Langasite beam resonators: Theoretical and experimental investigations

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***Abstract* — The goal of this paper is to investigate the possibility of temperature-compensated cuts for both kinds of vibrations in Langasite resonators. We propose theoretical and experimental investigations of vibrating beam resonators with a rectangular cross-section in extensional and flexural modes for Langasite. Measures of frequency-temperature effects in beam vibrating in length extension are given.**

I. INTRODUCTION

Piezoelectric resonator domain has undergone new developments by means of new materials for several years. In this context, Langasite (LGS) crystal has shown its interest in bulk and surface acoustic wave resonators [1-2]. LGS crystal belongs to the same crystal class as quartz (32) and its chemical composition is $La_3Ga_5SiO_{14}$. This material begins to be well known in term of elastic coefficients [3-6] but thermal sensitivities of these elastic coefficients are still in discussion [7-13]. Table 1, 2 and 3 sum up different sets of coefficients.

Temperature-compensated cuts have been investigated theoretically for quartz and $GaPO_4$ by analytical methods for both kinds of vibrations: length extension and flexion [14-15]. In this paper, modeling and measuring temperature effects in LGS vibrating beam resonators are reported. In addition to the well known thickness-shear Y-cut, temperature compensated cuts exist in LGS for length extensional and flexural modes. Experimental evidence of temperature compensated cut in LGS rectangular beam resonator vibrating in length extension is given.

TABLE I. LINEAR COMPLIANCES (10^{-12} m²/N).

S_{11}^E	S_{12}^E	S_{13}^E	S_{14}^E	S_{33}^E	S_{44}^E	S_{66}^E	Ref
8,75	-4,02	-1,88		5,31	21,99	25,54	[3]
8,73	-4,24	-1,66	-3,47	5,05	20,67	25,94	[4]
8,74	-4,24	-1,64	-3,40	5,02	20,34	26,07	[5]
8,78	-4,29	-1,66	-3,48	5,04	20,55	26,11	[6]

TABLE II. 1ST ORDER TEMPERATURE SENSITIVITIES OF LINEAR COMPLIANCES.

First order: $T_S^{(1)}$ (10^{-6}/°C)							Ref
S_{11}^E	S_{12}^E	S_{13}^E	S_{14}^E	S_{33}^E	S_{44}^E	S_{66}^E	
-24,4	-89,0	-2,51	-164	69,3	27,4	-42,2	[8]
-96,1	-337	165	-249	151	43,5	-175	[9]
-58,5	-156	-2,37	-430	61,7	-41,2	-109	[10]
-4,88	-79	81	-172	102	28,1	-29,2	[11]
-10,5	-117	109	-317	102	4,21	-45,1	[12]
-31.0	-146	109	-355	110	3.91	-69.7	[13]

TABLE III. 2ND ORDER TEMPERATURE SENSITIVITIES OF LINEAR COMPLIANCES.

Second order: $T_S^{(2)}$ (10^{-9}/°C)							Ref
S_{11}^E	S_{12}^E	S_{13}^E	S_{14}^E	S_{33}^E	S_{44}^E	S_{66}^E	
1257.6	3112	-666.47	2221	-239.6	144	1832	[9]
167	281	62.3	492	71.8	148	205	[13]

II. LENGTH EXTENSIONAL MODE

A. *Theoretical investigation: Basic model*

The analytical model of length-extensional vibration is built without taking into account the piezoelectric effect [14]. It concerns rectangular cross-section beams. In this model, the length of the beam is assumed much greater than its width and its thickness.

Fig. 1 shows the plate orientation according to (X, Y, Z) cristallographic axes.

Resonant frequencies are determined from the equation of motion. For a free-free beam, we get:

$$f_n = \frac{n}{2 \cdot l} \frac{1}{\sqrt{\rho \cdot s_{22}}} \tag{1}$$

where n is a positive integer, l the length of the beam, ρ the mass density of material and s_{22} the compliance along the Y axis.

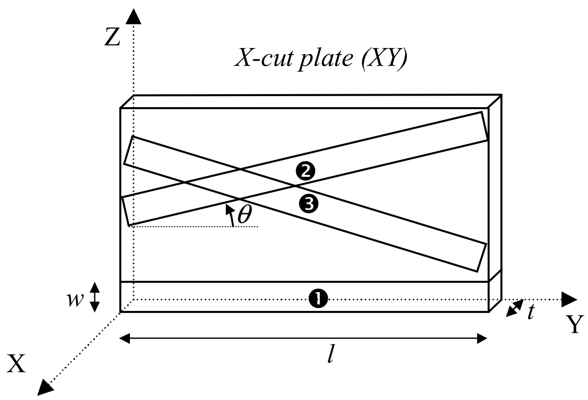

Figure 1. Beam orientations (X, Y, Z cristallographic axes).

Due to the unknown of the third order coefficients of LGS, the temperature dependence of resonant frequencies is defined by the parabolic equation:

$$f(T) = f(T_0)\left(1 + T_f^{(1)}(T - T_0) + T_f^{(2)}(T - T_0)^2\right) \qquad (2)$$

where f is the frequency at the temperature T, T_0 is the reference temperature for which the elastic constants are defined and $T_f^{(i)}$ (with i = 1, 2) are the first two temperature coefficients of frequency (TCFs). T_0 is set at 25 °C for all TCF calculus.

Analytical expressions are obtained for both TCFs by means of a method which consists of varying the elastic constants, beam dimensions and mass density as a function of temperature. $T_f^{(1)}$ and $T_f^{(2)}$ are given by (3)-(4) for a beam oriented along the Y crystallographic axis [14]:

$$T_f^{(1)} = \frac{1}{f(T_0)} \left.\frac{\partial f}{\partial T}\right)_{T=T_0} = \frac{1}{2}\left(\frac{w'}{w} + \frac{t'}{t} - \frac{s'_{22}}{s_{22}} - \frac{l'}{l}\right) \qquad (3)$$

$$T_f^{(2)} = \frac{1}{2!\,f(T_0)} \left.\frac{\partial^2 f}{\partial T^2}\right)_{T=T_0} = \frac{1}{4}\left[\frac{w''}{w} - \left(\frac{w'}{w}\right)^2 + \frac{t''}{t} - \left(\frac{t'}{t}\right)^2 - \frac{s''_{22}}{s_{22}} + \left(\frac{s'_{22}}{s_{22}}\right)^2 - \frac{l''}{l} + \left(\frac{l'}{l}\right)^2\right] + \frac{1}{2}\left(T_f^{(1)}\right)^2 \qquad (4)$$

where s_{22} is the compliance along the Y axis. w, t and l are width, thickness and length of the beam, respectively. " ' " and " '' " denote $\frac{\partial}{\partial T}$, $\frac{\partial^2}{\partial T^2}$, respectively.

The thermal expansion coefficients used in this paper are recalled in table 4.

TABLE IV. THERMAL EXPANSION COEFFICIENTS OF LANGASITE

(10^{-6}/°C)		(10^{-9}/°C^2)		Ref
$\alpha^{(1)}_{ll}$ or $\alpha^{(1)}_{ww}$	$\alpha^{(1)}_{tt}$	$\alpha^{(2)}_{ll}$ or $\alpha^{(2)}_{ww}$	$\alpha^{(2)}_{tt}$	
$\alpha^{(1)}_{11}$ = 5,68	$\alpha^{(1)}_{33}$ = 4,08	$\alpha^{(2)}_{11}$ = 5,43	$\alpha^{(2)}_{33}$ = 2,48	[7]

By means of thermal expansion coefficients $a^{(k)}_{mm}$ (with k = 1, 2 and mm= ww, ll, tt) and thermal coefficients of compliances $T^{(k)}_{Sij}$, we have:

$$T_f^{(1)} = \frac{1}{2}\left(\alpha_{ww}^{(1)} + \alpha_{tt}^{(1)} - T_{s22}^{(1)} - \alpha_{ll}^{(1)}\right) \qquad (5)$$

$$T_f^{(2)} = \frac{1}{4}\left[2\alpha_{ww}^{(2)} - \left(\alpha_{ww}^{(1)}\right)^2 + 2\alpha_{tt}^{(2)} - \left(\alpha_{tt}^{(1)}\right)^2 - 2T_{s22}^{(2)} + \left(T_{s22}^{(1)}\right)^2 - 2\alpha_{ll}^{(2)} + \left(\alpha_{ll}^{(1)}\right)^2\right] + \frac{1}{2}\left(T_f^{(1)}\right)^2 \qquad (6)$$

Table 5 presents the first order temperature compensated angles. Temperature effect minimization is found again for quartz crystal at θ = 5°. On the other hand, temperature-compensated cuts exist for $GaPO_4$ for both compensated angles θ = –8° and θ = –52.6°. For LGS, two cut angles are found by means of coefficients published in [13].

TABLE V. ORIENTATION ANGLES OF TEMPERATURE-COMPENSATED CUTS FOR RECTANGULAR BEAMS IN QUARTZ , $GaPO_4$ AND LGS: EXTENSIONNAL MODE

Rectangular beam	Quartz [16]	$GaPO_4$ [17]	LGS [17]
Length Extension (LE)	θ = 5° (minimization)	θ = -8° or θ = -52.6°	θ = -7° or θ = 62°

B. *Experimental results and discussion*

(XY) plates (X-cut) have been used. Rectangular blank resonators have been achieved according to the plate sizes (20×16×1 mm) by using a mechanical saw in order to vibrate in length extensional mode [15]. These beams have a rotation angle θ = 0° (see ❶ in Fig. 1), θ = 25° or θ = -25° (see ❷ and ❸ in Fig. 1, respectively).

The electrodes are deposited by sputtering method and the design of the electrodes allows vibrating modes in length extension. The electric field must be along the X axis for any kind of beams. For the (XY) beam, only one electrode is necessary per face (Fig. 2). Dimensions of the beams are 20 mm long, 2 mm width and 1 mm thick.

Frequency versus temperature behaviors have been measured for the three different cut angles (Fig. 3). At room temperature, relative frequency variations versus temperature are about 10 ppm/°C for a beam with θ = 0°, 39 ppm/°C for a beam with θ = 25° and -44 ppm/°C fo a beam with θ = -25°. Fig.3 gives the experimental proof of the existence of the compensated cut for length extensional mode in rectangular beam because the slope of the curves goes from a positive value to a negative value.

The model computed with temperature coefficients of [13] doesn't follow really the trend of the measurement. Significant difference between the model and the experimental dots can be observed. Assuming that the thermal expansion coefficients given in the literature are sufficiently reliable, new thermal coefficients of compliances have been fitted to follow the experimental measurements according to parabolic approximations ("Ts mod" curves in Fig. 3). $T_{Sij}^{(1)}$ and $T_{Sij}^{(2)}$ are modified and given in table 6.

Coefficients $T^{(i)}_{S11}$ (with i = 1, 2) allow to adjust the cut with an angle equal to 0° and $T^{(i)}_{S13}$ and $T^{(i)}_{S14}$ have been used for θ = -25° and θ = + 25°. New angles of compensated cuts have been computed with these new thermal coefficients. These angles are θ = -4.7° and θ = 60°.

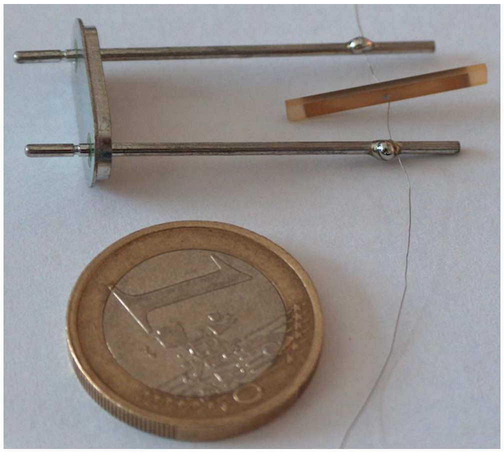

Figure 2. Resonator vibrating in length extensional mode. X-cut beam resonator: 20×2×1 mm, θ = 0°.

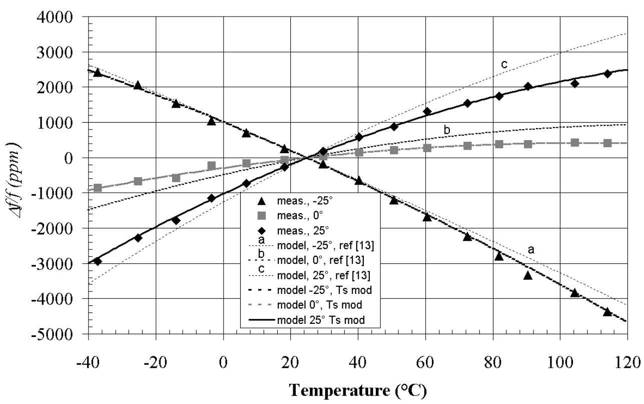

Figure 3. Normalized frequency versus temperature for several rotation angles θ.

Residual second order thermal coefficients of frequency are compared in Table 7. LGS and $GAPO_4$ have a similar behavior.

Table 8 gives motional parameters of three beam resonators vibrating in length extensional mode at atmospheric pressure and in vaccum. This kind of resonator can exhibit a quality factor of about 20 000 at atmospheric pressure.

TABLE VI. NEW TEMPERATURE SENSITIVITIES OF LINEAR COMPLIANCES FOR LENGTH EXTENSIONAL MODE.

	S_{11}	S_{12}	S_{13}	S_{14}	S_{33}	S_{44}	S_{66}
$T_{Sij}^{(1)}$ (10^{-6}/°C)	-16.2	-149	60	-310	110	3.91	-69.7
$T_{Sij}^{(2)}$ (10^{-9}/°C²)	125	281	-1300	420	71.8	148	205

TABLE VII. 2ND ORDER OF THERMAL COEFFICIENTS OF FREQUENCY FOR RECTANGULAR BEAM VIBRATING IN LENGTH EXTENSIONAL MODE.

	Quartz	$GaPO_4$			LGS		
	BT cut	AT cut	LE θ = -8°	LE θ = -52.6°	Y cut	LE θ = -4.7°	LE θ = 60°
$\lvert T^{(2)}_f \rvert$ *(10^{-9} /°C²)*	40	20	75	1.4	50	53	129

TABLE VIII. MOTIONAL PARAMETERS OF RECTANGULAR BEAM RESONATORS VIBRATING IN LENGTH EXTENSIONAL MODE.

	θ = -25°	θ = 25°	θ = 0°	θ = 0° ($7\cdot10^{-6}$ Torr)
Length (mm)	20	20	20	20
Frequency (Hz)	130 144	100 691	111 427	111 435
R (Ω)	2 112	1 300	550	515
L (H)	39	16	16	17
C (fF)	38	152	127	120
C_0 (pF)	1	9	68	60
Q	18 200	18 900	20 500	23 000

III. BASIC MODEL FOR FLEXURAL MODE

In the same way, a beam resonator vibrating in a flexural mode is studied. The derivation used in this model is based on the Bernoulli beam model where shear effects are neglected. These assumptions are valid when the beam length is much larger than its width and thickness. For these assumptions and this vibrating mode, the equation of motion (7) is classical:

$$Z^{(IV)} - \alpha^4 Z = 0 \qquad (7)$$

with $\alpha^4 = \frac{\rho S \omega^2}{EI}$ and "(IV)" denotes the fourth order derivative with respect to the length of the beam.
Z: displacement in the Z direction of the beam.
S: cross-sectional area of the beam
E: Young's modulus in the Y direction of beam, $E = 1/s_{22}$ for a beam oriented along the crystallographic Y-axis
I: Inertia of beam with $I = \frac{w \cdot t^3}{12}$
ρ: mass density of material
ω: pulsation, $\omega = 2\pi f$ where *f* is the flexural vibration frequency.

The resonant frequencies are:

$$f = \frac{\lambda^2}{2 \cdot \pi \cdot l^2} \sqrt{\left(\frac{I}{s_{22} \rho S} \right)} \qquad (8)$$

where λ is solution of an eigenfrequency equation that depends upon the boundary conditions.
For a clamped-free beam the eigenfrequency equation is:

$$1 + \cos(\lambda) cosh(\lambda) = 0 \qquad (9)$$

and for a clamped-clamped beam:

$$1-\cos(\lambda)cosh(\lambda)=0 \tag{10}$$

The first two TCFs $T_f^{(i)}$ (i = 1, 2) are given by:

$$T_f^{(1)}=\frac{1}{2}\left(\frac{w'}{w}+3\frac{t'}{t}-\frac{s'_{22}}{s_{22}}-3\frac{l'}{l}\right) \tag{11}$$

$$T_f^{(2)}=\frac{1}{4}\left[\frac{w''}{w}-\left(\frac{w'}{w}\right)^2+3\frac{t''}{t}-3\left(\frac{t'}{t}\right)^2 -\frac{s''_{22}}{s_{22}}+\left(\frac{s'_{22}}{s_{22}}\right)^2-3\frac{l''}{l}+3\left(\frac{l'}{l}\right)^2\right]+\frac{1}{2}\left(T_f^{(1)}\right)^2 \tag{12}$$

Table 9 presents the first order temperature compensated angles of LGS (computed from coefficients given in table 6) compared with quartz and $GaPO_4$. We found also two compensated cuts.

TABLE IX. ORIENTATION ANGLES OF TEMPERATURE-COMPENSATED CUTS FOR RECTANGULAR BEAMS IN QUARTZ , $GaPO_4$ AND LGS: FLEXURE

Rectangular beam	Quartz [16]	$GaPO_4$ [17]	LGS [17]
flexion	θ = 5° (minimization)	θ = -14.1° or θ = -53.7°	θ = -4° or θ = 60.3°

Theoretical second order TCFs are $-57\cdot10^{-9}$ /°C^2 for θ = -4° and $-127\cdot10^{-9}$ /°C^2 for θ = 60.3°.

IV. CONCLUSION

First order temperature compensated cuts in LGS for extensional mode and flexural modes have been demonstrated theoretically. In length extension, the cut angles are -4.7° and 60°. In flexion, we have θ = -4° and θ = 60.3°. Length extensional and flexural modes have a thermal behavior very similar. Experimental proof of the existence of compensated cuts in length extensional mode has been given.

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REFERENCES

[1] R. C. Smythe, R. C. Helmbold, G. E. Hague and K. A. Snow, “Langasite, Langanite, and Langatate Bulk-Wave Y-cut resonators,” IEEE Trans. on Ultrason., Ferroelec. and Freq. Contr., vol. 47, no. 2, March, pp. 355-360, (2001).

[2] E. Henry-Briot, E. Bigler, S. Ballandras, G. Marianneau and M. Solal, “Experimental measurements of velocities and temperature effects for SAW on Y-rotated and X-cuts of Langasite,” in Proc. Joint Meeting 53rd IEEE Ann. Freq. Cont. Symp and 13rd European Frequency and Time Forum, Besançon, France, 13-16 April, pp. 883-886, (1999).

[3] V.B. Grouzinenko and V.V. Bezdelkin, “Piezoelectric resonators from $La_3Ga_5SiO_{14}$ - single crystals,” in Proc 46tht IEEE Ann. Freq. Cont. Symp., Hershey, 27-29 May, printed by IEEE, Piscataway, NJ, pp. 707-712, (1992).

[4] J.A. Kosinski, R.A. Pastore, E. Bigler, M.P. da Cunha, D.C.Malocha and J. Detaint, “A review of langasite material constants from BAW and SAW data; toward an improved data set,” in Proc. 55th IEEE Ann. Freq. Cont. Symp., Seattle, Washington, 6-8 June, pp. 278-286, (2001).

[5] J.Schreuer, “Elastic and piezoelectric properties of $La_3Ga_5SiO_{14}$ and $LaGa_{5.5}Ta_{0.5}O_{14}$: an application of resonant ultrasound spectroscopy,” IEEE Trans. on Ultrason., Ferroelec. and Freq. Contr., vol. 49, no. 11, Nov., pp 1474-1479, (2002).

[6] H. Ogi, N. Nakamura, K. Sato, M.Hirao and S. Uda, “Elastic, anelastic and piezoelectric coefficients of langasite: resonance ultrasound spcetroscopy with laser-dopler interferometry,” IEEE Trans. on Ultrason., Ferroelec. and Freq. Contr., vol. 50, no. 5, March, pp. 553-560, (2003).

[7] D. C. Malocha, H. François-Saint-Cyr, K. Richardson, R. Helmbold, “Measurements of LGS, LGN and LGT thermal coefficients of expansion and density,” IEEE Trans. on Ultrason., Ferroelec. and Freq. Contr., vol. 49, no. 3, March, pp. 350-355, (2002).

[8] A.A. Kaminskii, I.M. Silvestrova, S.E. Sarkisov and G.A. Denisenko, “Investigation of trigonal $(La_{1-x}Nd_x)Ga_5SiO_{14}$ crystals; II. Spectral laser and electromechanical properties,” Phys. Stat. Sol. (A), vol. 80, no. 2, pp. 607-620, (1983).

[9] A.B. Ilyaev, B.S. Umarov, L.A. Shabanova and M.F. Dubovik, “Temperature dependence of electromechanical properties of LGS crystals,” Physica Status Solidi (A), vol. 98, no. 2, pp. K109-K114, (1986).

[10] I.M. Silvestrova, V.V. Bezdelkin, P.A. Senyushenkov, Y.V. Pisarevsky, “Present stage of $La_3Ga_5SiO_{14}$ - research,” in Proc. 47th IEEE Ann. Freq. Cont. Symp., Salt Lake City, Utah, 2-4 June, pp. 348-350, (1993).

[11] V.S. Naumov, I.I. Kalashnikova and S.S. Pashkov, “Stress-induced optical activity in piezoelectric crystals and internal stresses method of control in quartz, langasite, lithium niobate crystals,” in Proc. 48th IEEE Ann. Freq. Cont. Symp., Boston, Massachusetts, 1-3 June, pp. 40-42, (1994).

[12] R. M. Taziev, “Langasite: what temperature coefficients of material constants are correct,” in Proc. Joint Meeting 53rd IEEE Ann. Freq. Cont. Symp and 13rd European Frequency and Time Forum, Besançon, France, 13-16 April, pp. 835-838, (1999).

[13] R. Bourquin and B. Dulmet, “New sets of data for the thermal sensitivity of elastic coefficients of langasite and langatate,” in Proc. 20th European Frequency and Time Forum, Braunschweig, Germany, 27-30 March, to be published, (2006).

[14] L. Delmas, F. Sthal, E. Bigler, B. Dulmet and R. Bourquin, “Temperature-compensated cuts for length–extensional and flexural vibrating modes in $GaPO_4$ beam resonators,” IEEE Trans. on Ultrason., Ferroelec. and Freq. Contr., vol. 52, no. 4, April, pp. 666-671, (2005).

[15] L. Delmas, F. Sthal, E. Bigler, J.J. Boy, S. Galliou, R. Bourquin, "Experimental study of temperature effects in vibrating beam and thickness-shear resonators of $GaPO_4$ machined by ultrasonic milling", in Proc. IEEE Int. Ultrason., Ferroelec., and Freq. Cont. 50th Anniversary Joint Conf., Montreal, Canada, 24-27 August, pp. 625-629, (2004).

[16] “IRE standards on piezoelectric crystals, 1949,” Proc. IRE, vol. 37, pp. 1378-1395, Dec. 1949.

[17] ANSI/IEEE Std 176-1987, “IEEE standards on piezoelectricity”, 1987.