isoclinic point or the peak of any transition. Absorption cell A acted as our frequency discriminator: since the laser excited the same transition in cells A and B, the output of LIA-A was proportional to the frequency difference of that transition as manifested in the two cells. Holding the temperature of absorption cell A constant and changing the temperature of absorption cell B, we could thus upper-bound the temperature sensitivity of any absorption feature in the D_1 Rb spectrum. We calibrated the output of LIA-A by tuning the laser frequency and measuring the output voltage of LIA-A under open-loop conditions, which gave us df/dV_{LIA}. In the measurements, we had to exercise care to compensate for the diode laser's concomitant amplitude modulation [9], which produces an offset between the frequency the laser actually locks to and the local extremum in the absorption spectrum; this is discussed further in the Appendix.

Figure 3 shows the principal results of the present work. We first locked the laser to the reasonably well-isolated Rb^{87} D_1 transition shown as transition-D in Fig. 1. We then varied the temperature of Cell B and determined the frequency change of this resonant feature by examining the lock-in output from cell A. The results showed a temperature sensitivity of $4\times10^{-11}/^{o}C$. This result is consistent with the simple theory developed in Section II.

We then locked the laser to the Rb^{87} D_1 isoclinic point, which is midway between transitions A and B. Again we varied the temperature of Cell B and determined the frequency change of the isoclinic point by examining the lock-in output from cell A. The results now showed a temperature sensitivity of $-(2.0\pm1.4)\times10^{-12}/^{o}C$. Not only was the isoclinic point's average temperature sensitivity an order of magnitude lower than transition-D's, but at the 95% confidence level we could not distinguish a difference between the isoclinic point's temperature sensitivity and zero.

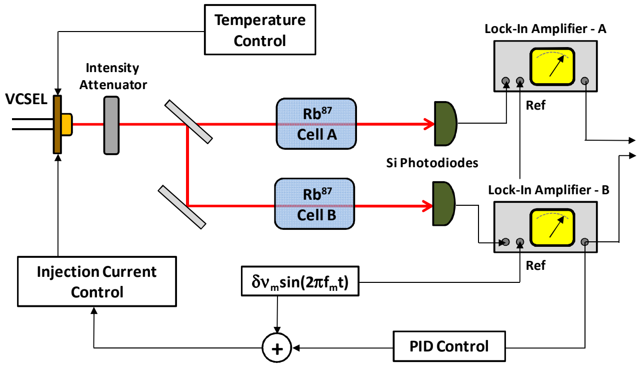

Figure 2. Block diagram of the experimental arrangment. The outputs of the two lock-in amplifiers go to our data acquisition system.

IV. SUMMARY

As noted in the introduction, compared to Doppler-free spectroscopy linear absorption spectroscopy (LAS) has clear advantages for ultraminiature atomic physics. Unfortunately, the spectral overlap of atomic transitions that comes with Doppler broadening gives rise to a temperature dependence in the peak frequencies of absorption features. Here, we have discussed the possibility of locking lasers to isoclinic points in atomic spectra, and have identified an isoclinic point in the D_1 spectrum of Rb^{87}. Further, we have performed a series of experiments to show that isoclinic points can indeed have much lower sensitivities to vapor temperature variations, and that they therefore offer the possibility of much better long-term performance for stabilized lasers.

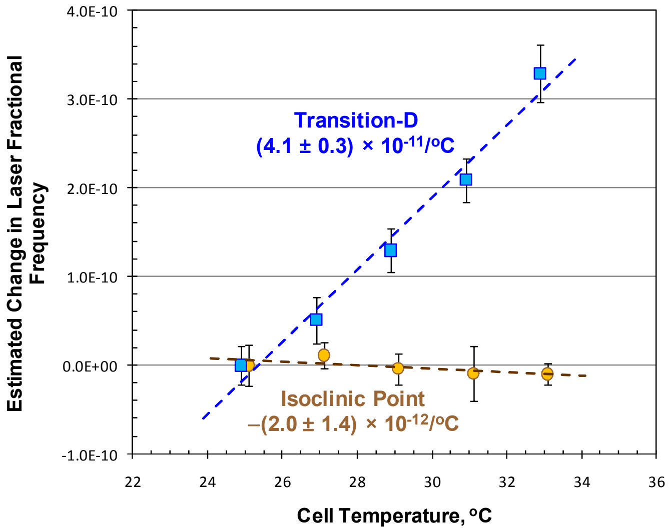

Figure 3. Temperature sensitivity of transition-D (as identified in Fig. 1) and the isoclinic point midway between transitions A and B.

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APPENDIX

In our experiments, we modulate the VCSEL diode laser's injection current, which gives us both frequency modulation of the laser (FM) and amplitude modulation of the laser (AM) [9]. When we demodulate the atomic absorption signal, we recover a lineshape (i.e., a lock-in output voltage) that is proportional to the first derivative of the LAS atomic lineshape; this derivative lineshape comes from our FM of the laser. In addition, however, we also recover a (unwanted) DC offset that is proportional to the laser's AM. Since the demodulated signal is the correction voltage that we feed back to lock the laser, any DC offset in the demodulated signal will shift the “locking point” of the laser from the lineshape extremum.

To be more specific, near the extremum of an absorption feature, we can approximate the lock-in amplifier signal as a linear function of the frequency offset ($\omega_L - \omega_J$): $V_{LIB} = \beta(\omega_L - \omega_J) + \delta_{AM}$. The proportionality coefficient β is the slope of the correction signal (which is related to the 2nd derivative of the lineshape and also the amount of absorbed light), and the DC offset due to the laser’s concomitant amplitude variations is δ_{AM}. If the number density of atoms in the vapor increases, the signal amplitude obviously will grow, implying that the slope β is a temperature-dependent quantity. Alternatively, we expect δ_{AM} to be independent of vapor temperature to first order, especially if the vapor is optically thin as it is in our experiments. Clearly, the laser will lock to a frequency that yields $V_{LIB} = 0$, or $\omega_L = \omega_J - \delta_{AM}/\beta$. Consequently, the locked laser’s frequency shifts with temperature depending on the ratio of δ_{AM} to β.

To eliminate this systematic temperature variation in our experiments, we removed resonance cell B from the laser beam path, and zeroed the output of lock-in amplifier B. Essentially, we added a DC bias voltage to LIA-B that was equal and opposite to δ_{AM}. We then placed resonance cell B back in the beam path; and to the extent that δ_{AM} is unaffected by the added surfaces, and is also independent of laser tuning, the “AM problem” was eliminated for locking to the isoclinic point and the peak of transition-D.

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