Autonomously Measuring an Atomic Clock's Allan Variance

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***Abstract*—In this work, we demonstrate that an atomic clock can autonomously assess its own frequency stability and integrity by comparing the phase of its output signal to a delayed version of itself in what is essentially an interferometric technique. Using a high-quality crystal oscillator as a surrogate delay line, we demonstrate that fractional frequency variations at the level of 10^{-12} are detectable, and that a Cs clock's short-term Allan deviation can be measured without reference to another standard. The paper concludes with a discussion of how an ambiguity in the method might be resolved, and how the method might be employed in the optical domain.**

I. Introduction

There are a number of situations in which a clock's ability to self-monitor would be advantageous: systems in which the clock is at a remote unattended site, deep space missions where it can take hours for a signal to propagate from the spacecraft to the Earth, space systems where the satellites may have to operate autonomously, and GNSS where rapid detection of clock problems can be crucial to safeguard lives. Notwithstanding the ubiquitous need for clock self-monitoring [1], there is no well-accepted, single technique for accomplishing it; though in general the methods that have been discussed most often fall into one of two broad categories: the "clock-comparison" method and the "signal-parameter" method.

In the clock-comparison method, the frequencies of two clocks are compared, and their beat frequency shows an anomaly if either clock fails. Problems with this approach include the fact that it requires two clocks of similar quality, and that there is always an ambiguity with regard to which clock has suffered the failure. In the signal-parameter method, various clock parameters are monitored that are related to the health of the clock. The major problem with this approach is that it monitors secondary indicators of clock health rather than the clock's frequency, which is of primary interest. Moreover, these secondary signals require calibration, which may be nonlinear; and one can never assess all parameters that could correlate with a clock problem.

In our "interferometric method," we compare a clock's signal with a delayed version of itself [2]. Specifically, the output signal of our interferometric system is proportional to the clock's fractional frequency deviation, Δy, averaged over the delay, Δt. The main advantage of this approach is that it requires only one clock, and that it is a direct measure of what is of primary interest: the clock's frequency.

II. The Interferometric Method

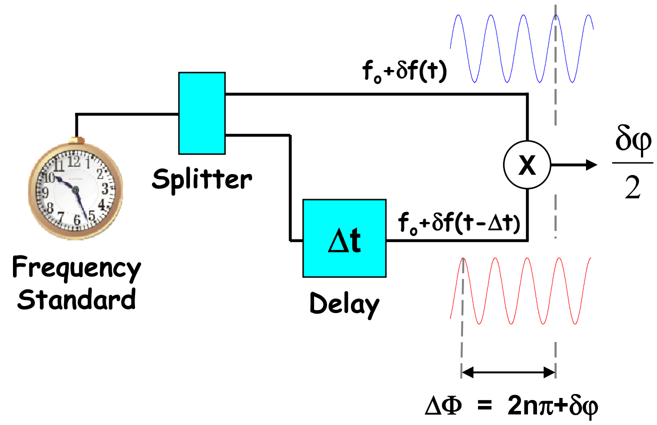

Figure 1. Block diagram of our basic interferometric approach to clock self-monitoring.

The basic idea behind our interferometric method for clock self-monitoring is illustrated in Fig. 1. There, the output signal of a precision frequency standard is split into two paths. One path goes to the RF port of a mixer and the other path, delayed by Δt, is sent to the mixer's LO port. The IF output of the mixer is proportional to the phase difference between the two input signals, which is given by

$$\delta\varphi = 2\pi\int_0^t \left(f_o + \delta f(x)\right)dx - 2\pi\int_0^{t-\Delta t}\left(f_o + \delta f(x)\right)dx\,; \qquad (1)$$

or with

$$\bar{y}(t) \equiv \frac{1}{\Delta t}\int_{t-\Delta t}^{t} \frac{\delta f(x)}{f_o}\,dx, \qquad (2)$$

Eq. (1) becomes

$$\delta\varphi = 2\pi f_o \Delta t\, \bar{y}(t) + \mathrm{Mod}[2\pi f_o \Delta t, 2\pi]. \qquad (3)$$

In this expression, $\bar{y}(t)$ is the fractional frequency deviation of the clock under test at time t, averaged over the delay time Δt. Thus, a time history of the mixer's output, $V_{IF}(t)$, can be converted to a time history of the clock's fractional frequency fluctuations.

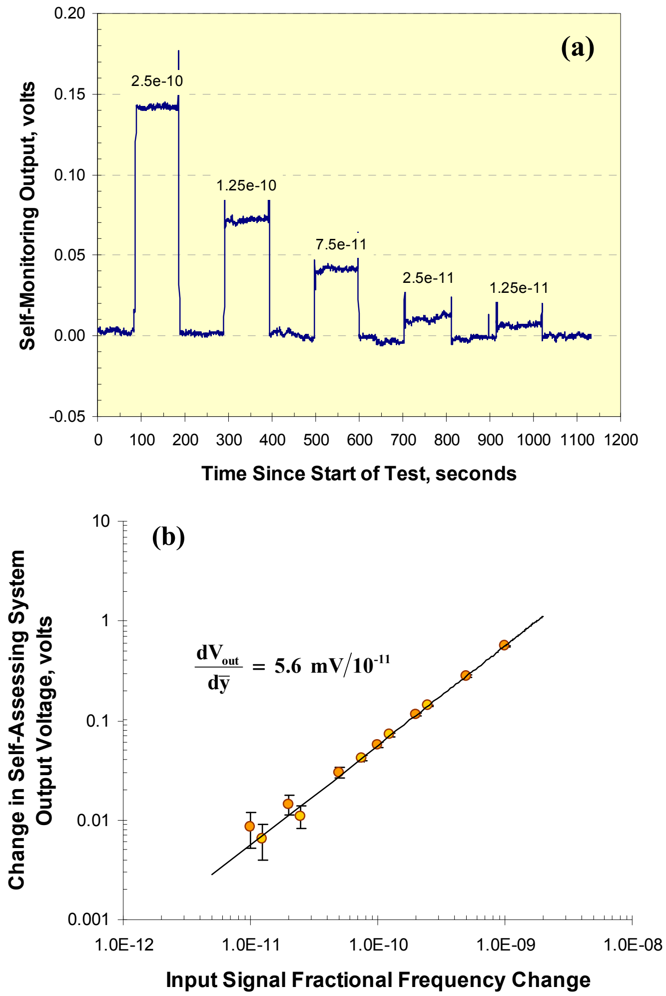

Figure 2. (a) Our system's output voltage as a function of input fractional frequency offset; we changed the BVA-OCXO's frequency in a square wave pattern roughly every 100 seconds. (b) Calibration curve of our self-assessment system, showing output voltage (i.e., V_{IF}) as a function of input fractional frequency change.

In our realization of an interferometric clock self-assessing system, we delay the signal via dispersion [3] using a high-Q crystal resonator as a surrogate delay line. Modeling the crystal with a simple transfer function:

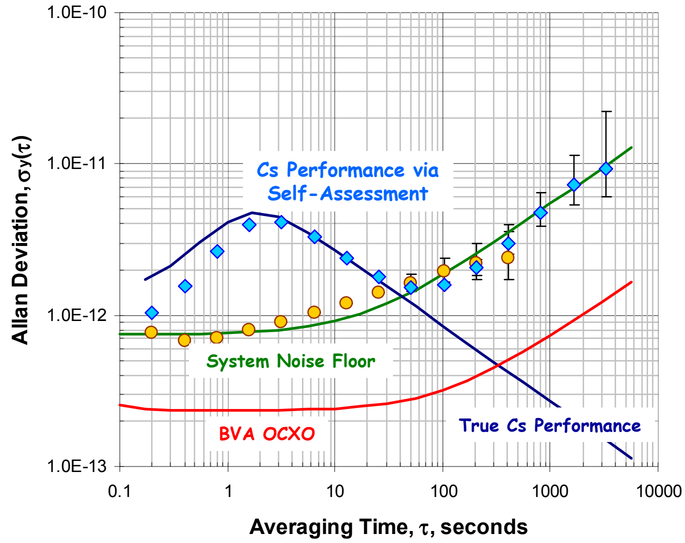

Figure 3. Allan deviation of Cs clock fractional frequency fluctuations measured with our interferometric system.

$$H(\omega) = \frac{\left(\omega_o/2Q\right)}{(\omega - \omega_o) + i\left(\omega_o/2Q\right)}, \qquad (4)$$

it is straight-forward to show that

$$\Delta t = d\theta/d\omega = (Q/\pi f_o)[1+(2Q(f-f_o)/f_o)^2]^{-1} \cong (Q/\pi f_o), \qquad (5)$$

where $f_o = \omega_o/2\pi$. For our SC-cut crystal with $Q = 3\times10^5$ and $f_o = 10$ MHz, we have $\Delta t \cong 10$ msec.

Figure 2a shows the output of our interferometric system after amplification and filtering when a BVA OCXO [4] was input to the system. Specifically, we changed the control voltage to the BVA OCXO every 100 seconds, and monitored the change in the interferometric system's output voltage, V_{IF}. Figure 2b shows the calibration curve for our interferometric system, and with a sensitivity of about 6 mV for a 10^{-11} fractional frequency change, it is clear that the interferometric method has the ability to detect small changes in the output frequency of a precision oscillator.

The filled circles in Fig. 3 show the Allan deviation of the interferometer's output (with the BVA OCXO as input) after calibrating the time-history of the mixer's IF output voltage: $\bar{y}(t) = V_{IF}(t)/(dV_{IF}/d\bar{y})$. The lowest curve shows the Allan deviation of the BVA OCXO, so that the filled circles clearly represent the noise floor of our interferometric self-assessing system. Note that at averaging times between one and ten seconds, the interferometric system has the ability to self-detect fractional frequency changes as low as 10^{-12}.

As a further test, we replaced the BVA OCXO with an hp5071 cesium atomic clock. Again, we recorded the time-history of the mixer's IF output voltage and converted this to a time-history of fractional frequency fluctuations. The resulting Allan deviation is shown as diamonds in Fig. 3, and the solid line passing through the diamonds at short averaging times is the Allan deviation of the Cs clock measured in a standard clock-comparison method. The difference between the diamonds and the solid line at very short times (less than one second) is due to the fact that the delay acts as a low-pass filter, so that the Allan deviation is underestimated for averaging times less than or roughly equal to Δt. Nevertheless, for averaging times between one and twenty seconds, the interferometric method's (autonomous) assessment of the Cs clocks' performance agrees very well with its performance assessed via the more standard method.

III. A Problem Using Dispersion for Delay… And Its Potential Solution

Regarding Eqs. (4) and (5), it should be clear that unless the fractional frequency change of the input (clock) signal, ω_c, is abnormally large (i.e., much greater than Q), there will be an ambiguity as to what may have caused a change in the mixer's output voltage. Specifically, it is difficult to tell whether a change in V_{IF} is due to a change in the frequency of the clock-under-test, $\Delta\omega_c$, or is due to a change in the dispersive element's resonant frequency, $\Delta\omega_o$, (e.g., a temperature or radiation induced shift in the crystal's resonance):

$$\delta\varphi = 2\pi\delta f_c \frac{\partial\theta}{\partial\omega_c} + 2\pi\delta f_o \frac{\partial\theta}{\partial\omega_o}, \qquad \left|\frac{\partial\theta}{\partial\omega_c}\right| \cong \left|\frac{\partial\theta}{\partial\omega_o}\right|. \quad (6)$$

Figure 4 illustrates a potential solution to this problem: in addition to self-assessing the performance of the precision clock interferometrically, we self-assess the performance of a "low-quality" crystal oscillator. (The two signals could take advantage of the same dispersive element for delay in a time-division fashion, periodically switching the delay's input from the clock-under-test to the low-quality crystal oscillator.) If both interferometric signals show an anomaly, then quite likely the precision frequency standard has not failed, and the anomaly should be associated with the dispersive element. Alternatively, if only the precision frequency standard path shows an anomaly, then the system would (autonomously) have good evidence that the precision clock has a problem.

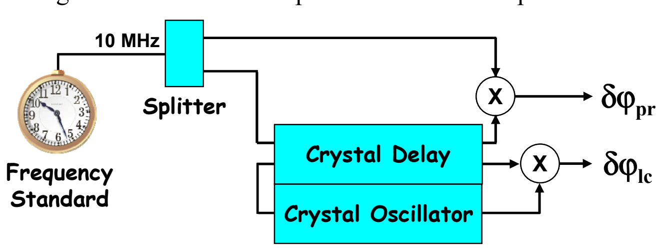

Figure 4. To overcome the problem of ambiguity when dispersion is used for delay, it might be possible to self-assess both the frequency standard of primary interest and a low-quality crystal oscillator using the same delay element. Two outputs would then be detected: a precision output, $\delta\varphi_{pr}$, and a low-quality output, $\delta\varphi_{lc}$; comparison of the two could help differentiate between a frequency standard problem and (for example) a temperature-induced shift in the resonant frequency of the crystal delay.

Though one might be tempted to think that the configuration of Fig. 4 is no better than the standard clock comparison method, it is important to note that the precision frequency standard and crystal oscillator in Fig. 4 are of vastly different quality. The low-quality crystal oscillator is simply present to assess the environment of the dispersive delay element (e.g., temperature, radiation, etc.) in terms of its effect on the dispersive delay's resonant frequency. In the clock-comparison method one routinely requires two standards of near-equal frequency stability.

IV. Slow Light

In addition to using the interferometric method for microwave frequency standards, we also want to point out its potential application to optical frequency standards. In recent years, much attention has focused on the phenomenon of slow light [5]. Briefly, taking advantage of coherences in atomic systems, it is possible to significantly alter a medium's index of refraction, and thereby slow the propagation of light by orders of magnitude. Consequently, one could employ a "slow light" medium as a surrogate delay line, and recombine the optical field with a delayed version of itself on a fast photodiode as illustrated in Fig. 5. The time-history of the photodiode's output would then provide a measure of the laser's fractional frequency fluctuations. Alternatively, if the laser was very stable, the photodiode's output would provide a time-history of index-of-refraction fluctuations in the slow light medium. For the latter case, it is worth noting that the loss of coherence in atomic media is of considerable interest, and the interferometric method discussed here might provide a new means for studying decoherence in macroscopic systems.

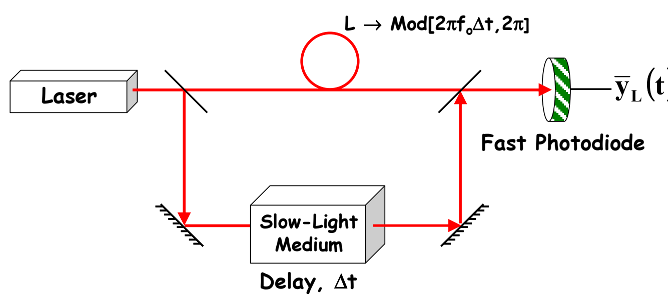

Figure 5. Use of a slow light medium as a delay for assessing the performance of optical fields.

V. Summary

In this work, we have discussed a means for clocks to self-assess their performance based on an interferometric technique: a clock's output signal is compared with a delayed version of itself. In our realization of the interferometric method, we achieved long delays by passing a clock's signal through a dispersive element and by taking advantage of group delay: $\Delta t = d\theta/d\omega$. In this way, we demonstrated that the interferometric technique can sense frequency variations of a clock signal at the 10^{-12} level, and we measured the short-term Allan deviation of a cesium atomic clock without reference to another oscillator.

We believe that the interferometric self-assessing method may have a number of important applications, including integrity monitoring of remote clocks, monitoring of the environment of precision clocks (via a low-quality crystal oscillator), and perhaps even self-assessment of optical frequency standards. We believe that there is much potential for the interferometric method, and we hope that our work will motivate others to investigate the technique further.

ACKNOWLEDGMENT

The authors would like to acknowledge the long and productive career of Walt Johnson, who passed away following the work described here. He will be sorely missed, both for his deep knowledge of rf and microwave electronics, and for his congenial attitude in collaborative efforts. This work was funded by U.S. Air Force Space and Missile Systems Center under Contract No. FA8802-09-C-0001.

REFERENCES

[1] E. Nunzi, L. Galleani, P. Tavella, and P. Carbone, “Detection of anomalies in the behavior of atomic clocks,” IEEE Trans. Instrum. & Meas., vol. 56(2), pp. 523-528, 2007.

[2] Y. C. Chan, W. A. Johnson, S. K. Karuza, A. M. Young, and J. C. Camparo, “Self-Monitoring and Self-Assessing Atomic Clocks,” IEEE Trans. Instrum. & Meas., in press.

[3] E. H. Young, “Discussion of time delay in reference to electrical waves,” IRE Trans. Ultrson. Eng., vol. 9(1), pp. 13-20, 1962.

[4] R. J. Besson, “A new electrodeless resonator design,” in Proc. 31st Annual Freq. Control Symp. (IEEE Press, Piscataway, NJ, 1977) pp. 147-152.

[5] R. W. Boyd and D. J. Gauthier, “‘Slow’ and ‘fast’ light,” in Prog. in Opt., ed. E. Wolf (Elsevier Science, Amsterdam, 2002) Vol. 43, pp. 497-530.