



# Thermal design and operational limits of two-phase micro-channel heat sinks



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## ABSTRACT

While the vast majority of published studies on two-phase micro-channel heat sinks have been focused on determination of pressure drop and heat transfer coefficient, very few studies have addressed the operational limits of these devices. This study provided a comprehensive methodology for thermal design of micro-channel heat sinks with saturated inlet conditions. This includes predictive methods for pressure drop and heat transfer coefficient using universal correlations that rely on large databases amassed from numerous sources, and which encompass many working fluids, and very broad ranges of hydraulic diameter, mass velocity, inlet pressure, and inlet quality. This is followed by predictive tools for thermal limits associated with dryout incipience and premature critical heat flux, as well as two-phase critical flow limit. The three limits are combined to define an envelope for acceptable heat sink performance. Using these tools, a parametric study is performed to determine the variation of maximum heat flux with total volumetric flow rate for different combinations of the channel's geometrical parameters for three working fluids, HFE-7100, R134a, and water. Then, the values of maximum heat flux are used to assess corresponding variations of pressure drop and maximum bottom wall temperature of the heat sink. It is shown that maximum heat flux is dominated by different limits for different flow rate ranges, and may be increased significantly, while decreasing bottom wall temperature, by using a large number of small channels. Furthermore, using deeper micro-channels is shown to increase maximum heat flux and decrease pressure drop, while producing a relatively weak adverse effect on bottom wall temperature.

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## 1. Introduction

### 1.1. High-flux two-phase thermal management

As heat dissipation from electronic and power devices began to escalate beyond the capabilities of air cooling technologies and, later, single-phase liquid cooling technologies, thermal management system designers shifted their attention to phase-change cooling technologies, which capitalize on full cooling potential of the working fluid, both sensible and latent [1].

Phase change cooling can be implemented in a variety of configurations, the simplest of which is pool boiling thermosyphons [2–6]. These systems offer several advantages, including simplicity of design, low cost, and, most importantly, passive circulation of

the working fluid with the aid of buoyancy. But low circulation speeds place upper limits on cooling performance of thermosyphons that are below the operating heat flux in many cutting-edge electronics applications.

These limits have spurred numerous research efforts aimed at achieving superior cooling performance by capitalizing on high coolant speed achieved with pumped liquid loops. Currently, attention is being placed on three primary competing pumped two-phase cooling schemes: jet-impingement, spray, and micro-channel [7]. Each of these schemes provides intrinsic cooling merits while also posing performance challenges and limits.

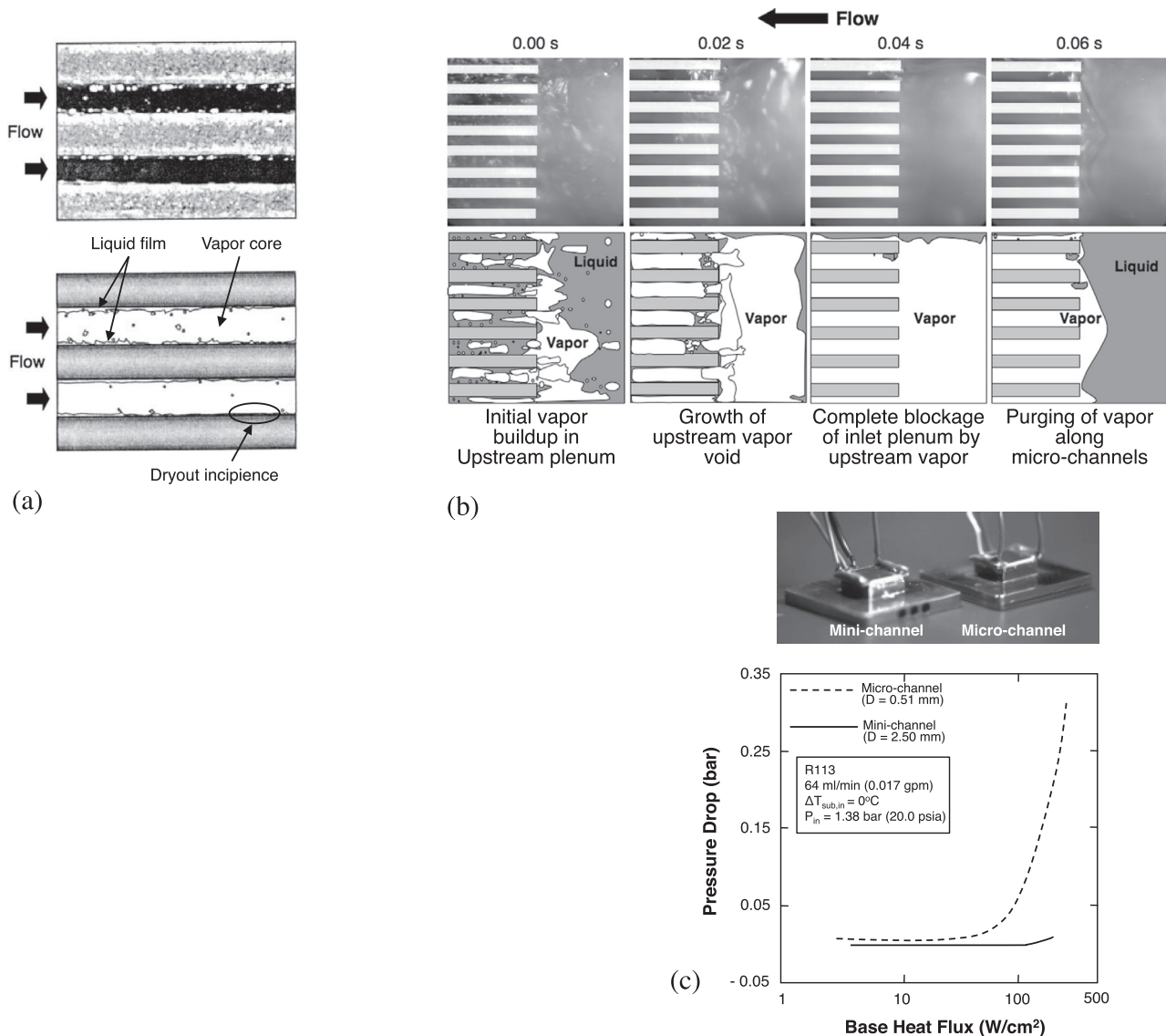
Jet impingement provides enormous cooling heat fluxes, but greatly increases coolant flow rate requirements as well as produces appreciable temperature gradients across the heated wall away from the impingement zone [8–10]. To mitigate large temperature gradients, multiple jets have been recommended [11–13] to create multiple impingement zones, but this tactic poses additional challenges, including increased coolant flow rate

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**Fig. 1.** Design limits for two-phase micro-channel heat sinks. (a) Dryout incipience in R134a for  $D_h = 348.9 \mu\text{m}$ ,  $x = 0.68$ ,  $G = 128 \text{ kg/m}^2 \text{ s}$ , and  $q'' = 31.6 \text{ W/cm}^2$  [54]. (b) Premature CHF and flow oscillations in HFE-7100 for  $D_h = 415.9 \mu\text{m}$ ,  $T_{in} = 0 \text{ }^\circ\text{C}$ ,  $G = 670 \text{ kg/m}^2 \text{ s}$ , and  $q'' > 250.0 \text{ W/cm}^2$  [56]. (c) Pressure drop escalation in R113 at high two-phase Mach number in micro-channel heat sink compared to low pressure drop in mini-channel heat sink for identical inlet conditions [57].

premature critical heat flux (CHF) when dissipating high heat fluxes [21]. In fact, these effects set important performance limits that must be carefully assessed when using a two-phase micro-channel heat sink.

The present study concerns predictions of pressure drop and two-phase heat transfer coefficient for two-phase micro-channel heat sinks, including careful assessment of design envelope dictated by the afore-mentioned limits.

### 1.2. Predictive tools for pressure drop and heat transfer coefficient in two-phase micro-channel heat sinks

Investigators have employed a variety of methods to predict flow boiling pressure drop in micro-channels, which, as discussed in a recent review article by the present authors [25], include theoretical formulations based on the homogeneous equilibrium model [26–31] or empirical and semi-empirical correlations [32–39]; only a few theoretical treatments using the control volume method are presently available (e.g., [40]). Broad utilization of the separated flow model in most recent micro-channel studies

is the result of many experimental studies that have shown the dominance of the annular flow pattern for most operating conditions. It should be also noteworthy to mention that the majority of empirical and semi-empirical correlations are based on the original formulation of Lockhart and Martinelli [41] for separated flow.

The authors' recent review article [42] revealed that the vast majority of available predictive methods for two-phase heat transfer coefficient for micro-channel flows are empirical in nature, in which heat transfer is represented by correlations for nucleate boiling, convective boiling, or both [43–48]. Here too, very few theoretical treatments (e.g., [49]) are presently available.

### 1.3. Dryout and dryout incipience in micro-channels

Critical heat flux is by far the most crucial design and safety parameter for any two-phase scheme used to cool heat-flux-controlled devices, including micro-channel heat sinks. It is the outcome of a substantial reduction in local heat transfer coefficient, brought about by interruption of liquid access to the heated wall. CHF is associated with an unsteady rise in the wall tempera-

ture, the rate of which is dependent on thermophysical properties of the coolant, channel diameter and length, and inlet pressure, inlet quality, and flow rate [50].

*Departure from Nucleate Boiling (DNB)* is the more catastrophic form of CHF, which is generally encountered with highly subcooled inlet conditions, high mass velocities, and short channels [50–53]. It is triggered by vapor blanket formation along the wall, even where liquid is abundant in the channel core, and manifest by fast escalation in the wall temperature.

*Dryout* is typically encountered with saturated or slightly subcooled inlet conditions, low mass velocities, and large length-to-diameter ratios, conditions that culminate in early transition to annular flow [54,55]. The annular film is gradually consumed along the channel by interfacial evaporation, and dryout occurs where the film is fully evaporated. Small diameter in micro-channel heat sinks has been shown to cause significant bubble confinement as well as upstream transition to annular flow, which is why CHF in micro-channel heat sinks is associated mostly with dryout rather than DNB [54,55].

Another cooling limit that is especially important to cooling temperature sensitive electronic devices is *dryout incipience*, which is initiated by localized dryout of portions of the annular liquid film, causing substantial reduction in the heat transfer coefficient even upstream of the location of complete dryout. This phenomenon is depicted in Fig. 1(a) for flow boiling of R134a in a micro-channel heat sink [54].

#### 1.4. Premature CHF

Premature CHF is another important design limit inherent to two-phase micro-channel heat sinks, which is triggered by flow instabilities and oscillations. Fig. 1(b) illustrates this phenomenon for the upstream region of a micro-channel heat sink using HFE-7100 as working fluid [56]. It is especially problematic at low mass velocities, which produce high vapor void fractions along the channels. The resulting high pressure drop offers appreciable resistance to the momentum of incoming liquid in the inlet plenum, and causes backflow of vapor from the micro-channels to the inlet plenum. This action blocks incoming liquid from entering the micro-channels, culminating in dryout in the micro-channels. As discussed in [56], two effective means to overcoming premature CHF are to (i) increase mass velocity, to resist the vapor backflow, and (ii) increase inlet subcooling, to reduce the size of vapor void in the inlet plenum.

#### 1.5. Two-phase critical flow

Two-phase flow in micro-channel heat sinks has recently received significant attention for high-flux applications. Small diameter in these devices is key to achieving high heat transfer coefficients. However, small diameter can greatly increase pressure drop, which raises the likelihood of choking.

Pressure drop problems arising from large property variations along the micro-channel heat sink were first addressed and modeled by Bowers and Mudawar [57]. They compared flow boiling of R113 in two heat sinks, one containing circular micro-channels with  $D = 0.51$  mm, and the other mini-channels with  $D = 2.50$  mm, but both having fairly similar wetted areas. With increasing heat flux, they captured a sharp escalation in pressure drop for the micro-channel heat sink compared to a mild increase for the mini-channel, as shown in Fig. 1(c). High pressure drop in the micro-channel heat sink was attributed to significant compressibility and flashing, caused by large variations of specific volume and enthalpy, respectively, with pressure. The combination of compressibility and flashing also led to large values of two-phase Mach number, raising the potential for two-phase choking. It is

important to note that choking is far more prevalent in two-phase than single-phase systems because critical (sonic) speed for a two-phase liquid–vapor mixture is much smaller than that of pure liquid or pure vapor. This implies choking may constitute a serious design limit for two-phase micro-channel heat sinks. In a recent study, the present authors examined relationships among two-phase critical flow, premature CHF, and CHF for micro-channel heat sinks, and showed how choking can trigger premature CHF and/or CHF at low pressures, especially for very small channel diameters [58].

#### 1.6. Objectives of study

This study is the culmination of a series of studies that have been recently performed at the Purdue University Boiling and Two-Phase Flow Laboratory (PU-BTPFL) in pursuit of accurate predictive tools for two-phase micro-channel heat sinks. They include universal correlations for two-phase pressure drop [59], dryout incipience [60], and two-phase heat transfer coefficient [61], as well as predictive methods for premature CHF [62] and two-phase critical flow [58]. A clear distinction must be made between universal correlations and the majority of correlations available from the literature. Most correlations are limited to a single working fluid or a few fluids, and narrow ranges of geometrical or operating parameters, which greatly compromises their effectiveness as design tools. On the other hand, universal correlations derive

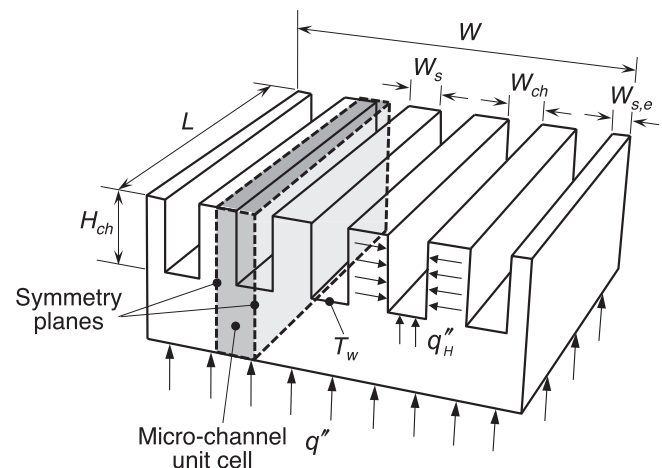


Fig. 2. Schematic of micro-channel heat sink with three-sided wall heating.

Table 1

Design parameters and constraints used in the present study.

	Parameter	Value
Heat sink	Flow direction	Horizontal
	Heated width, $W$	1 cm
	Heated length, $L$	1 cm
	Base area, $A = W \times L$	1 cm <sup>2</sup>
	Flow geometry	Horizontal rectangular
	Channel height, $H_{ch}$	400, 800 $\mu$ m
	Channel width, $W_{ch}$	$40 \leq W_{ch} \leq 1000$ $\mu$ m
	Width of solid sidewall, $W_s$	$W_s \geq 40$ $\mu$ m
Operating conditions	Width of endwall, $W_{s,e}$	$0.5 W_s \leq W_{s,e} \leq 1.0 W_s$
	Thermal conductivity of oxygen-free copper, $k$	391 W/m·K
	Channel inlet pressure, $P_{in}$	3 bar
	Channel inlet quality, $x_{in}$	0.05
	Working fluids	HFE-7100, R134a, water

their effectiveness and predictive accuracy from reliance on large databases amassed from numerous sources, and encompassing many working fluids, and very broad ranges of hydraulic diameter, mass velocity, inlet pressure, and inlet quality.

Using the above-mentioned design tools for saturated flow boiling in micro-channel heat sinks, the primary objectives of the present study are to:

- (1) Provide optimum predictive methods for total pressure drop across micro-channel heat sink, which dictates pumping power requirements.
- (2) Provide optimum predictive methods for two-phase heat transfer coefficient in micro-channel heat sinks, which is required to predict maximum heat sink temperature, and ensure that the heat sink design is capable of maintaining device temperature safely below its maximum allowable limit.
- (3) Provide optimum predictive methods for heat sink heat transfer limits of dryout incipience and premature CHF, and flow limit of two-phase critical flow.
- (4) Conduct a parametric study to assess various influences on pressure drop, maximum heat sink temperature, and both heat transfer and flow limits in pursuit of superior heat sink design.

## 2. System parameters and predictive methods

### 2.1. Heat sink geometry, working fluids, and operating conditions

Fig. 2 shows a schematic of the two-phase micro-channel heat sink geometry examined in this study, which is also the configuration most widely used in practical electronic cooling packages. The heat sink has a fixed base area of width  $W = 1$  cm and length  $L = 1$  cm, and contains multiple micro-channels. The parametric study will allow for varying dimensions of channel height,  $H_{ch}$ , channel width,  $W_{ch}$ , width of solid wall between micro-channels,  $W_s$ , and width of endwall,  $W_{s,e}$ . The heated width of heat sink,  $W$ , can be related to the number of micro-channels,  $N$ , in the heat sink according to

$$W = NW_{ch} + (N - 1)W_s + 2W_{s,e}. \quad (1)$$

In this parametric study, the maximum value of  $W_{ch}$  is set at 1000  $\mu\text{m}$ , and minimum values of both  $W_{ch}$  and  $W_s$  at 40  $\mu\text{m}$ . Oxygen-free copper, with  $k = 391$  W/m-K, is assumed for heat sink material. All calculations are conducted using a fixed channel inlet pressure of 3 bar, and a fixed inlet quality value of 0.05, meaning the flow enters the micro-channels as a saturated two-phase mixture. To account for performance variations resulting from thermophysical properties, three different working fluids are considered: HFE-7100, R134a, and water. Values or ranges of the heat sink's geometrical parameters are provided in Table 1, while Table 2 summarizes saturated thermophysical properties of the three working fluids corresponding to the inlet pressure of 3 bar. Despite the specific limits adopted in this study and specific fluids examined, the primary goal is to assess the influence of key geometrical

parameters on heat sink performance, while, simultaneously, provide a systematic methodology the reader may follow to address other fluids or operating conditions of interest.

In what follows, predictive methods are provided for pressure drop and heat transfer coefficient, followed by important limits associated with dryout incipience, premature CHF, and two-phase critical flow.

### 2.2. Total pressure drop

Total pressure drop between the heat sink's upstream and downstream plenums is the sum of pressure drop contributions of inlet contraction,  $\Delta P_c$ , saturated flow boiling region,  $\Delta P_{tp}$ , and outlet expansion,  $\Delta P_e$ ,

$$\Delta P_{tot} = \Delta P_c + \Delta P_{tp} + \Delta P_e. \quad (2)$$

The contraction pressure loss and expansion recovery at the inlet and outlet of the micro-channels are determined according to [63]

$$\Delta P_c = \frac{G^2 v_f}{2} \left[ \left( \frac{1}{C_c} - 1 \right)^2 + (1 - \sigma_c^2) \right] \left[ 1 + \frac{v_{fg} x_{in}}{v_f} \right] \quad (3)$$

$$\text{and} \quad \Delta P_e = G^2 \sigma_c (\sigma_c - 1) v_f \left[ 1 + \frac{v_{fg} x_{out}}{v_f} \right], \quad (4)$$

respectively. In the above equations, plenum height is set equal to the micro-channel height, thus the area ratio is given by

$$\sigma_c = \frac{W_{ch} N}{W}. \quad (5)$$

Additionally, the flow quality is assumed equal to the thermodynamic equilibrium quality, and this assumption is used in all subsequent equations. Based on recommendations by Schmidt and Friedel [64] and Abdelal et al. [65], the vena-contracta effect can be neglected for saturated inlet conditions, allowing the contraction coefficient in Eq. (3) to be set at  $C_c = 1$ .

For horizontal channels, gravitational pressure drop is neglected and the two-phase pressure drop can be expressed as the sum of accelerational and frictional components,

$$\Delta P_{tp} = \Delta P_{tp,A} + \Delta P_{tp,F}. \quad (6)$$

The accelerational pressure gradient is given by

$$-\left( \frac{dP}{dz} \right)_A = G^2 \frac{d}{dz} \left[ \frac{v_g x^2}{\alpha} + \frac{v_f (1-x)^2}{(1-\alpha)} \right], \quad (7)$$

where the void fraction,  $\alpha$ , is expressed in terms of quality using Zivi's relation [66],

$$\alpha = \left[ 1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_g}{\rho_f} \right)^{2/3} \right]^{-1}. \quad (8)$$

The two-phase frictional pressure drop,  $\Delta P_{tp,F}$ , is both a major contributor to total pressure drop and the most elusive of all pressure drop components. In the present study,  $\Delta P_{tp,F}$  is determined using the authors' universal pressure drop correlation [59] for mini/micro-channels. This universal correlation is derived from a

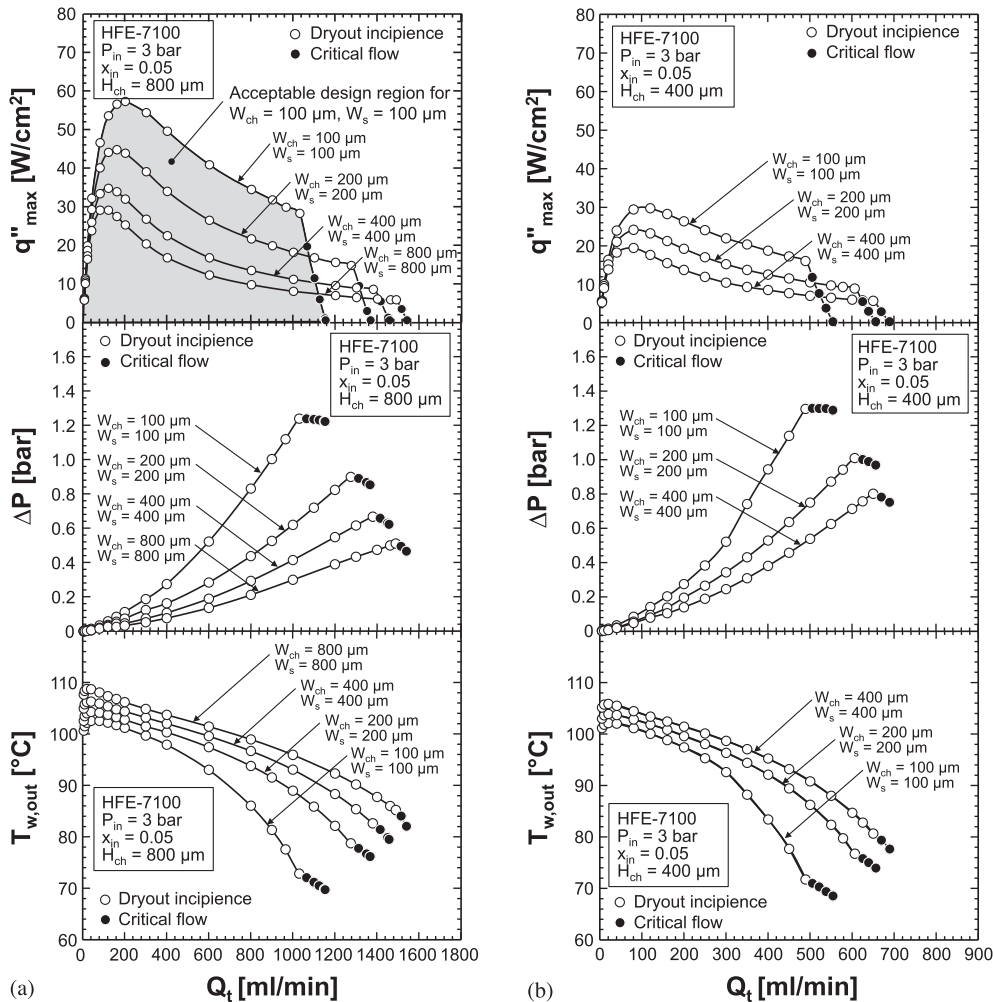
**Table 2**  
Thermophysical properties of saturated HFE-7100, R134a, and water at  $P_{sat} = 3$  bar.

	$T_{sat}$ [°C]	$\rho_f$ [kg/m <sup>3</sup> ]	$\rho_g$ [kg/m <sup>3</sup> ]	$\mu_f$ [kg/m s]	$h_f$ [kJ/kg]	$h_{fg}$ [kJ/kg]	$k_f$ [W/m K]	$c_{p,f}$ [J/kg K]	$\sigma$ [mN/m]
HFE-7100	98.7	1258	27.7	$2.48 \times 10^{-4}$	140.1	98.4	0.0545	1330	7.5
R134a	0.70	1293	14.8	$2.64 \times 10^{-4}$	200.9	198.1	0.0917	1343	11.5
Water	133.5	932	1.65	$2.07 \times 10^{-4}$	561.4	2164	0.6837	4269	52.2



**Table 3**  
Summary of key predictive methods used in the present study.

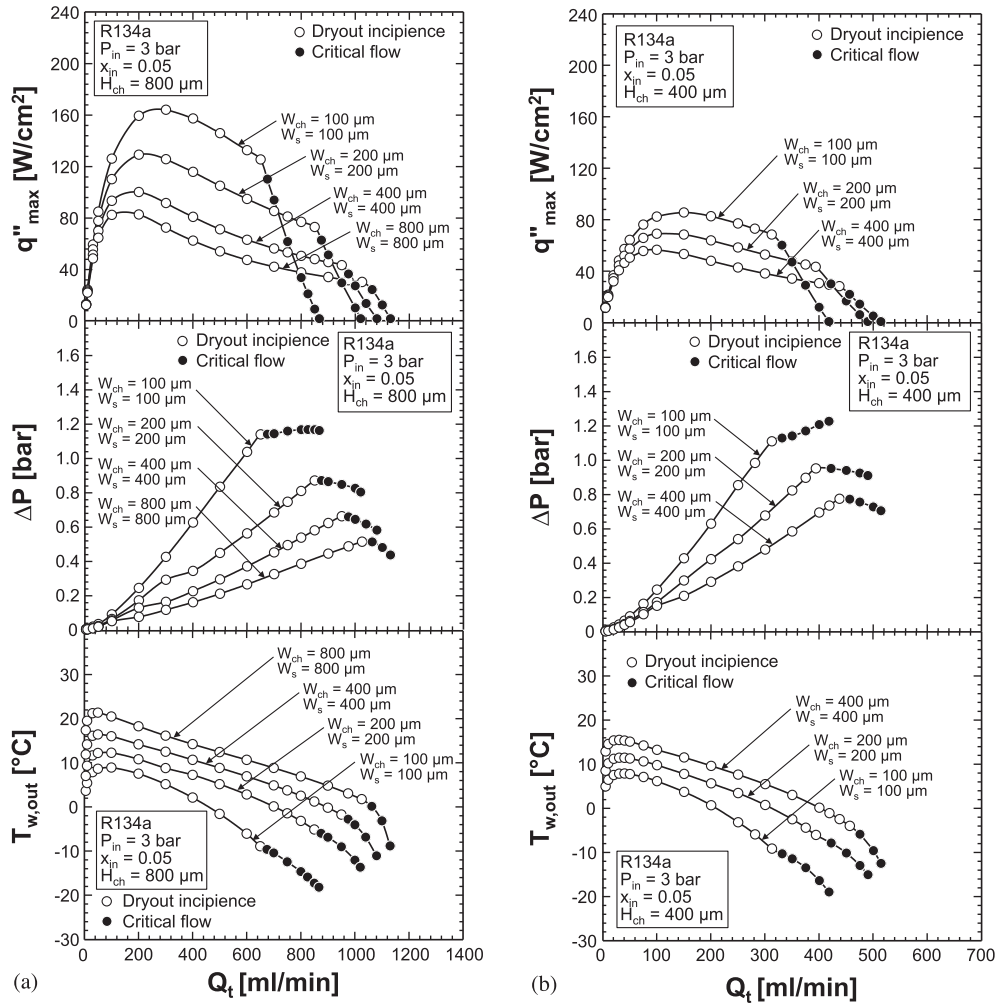
	Equation	Remarks
Frictional pressure gradient [59]	$\left(\frac{dp}{dz}\right)_F = \left(\frac{dp}{dz}\right)_f \phi_f^2$ <p>where <math>\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}</math>, <math>X^2 = \frac{(dp/dz)_g}{(dp/dz)_l}</math>, and C is determined from Eqs. (16a) and (16b)</p>	<ul style="list-style-type: none"> <li>• <math>0.349 &lt; D_h &lt; 5.35</math> mm</li> <li>• Fluids: FC-72, R12, R134a, R22, R245fa, R410A, ammonia, CO<sub>2</sub>, and water</li> <li>• Universal correlation based on 2378 data points from 16 sources</li> </ul>
Heat transfer coefficient for saturated flow boiling [61]	$h_{tp} = (h_{nb}^2 + h_{cb}^2)^{0.5}$ <p>where <math>h_{nb}</math> and <math>h_{cb}</math> are determined from Eqs. (24a) and (24b), respectively</p>	<ul style="list-style-type: none"> <li>• <math>0.19 &lt; D_h &lt; 6.5</math> mm</li> <li>• Fluids: FC-72, R11, R113, R123, R1234yf, R1234ze, R134a, R152a, R22, R236fa, R245fa, R32, R404A, R407C, R410A, R417A, CO<sub>2</sub>, and water</li> <li>• Universal correlation based on 10,805 pre-dryout data points from 37 sources</li> </ul>
Dryout incipience [60]	$x_{di} = 1.4We_{fo}^{0.03} p_R^{0.08} - 15.0(Bo_{FF}^{0.15} Ca^{0.35} \left(\frac{\rho_g}{\rho_l}\right)^{0.06}$	<ul style="list-style-type: none"> <li>• <math>0.51 &lt; D_h &lt; 6.0</math> mm</li> <li>• Fluids: FC-72, R113, R1234yf, R1234ze, R134a, R22, R245fa, R290, R32, R407C, R410A, CO<sub>2</sub>, and water</li> <li>• Universal correlation based on 997 data points from 26 sources</li> </ul>
Premature critical heat flux [62]	$q''_{p-CHF} = 33.43Gh_{fg} \left(\frac{\rho_g}{\rho_l}\right)^{1.11} We_L^{-0.21} \left(\frac{L}{D_h}\right)^{-0.36}$	<ul style="list-style-type: none"> <li>• <math>D_h = 341</math> <math>\mu</math>m for water (three-sided heated)</li> <li>• <math>D = 2.54</math> mm, <math>510</math> <math>\mu</math>m for R113</li> </ul>
Two-phase critical flow [58]	$G_c = \left\{ - \left[ x \frac{dV_c}{dP} + (1-x) \frac{dV_l}{dP} \right] \right\}^{-0.5}$	<ul style="list-style-type: none"> <li>• <math>1.33 &lt; D &lt; 5.4</math> mm</li> <li>• Fluids: water, R134a, air–water</li> <li>• Choking data through pipes, short tube orifices, and short nozzles</li> </ul>



**Fig. 3.** Variations of maximum base heat flux and corresponding values of total pressure drop and bottom wall temperature at channel exit with total volume flow rate for different channel widths and sidewall thicknesses using HFE-7100 as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

database consisting of 2378 data points amassed from 16 sources. The database consists of 9 working fluids, hydraulic diameters from 0.349 to 5.35 mm, mass velocities from 33 to 2738 kg/m<sup>2</sup> s,

liquid-only Reynolds numbers from 156 to 28,010, qualities from 0 to 1, reduced pressures from 0.005 to 0.78, and both single-channels and multi-channel heat sinks. Formulation of pressure



**Fig. 4.** Variations of maximum base heat flux and corresponding values of total pressure drop and bottom wall temperature at channel exit with total volume flow rate for different channel widths and sidewall thicknesses using R134a as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

drop in this correlation is based on the Lockhart-Martinelli separated flow model [41], where the two-phase frictional pressure gradient is expressed as the product of the frictional pressure gradient for the liquid phase and a two-phase pressure drop multiplier.

$$\left(\frac{dP}{dz}\right)_F = \left(\frac{dP}{dz}\right)_f \phi_f^2, \quad (9)$$

$$\text{where } \phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2}, \quad (10)$$

and  $X$  is the Lockhart–Martinelli parameter, which is the ratio of frictional pressure gradient for liquid to that for vapor, based on actual flow rates of the individual phases.

$$X^2 = \frac{(dP/dz)_f}{(dP/dz)_g}, \quad (11)$$

$$\text{where } -\left(\frac{dP}{dz}\right)_f = \frac{2f_f v_f G^2 (1-x)^2}{D_h} \quad (12a)$$

$$\text{and } -\left(\frac{dP}{dz}\right)_g = \frac{2f_g v_g G^2 x^2}{D_h}. \quad (12b)$$

The friction factor for phase  $k$  ( $k = f$  for liquid or  $g$  for vapor) in Eqs. (12a) and (12b) is given by

$$f_k = 16 Re_k^{-1} \quad \text{for } Re_k < 2000, \quad (13a)$$

$$f_k = 0.079 Re_k^{-0.25} \quad \text{for } 2000 \leq Re_k < 20,000, \quad (13b)$$

$$\text{and } f_k = 0.046 Re_k^{-0.2} \quad \text{for } Re_k \geq 20,000, \quad (13c)$$

$$\text{where } Re_k = Re_f = \frac{G(1-x)D_h}{\mu_f} \quad \text{for liquid}, \quad (14a)$$

$$\text{and } Re_k = Re_g = \frac{GxD_h}{\mu_g} \quad \text{for vapor}. \quad (14b)$$

For laminar flow in a rectangular channel, the two-phase friction factor can be expressed as a function of the channel's aspect ratio [67],

$$f_k Re_k = 24 \left( 1 - 1.3553\beta + 1.9467\beta^2 - 1.7012\beta^3 + 0.9564\beta^4 - 0.2537\beta^5 \right). \quad (15)$$

For flow boiling in micro-channels, the function  $C$  in Eq. (10) for the two-phase pressure drop multiplier is calculated using

$$C = C_{non-boiling} \left[ 1 + 60 We_{fo}^{0.32} \left( Bo \frac{P_H}{P_F} \right)^{0.78} \right] \quad \text{for } Re_f \geq 2000 \quad (16a)$$

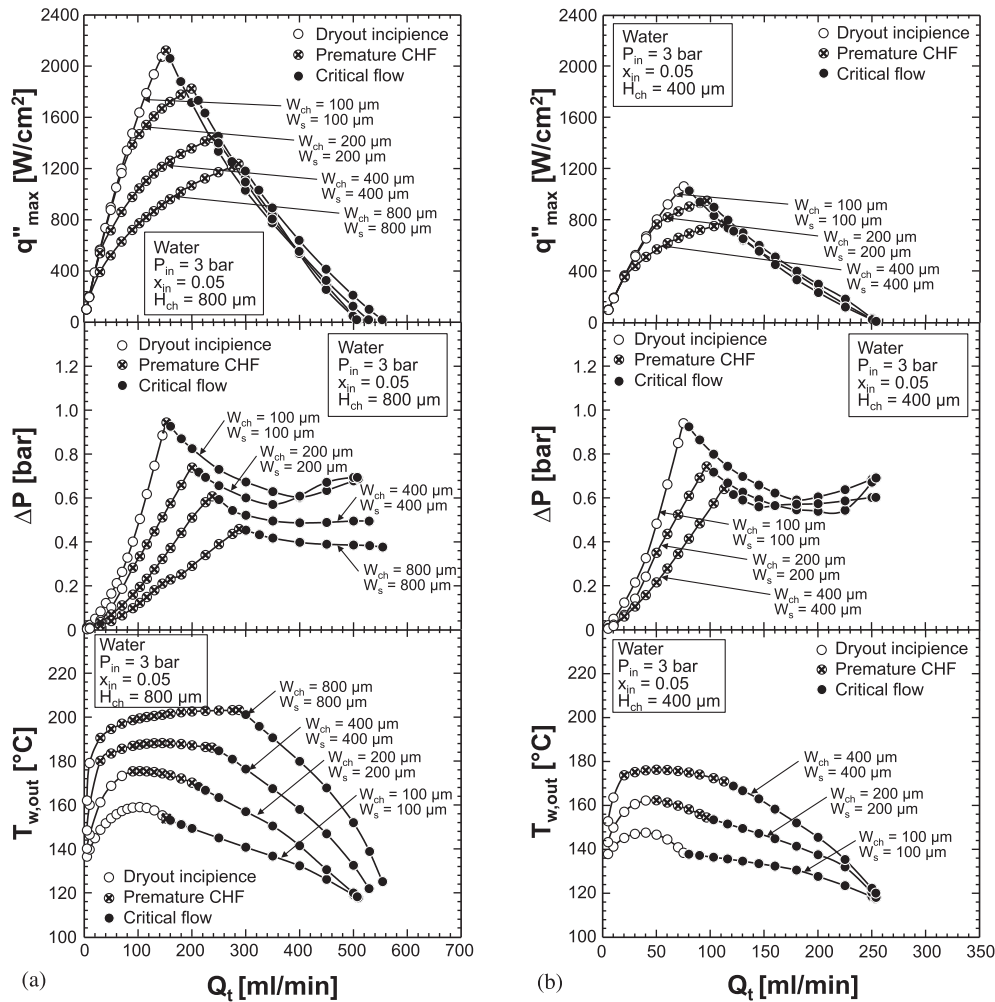


Fig. 5. Variations of maximum base heat flux and corresponding values of total pressure drop and bottom wall temperature at channel exit with total volume flow rate for different channel widths and sidewall thicknesses using water as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

$$\text{and } C = C_{non-boiling} \left[ 1 + 530 We_{fo}^{0.52} \left( Bo \frac{P_H}{P_F} \right)^{1.09} \right] \text{ for } Re_f < 2000, \quad (16b)$$

where  $P_H/P_F$  is the ratio of the flow channel's heated to wetted perimeters, given by  $P_H/P_F = (W_{ch} + 2H_{ch}) / (2W_{ch} + 2H_{ch})$  for three-sided heating as shown in Fig. 2. In Eqs. (16a) and (16b), the liquid-only Weber number is defined as

$$We_{fo} = \frac{G^2 D_h}{\rho_f \sigma}, \quad (17)$$

and Boiling number expressed in terms of  $q''_H$ , the effective heat flux averaged over the heated perimeter of the channel,

$$Bo = \frac{q''_H}{G h_{fg}}. \quad (18)$$

Also in Eqs. (16a) and (16b), the function  $C_{non-boiling}$  was derived by the authors for non-boiling (adiabatic and condensing) flows [68] for different combinations of liquid and vapor states according to

$$C_{non-boiling} = 0.39 Re_{fo}^{0.03} Su_{go}^{0.10} \left( \frac{\rho_f}{\rho_g} \right)^{0.35} \text{ for } Re_f \geq 2000, \quad (19a)$$

$$Re_g \geq 2000(tt),$$

$$C_{non-boiling} = 8.7 \times 10^{-4} Re_{fo}^{0.17} Su_{go}^{0.50} \left( \frac{\rho_f}{\rho_g} \right)^{0.14} \text{ for } Re_f \geq 2000, \quad (19b)$$

$$Re_g < 2000(tv),$$

$$C_{non-boiling} = 0.0015 Re_{fo}^{0.59} Su_{go}^{0.19} \left( \frac{\rho_f}{\rho_g} \right)^{0.36} \text{ for } Re_f < 2000, \quad (19c)$$

$$Re_g \geq 2000(vt),$$

$$\text{and } C_{non-boiling} = 3.5 \times 10^{-5} Re_{fo}^{0.44} Su_{go}^{0.50} \left( \frac{\rho_f}{\rho_g} \right)^{0.48} \quad (19d)$$

$$\text{for } Re_f < 2000, Re_g < 2000(vv).$$

The two-phase pressure drop is determined by integrating the relations for accelerational and frictional pressure gradients numerically according to

$$\Delta P_{tp} = \int_0^{L_{tp}} \left[ - \left( \frac{dP}{dz} \right)_A - \left( \frac{dP}{dz} \right)_F \right] dz. \quad (20)$$

In this study, local saturation temperature as well as thermo-physical properties of liquid and vapor are based on local saturation pressure. Thermo-physical properties for R134a and water are obtained using REFPROP 8.0 software from NIST [69], and those for HFE-7100 from 3 M Company.



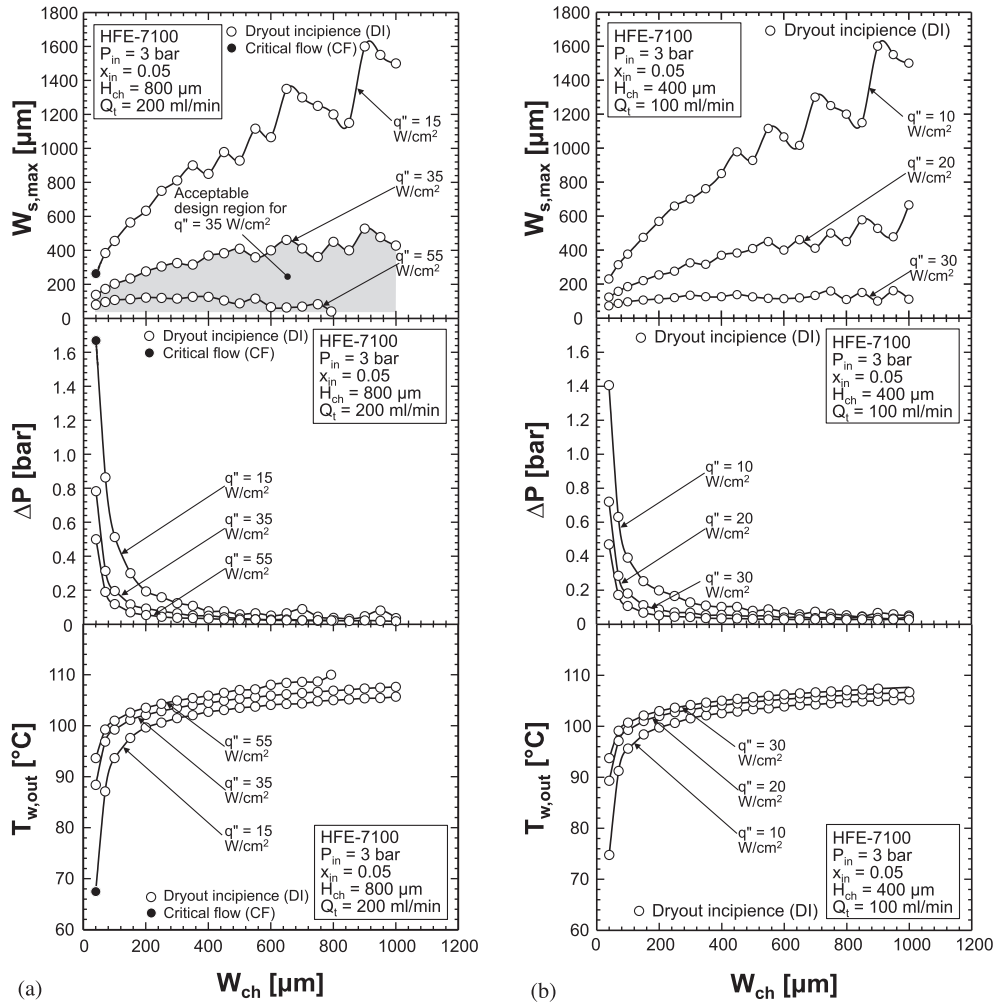


Fig. 6. Variations of maximum sidewall width and corresponding values of total pressure drop and bottom wall temperature at channel exit with channel width for different base heat fluxes using HFE-7100 as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

2.3. Heat transfer in micro-channels

For a rectangular micro-channel heat sink with three-sided heating (i.e., with a perfectly insulating top cover plate), applying the fin analysis method to the micro-channel unit cell shown in Fig. 2 yields the following relation for temperature along the plane of the micro-channel’s bottom wall,

$$T_w = \frac{q''(W_{ch} + W_s)}{h_{tp}(W_{ch} + 2\eta H_{ch})} + T_{sat} = \frac{q''(W_{ch} + 2H_{ch})}{h_{tp}(W_{ch} + 2\eta H_{ch})} + T_{sat}, \quad (21)$$

where the fin efficiency and fin parameter are defined, respectively, as [70]

$$\eta = \frac{\tanh(mH_{ch})}{mH_{ch}} \quad (22a)$$

$$\text{and } m = \sqrt{2h_{tp}/(kW_s)}. \quad (22b)$$

The saturated flow boiling heat transfer coefficient is determined using the authors’ universal heat transfer correlation for saturated flow boiling in mini/micro-channels [61]. This correlation is based on a consolidated database consisting of 10,805 pre-dryout data points amassed from 31 sources, which consists of 18 working fluids, hydraulic diameters of 0.19–6.5 mm, mass velocities of 19–1608 kg/m<sup>2</sup>s, liquid-only Reynolds numbers of 57–49,820,

qualities of 0–1, and reduced pressures of 0.005–0.69. The correlation is constructed by superpositioning the contributions of nucleate boiling and convective boiling according to

$$h_{tp} = (h_{nb}^2 + h_{cb}^2)^{0.5}, \quad (23)$$

$$\text{where } h_{nb} = \left[ 2345 \left( Bo \frac{P_H}{P_F} \right)^{0.70} P_R^{0.38} (1-x)^{-0.51} \right] \left( 0.023 Re_f^{0.8} Pr_f^{0.4} \frac{k_f}{D_h} \right) \quad (24a)$$

$$\text{and } h_{cb} = \left[ 5.2 \left( Bo \frac{P_H}{P_F} \right)^{0.08} We_{fo}^{-0.54} + 3.5 \left( \frac{1}{X_{tt}} \right)^{0.94} \left( \frac{\rho_g}{\rho_f} \right)^{0.25} \right] \times \left( 0.023 Re_f^{0.8} Pr_f^{0.4} \frac{k_f}{D_h} \right). \quad (24b)$$

The reduced pressure,  $P_R$ , in Eq. (24a) and Lockhart-Martinelli parameter,  $X_{tt}$ , in Eq. (24b) are defined, respectively, as

$$P_R = \frac{P}{P_{crit}} \quad (25)$$

$$\text{and } X_{tt} = \left( \frac{\mu_f}{\mu_g} \right)^{0.1} \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{\rho_g}{\rho_f} \right)^{0.5}. \quad (26)$$

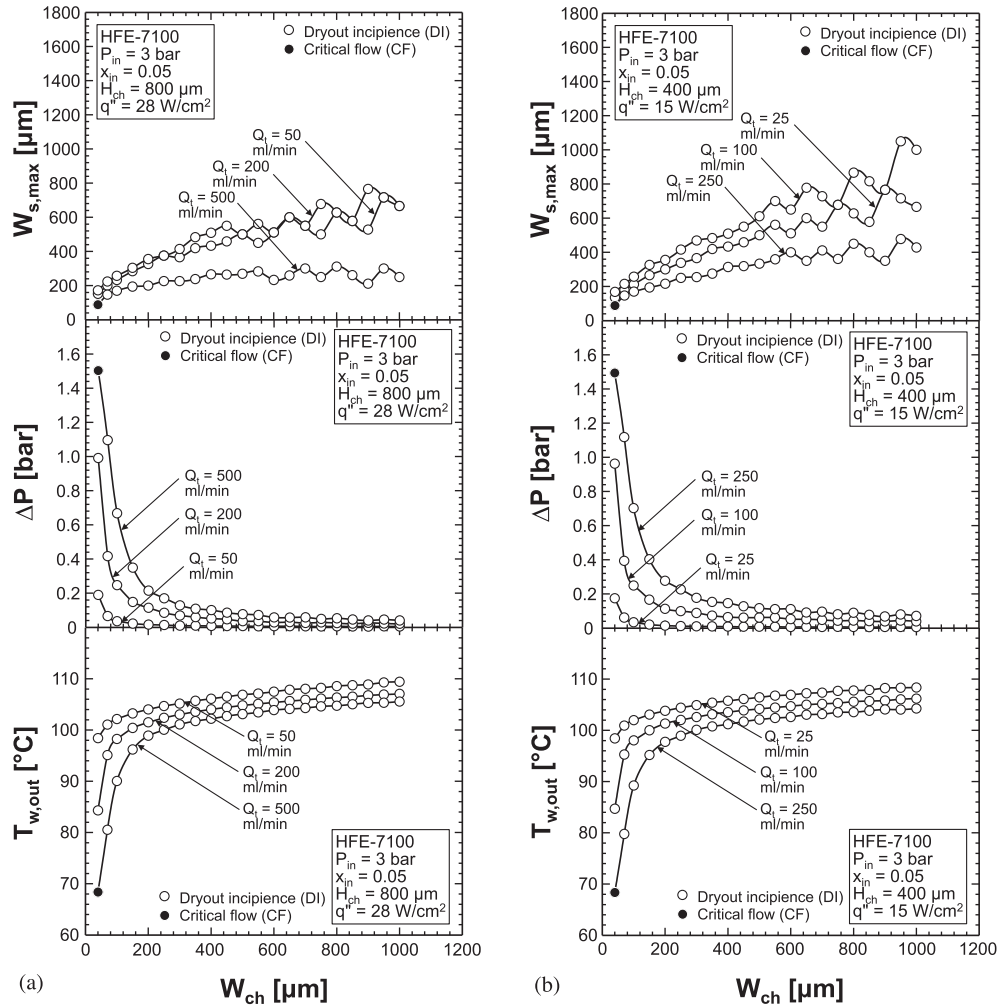


Fig. 7. Variations of maximum sidewall width and corresponding values of total pressure drop and bottom wall temperature at channel exit with channel width for different total volume flow rates using HFE-7100 as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

## 2.4. Heat transfer and flow limits

### 2.4.1. Dryout incipience

CHF is arguably the most important thermal design parameter for micro-channel heat sinks intended for cooling heat-flux controlled devices, and exceeding this limit can lead to catastrophic device failure due to overheating. As discussed earlier, CHF in micro-channels is triggered mostly by complete dryout of the annular liquid film. An important precursor to this event is dryout incipience, corresponding to localized dryout of the annular film, which occurs upstream of the point of complete dryout. Dryout incipience therefore serves as a very effective thermal design limit for heat flux, including a safety margin. The quality  $x_{di}$  corresponding to dryout incipience is determined from the authors' recent universal correlation [60] consisting of 997 data points from 26 source, which consists of 13 working fluids, hydraulic diameters from 0.51 to 6.0 mm, mass velocities from 29 to 2303 kg/m<sup>2</sup> s, liquid-only Reynolds numbers from 125 to 53,770, Boiling numbers from  $0.31 \times 10^{-4}$  to  $44.3 \times 10^{-4}$ , and reduced pressures from 0.005 to 0.78. The dryout incipience quality is given by

$$x_{di} = 1.4We_{fo}^{0.03}P_R^{0.08} - 15.0\left(Bo\frac{P_H}{P_F}\right)^{0.15}Ca^{0.35}\left(\frac{\rho_g}{\rho_f}\right)^{0.06}, \quad (27)$$

where  $Ca$  is the capillary number, defined as

$$Ca = \frac{\mu_f G}{\rho_f \sigma}. \quad (28)$$

### 2.4.2. Premature critical heat flux

As discussed earlier, premature CHF is associated with appreciable flow reversal and instabilities. In the present study, it is calculated using the premature CHF correlation by Qu and Mudawar [62]. This correlation is based on water data for a micro-channel heat sink with  $D_h = 341 \mu\text{m}$  and three-sided wall heating, and R113 data for two separate heat sinks, one containing circular mini-channels with  $D = 2.54 \text{ mm}$ , and the other circular micro-channels with  $D = 0.51 \text{ mm}$ . This correlation allows the prediction of heat flux corresponding to premature CHF averaged over the heated perimeter of channel,

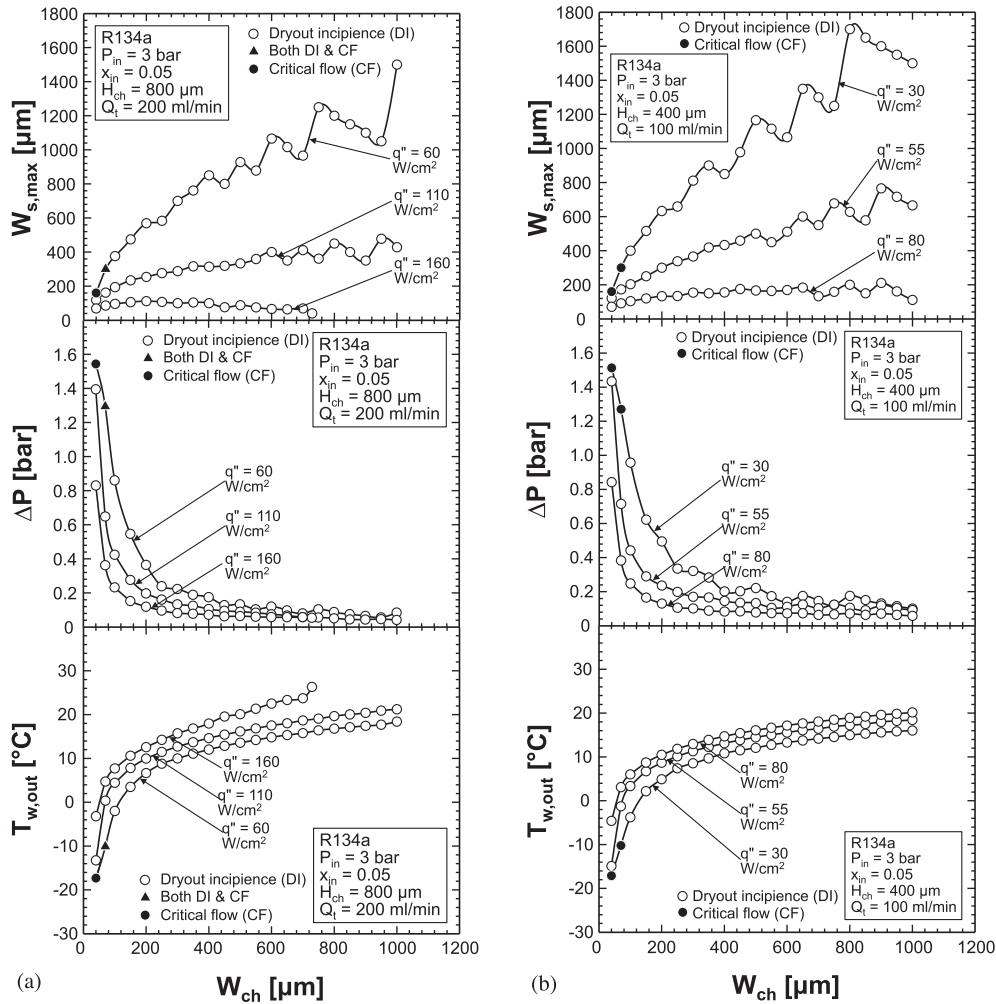
$$q''_{p-CHF} = 33.43Gh_{fg}\left(\frac{\rho_g}{\rho_f}\right)^{1.11}We_L^{-0.21}\left(\frac{L}{D_h}\right)^{-0.36}, \quad (29)$$

where  $We_L$  is Weber number based on the channel's heated length, defined as

$$We_L = \frac{G^2 L}{\rho_f \sigma}. \quad (30)$$

### 2.4.3. Two-phase critical flow (two-phase choking)

In a previous study by the authors [58] addressing the relationship between two-phase critical flow, premature CHF, and CHF in



**Fig. 8.** Variations of maximum sidewall width and corresponding values of total pressure drop and bottom wall temperature at channel exit with channel width for different base heat fluxes using R134a as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

micro-channel heat sinks, the Homogeneous Frozen Model (HFM) derived by Wallis [71] for homogeneous flow with negligible flashing, showed the most superior predictions of experimental critical flow data. In the present study, the HFM is used to predict mass velocity corresponding to critical flow (also commonly referred to as two-phase choked flow) according to

$$G_c = \left\{ - \left[ x \frac{dv_g}{dP} + (1-x) \frac{dv_f}{dP} \right] \right\}^{-0.5} \quad (31)$$

### 2.5. Overall calculation methodology

Summarized in Table 3 are key predictive methods used in the present study along with the applicable range of channel diameter and working fluids. Notice that the correlations for frictional pressure gradient [59], saturated flow boiling heat transfer [61], and dryout incipience [62] are generalized universal predictive tools intended specifically for mini/micro-channels and applicable to very broad ranges of operating parameters and many working fluids. Using a finite difference scheme, relevant equations are solved numerically by dividing the channel length into small  $\Delta z$  increments and marching forward starting with known micro-channel inlet conditions. Local values of  $x_{di}$  and  $G_c$  are calculated at every node along the micro-channel based on local thermophysical properties using Eqs. (27) and (31), respectively. Occurrence of dryout

incipience or two-phase critical flow is checked by comparing local values of  $x$  and  $x_{di}$ , and  $G$  and  $G_c$ , respectively, at each node. The occurrence of premature CHF is determined by comparing  $q''_H$  and  $q''_{P-CHF}$ , where thermophysical properties are based on the channel's outlet pressure.

## 3. Thermal design of micro-channel heat sinks for HFE-7100, R134a, and water

### 3.1. Maximum heat flux and corresponding pressure drop and wall temperature

Figs. 3–5 show variations of maximum base heat flux,  $q''_{max}$ , and corresponding values of total pressure drop,  $\Delta P$ , and bottom wall temperature at channel exit,  $T_{w,out}$ , (heat sink's highest bottom wall temperature) with total volume flow rate,  $Q_t$ , for different channel widths and sidewall thicknesses using HFE-7100, R134a, and water, respectively, as working fluids. As discussed earlier, cooling performance of the micro-channel heat sink is governed by two heat transfer limits, dryout incipience and premature CHF, and one flow limit, two-phase critical flow. Only one of the three limits will be dominant for a given combination of  $H_{ch}$ ,  $W_{ch}$ ,  $W_s$ , and  $Q_t$ , and the dominant limit is therefore used to determine  $q''_{max}$ .

For HFE-7100, Fig. 3(a) and (b), and R134a, Fig. 4(a) and (b),  $q''_{max}$  is limited by dryout incipience for low  $Q_t$ , and two-phase critical

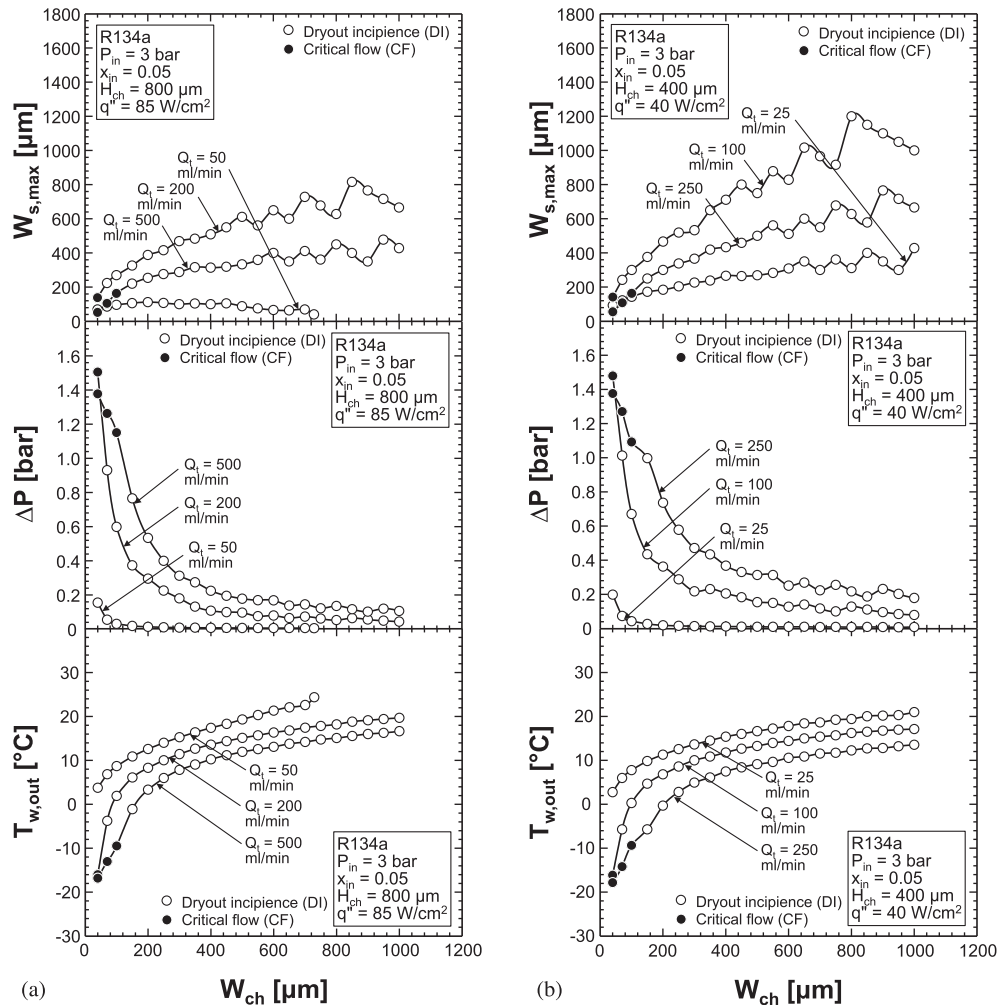


Fig. 9. Variations of maximum width of sidewall width and corresponding values of total pressure drop and bottom wall temperature at channel exit with channel width for different total volume flow rates using R134a as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

flow for high  $Q_t$ . As  $Q_t$  is increased within the dryout incipience dominant region,  $q''_{max}$  increases at a steep rate up to a certain point then decreases gradually. Within the critical flow dominant region,  $q''_{max}$  decreases very sharply. Notice that, for a given fluid and prescribed set of operating conditions, the acceptable design range for safe heat sink operation is confined to the region below the line corresponding to  $q''_{max}$ , which is depicted by the gray area in Fig. 3(a) corresponding to HFE-7100 with  $P_{in} = 3 \text{ bar}$ ,  $x_{in} = 0.05$ ,  $H_{ch} = 800 \mu\text{m}$ , and  $W_{ch} = W_s = 100 \mu\text{m}$ . Notice in Figs. 3(a), (b) and 4(a), (b) how smaller values of  $W_{ch}$  and  $W_s$  increase  $q''_{max}$  but promote transition to the critical flow dominant region at a smaller  $Q_t$ . This is further confirmation that smaller hydraulic diameter is advantageous to micro-channel cooling.

As shown in Fig. 3(a) and (b) for HFE-7100, and 4(a) and 4(b) for R134a,  $\Delta P$  calculated by setting  $q'' = q''_{max}$  increases monotonically with increasing  $Q_t$  within the dryout incipience dominant region but generally decreases in the critical flow dominant region. Excepting very low  $Q_t$  values,  $T_{w,out}$  calculated by setting  $q'' = q''_{max}$  decreases monotonically with increasing  $Q_t$  within both the dryout incipience and critical flow dominant regions. Figs. 3(a), (b) and 4(a), (b) clearly show the cooling advantage of decreasing  $W_{ch}$  and  $W_s$  by decreasing  $T_{w,out}$ . However, the advantage of smaller hydraulic diameter is realized with a higher  $\Delta P$  penalty.

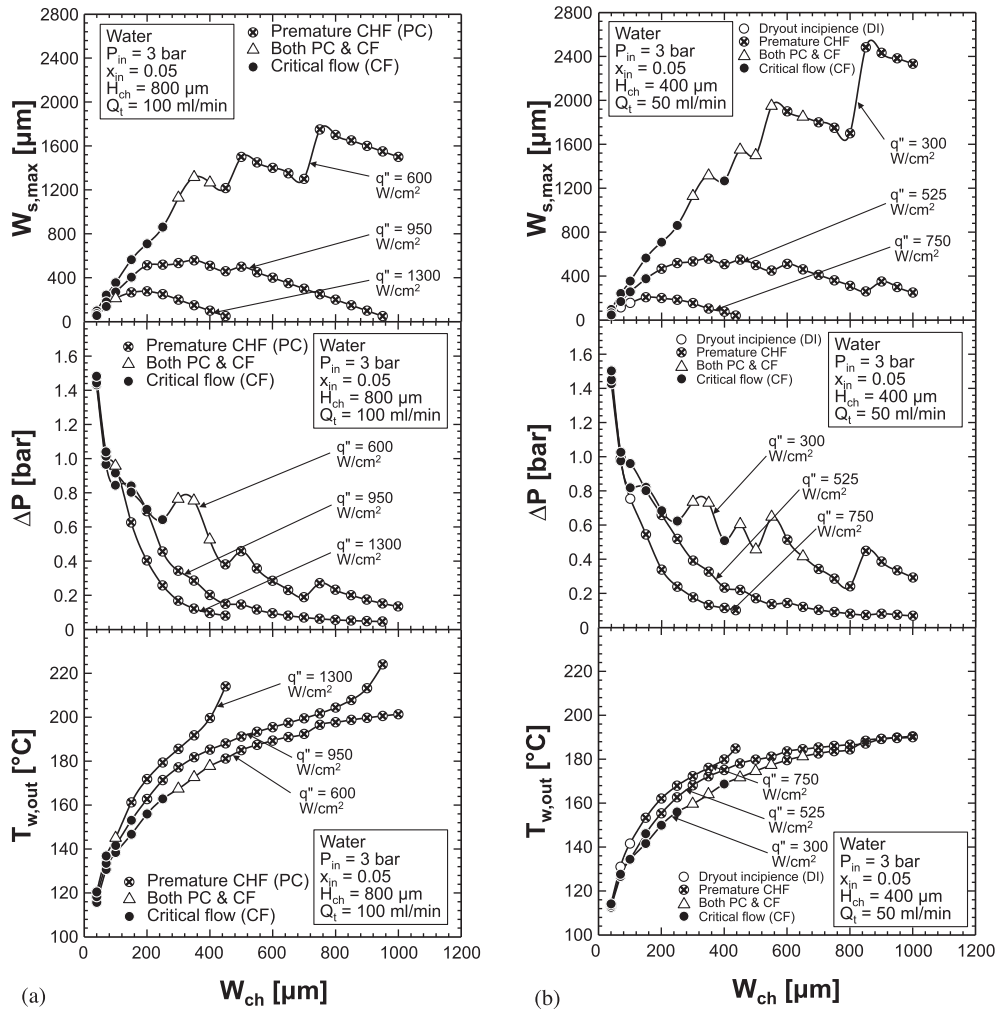
Fig. 5(a) and (b) shows that, unlike HFE-7100 and R134a, where  $q''_{max}$  is limited by either dryout incipience or critical flow,  $q''_{max}$  for water can also be limited by premature CHF, which is smaller than

$q''_{max}$  corresponding to dryout incipience. Decreasing  $W_{ch}$  and  $W_s$  increases  $q''_{max}$  for both the dryout incipience and premature CHF dominant regions corresponding to lower  $Q_t$  values, but the influence  $W_{ch}$  and  $W_s$  on  $q''_{max}$  is minimal at high  $Q_t$ . Similar to HFE-7100 and R134a, decreasing  $W_{ch}$  and  $W_s$  provides better cooling for water, evidenced by lower  $T_{w,out}$ , and this is advantage is realized with a larger pressure drop penalty.

For all three fluids, the larger channel height of  $H_{ch} = 800 \mu\text{m}$  (i.e., deeper micro-channels), Figs. 3(a), 4(a), and 5(a), yields appreciable enhancement in cooling performance, nearly doubling peak  $q''_{max}$  compared to  $H_{ch} = 400 \mu\text{m}$ , Figs. 3(b), 4(b), and 5(b), while producing a relatively weak adverse effect on  $T_{w,out}$ . Another advantage of the deeper micro-channel is greatly decreasing  $\Delta P$ .

Overall,  $q''_{max}$  for water is far greater than for both HFE-7100 and R134a without suffering appreciable pressure drop, while R134a achieves lower  $T_{w,out}$  compared to both HFE-7100 and water.

The trends of  $q''_{max}$  and corresponding  $\Delta P$  and  $T_{w,out}$  provide important guidelines when addressing different aspects of micro-channel performance, namely, upper flux limit, pumping power penalty, and cooling effectiveness. Overall, this design methodology allows a cooling system designer to achieve the best compromise when selecting optimum ranges of heat sink geometrical parameters and operating condition, in terms of safely dissipating the heat, maintaining device temperature below an acceptable limit, and minimizing pumping power.



**Fig. 10.** Variations of maximum sidewall width and corresponding values of total pressure drop and bottom wall temperature at channel exit with channel width for different base heat fluxes using water as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

**3.2. Maximum sidewall width and corresponding pressure drop and wall temperature**

The results discussed above are for sidewall thickness equal to channel width,  $W_s = W_{ch}$ . However, sidewall thickness represents another independent parameter that has a significant influence on pressure drop and thermal design. For fixed  $W_{ch}$ , Eq. (1) shows that increasing  $W_s$  decreases the number of micro-channels,  $N$ , in the heat sink, therefore increasing mass velocity,  $G$ . Therefore, the highest  $G$  is achieved when sidewall thickness is equal to  $W_{s,max}$ . Fig. 6(a) and (b) show, for HFE-7100, variations of maximum sidewall width,  $W_{s,max}$ , and corresponding values of  $\Delta P$  and  $T_{w,out}$  (calculated by setting  $W_s = W_{s,max}$ ) with  $W_{ch}$  for different values of  $q''$ ; Fig. 7(a) and (b) show variations of  $W_{s,max}$  for different values of  $Q_t$ . Similar plots are shown for R134a in Figs. 8(a), (b) and 9(a), (b) and for water in Figs. 10(a), (b) and 11(a), (b).

For HFE-7100 and R134a,  $W_{s,max}$  is limited mostly by dryout incipience for most  $W_{ch}$  values, and by critical flow for very small  $W_{ch}$ . For water,  $W_{s,max}$  is limited mostly by two-phase critical flow for small  $W_{ch}$  values, and premature CHF for larger  $W_{ch}$  values. However, as shown in Fig. 11(a) and (b),  $W_{s,max}$  for high  $Q_t$  is limited by critical flow over the entire range of  $W_{ch}$ .

As shown in Figs. 6(a), (b), and 7(a), (b) for HFE-7100, Figs. 8(a), (b), and 9(a), (b) for R134a, and Figs. 10(a), (b), and 11(a), (b) for water,  $W_{s,max}$  generally increases with increasing  $W_{ch}$ . However,

there are some exceptions to this trend, which include higher  $q''$  cases for HFE-7100, Fig. 6(a), R134a, Fig. 8(a), and water, Fig. 10 (a) and (b), where  $W_{s,max}$  increases to a maximum then decreases to the minimum value of  $40 \mu\text{m}$  corresponding to the  $W_s$  constraint given in Table 1. A similar trend is observed for lower  $Q_t$  cases for R134a, Fig. 9(a), and water, Fig. 11(a) and (b). Notice that the fluctuations in parametric trends of  $W_{s,max}$ , especially for high  $W_{ch}$  values, are associated with the smaller number of channels in the heat sink causing appreciable changes in  $W_{s,max}$  value with respect to  $W_{ch}$ . The acceptable design range is limited by the region below the line corresponding to  $W_{s,max}$  for a given set of operating conditions. This is represented by the gray area in Fig. 6(a) for HFE-7100 with  $H_{ch} = 800 \mu\text{m}$  and  $q'' = 35 \text{ W/cm}^2$ .

Figs. 6(a), (b), and 7(a), (b) for HFE-7100, Figs. 8(a), (b), and 9 (a), (b) for R134a, and Figs. 10(a), (b), and 11(a), (b) for water, all show increasing  $W_{ch}$  causes a decrease in  $\Delta P$  coupled with an increase in  $T_{w,out}$  corresponding to  $W_{s,max}$ . These trends are the result of the increase in  $W_{ch}$  decreasing the number of micro-channels,  $N$ , which also decreases mass velocity,  $G$ , in the micro-channels. Notice in the same figures how two-phase critical flow is encountered at smaller values of  $W_{ch}$ , where  $\Delta P$  is highest and  $T_{w,out}$  lowest.

Fig. 6(a) and (b) for HFE-7100, Fig. 8(a) and (b) for R134a, and Fig. 10(a) and (b) for water show the influence of  $q''$  on  $W_{s,max}$  for fixed  $Q_t$  values corresponding to the highest  $q''_{max}$  value from



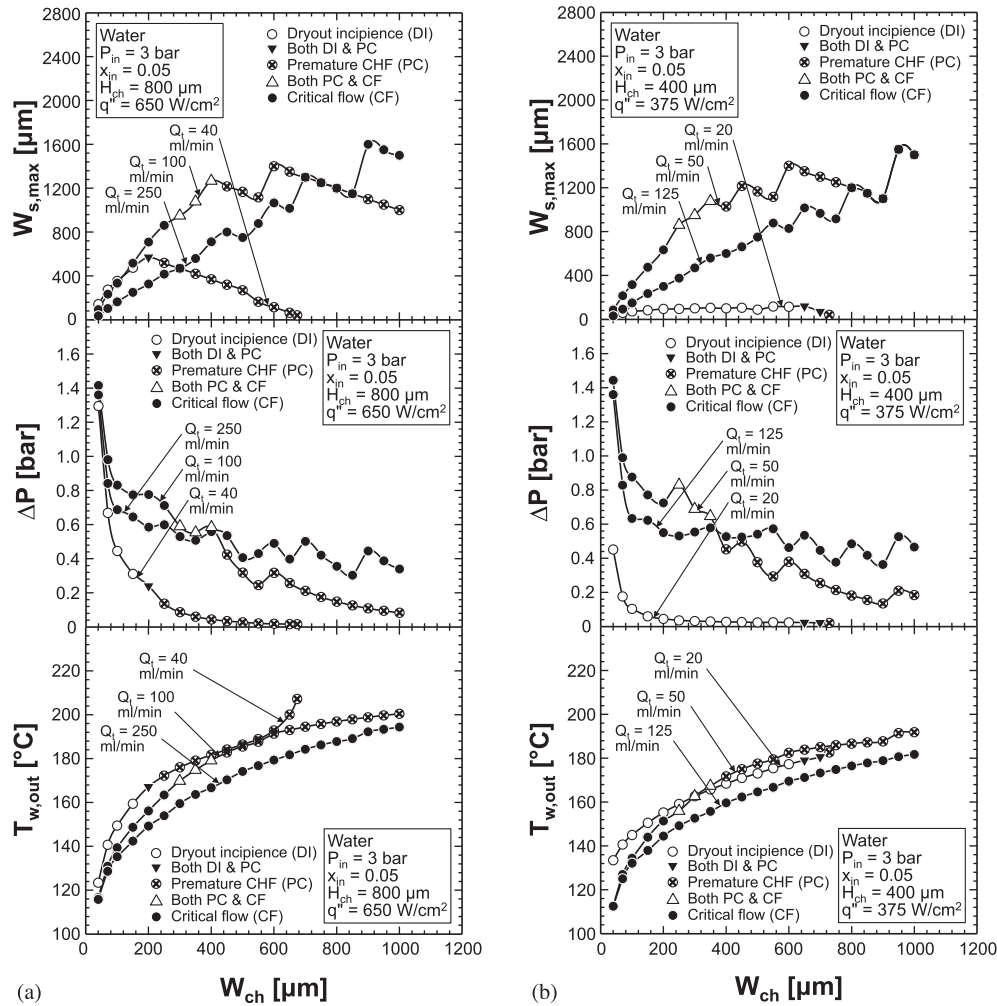


Fig. 11. Variations of maximum sidewall width and corresponding values of total pressure drop and bottom wall temperature at channel exit with channel width for different total volume flow rates using water as working fluid: (a)  $H_{ch} = 800 \mu\text{m}$  and (b)  $H_{ch} = 400 \mu\text{m}$ .

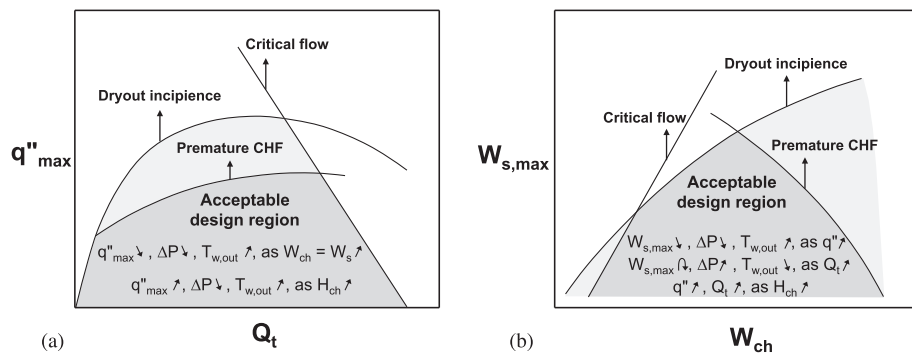


Fig. 12. Qualitative representation of acceptable design regions for saturated flow boiling in micro-channel heat sinks for (a) maximum heat flux and (b) maximum sidewall thickness. The premature CHF limit in both plots is specific to water.

Fig. 3(a) and (b) for HFE-7100, Fig. 4(a) and (b) for R134a, and Fig. 5 (a) and (b) for water, respectively. These figures show  $W_{s,max}$  decreases with increasing  $q''$ , which, for fixed  $W_{ch}$  and  $Q_t$ , increases  $N$  and decreases  $G$ , resulting in smaller  $\Delta P$  and higher  $T_{w,out}$ .

Fig. 7(a) and (b) for HFE-7100, Fig. 9(a) and (b) for R134a, and Fig. 11(a) and (b) for water show the influence of  $Q_t$  on  $W_{s,max}$  for fixed  $q''$ . The  $q''$  value in Fig. 7(a) and (b) for HFE-7100 is approximately half the highest  $q''_{max}$  value in Figs. 3(a) and (b), respectively. Similarly, the  $q''$  value in Fig. 9(a) and (b) for R134a is approxi-

mately half of highest  $q''_{max}$  value in Fig. 4(a) and (b), respectively. For water, the  $q''$  value in Fig. 11(a) and (b) is approximately one-third the highest  $q''_{max}$  value in Fig. 5(a) and (b), respectively. These figures show that the variation of  $W_{s,max}$  with respect to  $Q_t$  is not monotonic;  $W_{s,max}$  increases initially with increasing  $Q_t$  then decreases.

Overall, the  $W_{s,max}$  plots for the three fluids show that, for fixed  $W_{ch}$  and  $W_s$ , the larger  $H_{ch}$  (i.e., deeper micro-channels) allows for much higher  $q''$  corresponding to relatively small changes in  $\Delta P$

and  $T_{w,out}$ . The larger  $H_{ch}$  also facilitates increasing  $Q_t$  corresponding to small changes in  $\Delta P$  and  $T_{w,out}$ .

The parametric trends for  $q''_{max}$  and  $W_{s,max}$  are qualitatively summarized in Fig. 12(a) and (b), respectively. In general, micro-channels with smaller  $W_{ch}$  and  $W_s$  increase  $q''_{max}$ , and  $W_{s,max}$  decreases with increasing  $q''$ . Additionally, increasing  $H_{ch}$  allows for much higher  $q''$  corresponding to relatively small changes in  $\Delta P$  and  $T_{w,out}$ . Increasing  $H_{ch}$  also facilitates increasing  $Q_t$  corresponding to small changes in  $\Delta P$  and  $T_{w,out}$ . These figures also illustrate important flow and heat transfer limits encountered in micro-channel heat sinks, and the acceptable design region circumscribed by these limits. Fig. 12(a) shows  $q''_{max}$  is limited by dryout incipience at low  $Q_t$ , and two-phase critical flow at high  $Q_t$ . For water, premature CHF is an important limit that falls below the dryout incipience limit. Fig. 12(b) shows  $W_{s,max}$  is limited by two-phase critical flow, dryout incipience, and premature CHF, with critical flow generally occurring at smaller  $W_{ch}$  values, and premature CHF at larger  $W_{ch}$  values, especially for water.

### 3.3. Future research needs

It is important to emphasize that the accuracy of models and correlations employed in the present study are entirely dependent on availability of databases, especially those employing advanced diagnostic methods to capture interfacial behavior for different fluids and broad ranges of operating conditions. Overall, more data are needed to improve predictive relations for both premature CHF and critical flow. Detailed discussions of advanced diagnostic methods for measurement of temperature, interfacial velocity, and liquid velocity in two-phase flows are available in [72–75], [74–77], and [76–78], respectively.

## 4. Conclusions

This study provided a comprehensive methodology for thermal design of micro-channel heat sinks with saturated inlet conditions. Predictive methods for pressure drop and heat transfer were summarized. Also presented were thermal limits associated with dryout incipience and premature critical heat flux, as well as two-phase critical flow limit. The three limits were combined to define an envelope for acceptable heat sink performance. Special emphasis was placed on utilizing universal correlations that rely on large databases amassed from numerous sources, and which encompass many working fluids, and very broad ranges of hydraulic diameter, mass velocity, inlet pressure, and inlet quality. Key findings from the study are as follows:

1. A parametric study was performed to determine the variation of maximum heat flux,  $q''_{max}$ , with total volumetric flow rate,  $Q_t$ , for different combinations of channel geometrical parameters ( $W_{ch}$ ,  $W_s$ ,  $H_{ch}$ ) for three working fluids, HFE-7100, R134a, and water. Then  $q''_{max}$  values were used to assess corresponding variations of pressure drop,  $\Delta P$ , and maximum bottom wall temperature,  $T_{w,out}$ . For both HFE-7100 and R134a, the parametric study shows  $q''_{max}$  is limited by dryout incipience for low  $Q_t$ , and two-phase critical flow for high  $Q_t$ . For water,  $q''_{max}$  is also limited by premature CHF for intermediate values of  $Q_t$ . For fixed  $Q_t$ ,  $q''_{max}$  can be increased significantly, and  $T_{w,out}$  decreased by decreasing  $W_{ch}$  and  $W_s$  (i.e., by using a larger number of smaller channels), albeit at the expense of higher  $\Delta P$ . Increasing  $H_{ch}$  (i.e., using deeper micro-channels) serves to increase  $q''_{max}$  and decrease  $\Delta P$ , while producing a relatively weak adverse effect on  $T_{w,out}$ .
2. For HFE-7100 and R134a,  $W_{s,max}$  is limited mostly by dryout incipience for most  $W_{ch}$  values, and by critical flow for very small  $W_{ch}$ . For water,  $W_{s,max}$  is limited mostly by two-phase critical

flow for small  $W_{ch}$  values, and premature CHF for larger  $W_{ch}$  values. Increasing  $W_{ch}$  causes a decrease in  $\Delta P$  coupled with an increase in  $T_{w,out}$  corresponding to  $W_{s,max}$ . These trends are the result of the increase in  $W_{ch}$  decreasing the number of micro-channels, which also decreases mass velocity,  $G$ , in the micro-channels.

3. Despite the specific fluids and limits examined, this study provides a systematic methodology the reader may follow to address other fluids or operating conditions of interest. The trends of  $q''_{max}$  and corresponding trends of  $\Delta P$  and  $T_{w,out}$  provide an effective means for achieving the best compromise when selecting optimum ranges of heat sink geometrical parameters and operating conditions, in terms of safely dissipating the heat, maintaining acceptable device temperature, and minimizing pumping power.

## Acknowledgements

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