Experimental Investigation and Theoretical Model for Subcooled Flow Boiling Pressure Drop in Microchannel Heat Sinks

This study examines the pressure drop characteristics of subcooled two-phase microchannel heat sinks. A new model is proposed, which depicts the subcooled flow as consisting of a homogeneous two-phase flow layer near the heated walls of the microchannel and a second subcooled bulk liquid layer. This model is intended for conditions where subcooled flow boiling persists along the entire microchannel and the outlet fluid never reaches bulk saturation temperature. Mass, momentum, and energy control volume conservation equations are combined to predict flow characteristics for thermodynamic equilibrium qualities below zero. By incorporating a relation for apparent quality across the two-phase layer and a new criterion for bubble departure, this model enables the determination of axial variations in two-phase layer thickness and velocity as well as pressure drop. The model predictions are compared with HFE 7100 pressure drop data for four different microchannel sizes with hydraulic diameters of 176–416 μm, mass velocities of 670–5550 kg/m²s, and inlet temperatures of 0°C and −30°C. The pressure drop database is predicted with a mean absolute error of 14.9%. [DOI: 10.1115/1.3144146]

1 Introduction

Today, there is an urgent need for innovative thermal management solutions that can tackle the high flux heat dissipation of high-end electronic and power devices. Heat removal rates from microprocessors surpassed 100 W/cm², while specialized devices in defense applications such as lasers, microwave systems, and radars are expected to exceed 1000 W/cm² [1]. Aside from dissipating the heat, device temperatures must be maintained below levels that are dictated by material and reliability concerns.

Most recent high flux cooling studies have been focused on phase-change schemes such as microchannel heat sinks and jet impingement. Yet, even with the high heat transfer coefficients achievable with these cooling schemes, very high flux heat dissipation renders the task of maintaining acceptable temperatures exceedingly difficult. An effective solution to this problem is to greatly lower the temperature of the coolant using direct or indirect refrigeration cooling. Vapor compression systems are ideally suited for this purpose as they provide adequate cooling capacity at temperatures as low as −100°C. Recently, the authors of the present study examined the performance of a direct refrigeration cooling system in which the electronic device was cooled by refrigerant flowing through a miniature microchannel evaporator [2–4].

The dominant heat transfer mechanism in microchannel flows remains quite elusive. Recent studies in this area point to one of two fundamentally different mechanisms. Commonly cited in the vast majority of recent articles, the first mechanism is described as annular film evaporation evidenced by decreasing value of the convective heat transfer coefficient with increasing quality as well as dependence on mass velocity [5–12]. The second mechanism is described as dominated by nucleate boiling evidenced by the dependence of the heat transfer coefficient on heat flux but not mass velocity or quality [13–18]. As discussed in this study, the discrepancy between the two mechanisms may be explained by different coolants, operating conditions, and/or channel sizes used by different investigators yielding different dominant mechanisms.

The present study examines an indirect refrigeration cooling system, where the primary coolant flowing through the microchannel heat sink rejects its heat via a heat exchanger to refrigerant of a separate vapor compression loop. With a high degree of subcooling, flow boiling behavior of the primary coolant inside the microchannel heat sink is categorically different from the annular flow mechanism discussed in the vast majority of articles written about two-phase microchannel heat sinks. Those earlier studies involve high void fractions and saturated flow boiling. The present study concerns highly subcooled flow boiling, where sub-zero quality is preserved even as the fluid exits the microchannels. Here, vapor is formed by nucleation at the wall with a highly subcooled liquid core flow. Severe departure from thermodynamic equilibrium renders modeling of two-phase flow in this case highly elusive.

Pressure drop is a key parameter in the design of a two-phase cooling system. Pressure drop in subcooled flow boiling consists of frictional and accelerational components and the magnitude of each is highly dependent on void fraction. Departure from thermodynamic equilibrium greatly complicates the task of relating void fraction to flow quality, which is the primary reason behind the difficulty predicting pressure drop in subcooled flow boiling.

Nonequilibrium effects in subcooled flow boiling, especially the prediction of void fraction, is a very complex topic that was historically tackled only in a handful of macrochannel publications dating back to the 1960s and 1970s, e.g., Refs. [19–22]. A recent study by Hoffman and Wong [23] showed these models yield poor predictions of pressure drop.

The present study is the first attempt at developing a theoretical control-volume-based model for subcooled flow boiling pressure drop. Control-volume-based models were quite effective at predicting both pressure drop and heat transfer in saturated flow boiling in microchannel heat sinks [24,25] as well as flow boiling critical heat flux [26]. The model proposed in this study is particularly suited for situations involving severe nonequilibrium between the vapor and liquid phases.
2 Experimental Methods

2.1 Flow Loop and Test Section. Figure 1 shows a schematic of the two-phase facility developed for this study. The facility consisted of two main subsystems, a primary loop in which the microchannel test section was inserted and a refrigeration loop that was used to reduce the temperature of the working fluid, HFE 7100, in the primary loop. This 3M Novec fluid has very low freezing point below −100°C and a relatively moderate boiling point of 60°C at atmospheric pressure. It has excellent dielectric properties, zero ozone depletion potential, and unusually low global warming potential, which is very inert, and its surface tension is much smaller than that of water. Table 1 provides key thermal properties of HFE 7100.

As shown in Fig. 1, HFE 7100 is circulated through the primary loop with the aid of a centrifugal pump. This primary coolant is pumped from a reservoir through a heat exchanger where its temperature is reduced by heat transfer to the secondary refrigeration loop. The HFE 7100 then passes through a filter followed by a Coriolis mass flow meter before entering the microchannel test section. Throttling valves situated both upstream and downstream of the test section are used to control both flow rate and exit pressure. The HFE 7100 then returns to the reservoir, completing a full cycle.

Figure 2 depicts the construction of the microchannel test section. Microchannels with rectangular cross section are cut into the top surface of an oxygen-free copper block with the aid of thin carbide blades. The large lower section of the copper block is bored to accommodate four high-power-density cartridge heaters. The smaller top portion of the copper block is inserted into a rectangular housing made from G-11 fiberglass plastic that features coolant inlet and outlet ports, microchannel inlet and outlet plenums, and both pressure and temperature instrumentation ports. The microchannels are formed after the grooves in the top surface of the copper block are sealed with a transparent plastic cover plate. All outer surfaces of the copper block are carefully insulated to minimize heat loss to the ambient.

Table 1 Summary of thermophysical properties of HFE 7100

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$k_f$ (W/m K)</th>
<th>$\mu_f$ (kg/m s)</th>
<th>$c_p,f$ (J/kg K)</th>
<th>$\sigma$ (mN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−30°C</td>
<td>0.0796</td>
<td>14.74 × 10⁻⁴</td>
<td>1073.0</td>
<td>18.2</td>
</tr>
<tr>
<td>0°C</td>
<td>0.0737</td>
<td>8.26 × 10⁻⁴</td>
<td>1133.0</td>
<td>15.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P$ (bar)</th>
<th>$T_{sat}$ (°C)</th>
<th>$h_f$ (kJ/kg)</th>
<th>$h_f$ (kJ/kg)</th>
<th>$\rho_f$ (kg/m³)</th>
<th>$\rho_g$ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 bar</td>
<td>59.63</td>
<td>92.76</td>
<td>111.7</td>
<td>1372.7</td>
<td>9.58</td>
</tr>
<tr>
<td>3.5 bars</td>
<td>104.41</td>
<td>145.5</td>
<td>97.61</td>
<td>1238.9</td>
<td>32.14</td>
</tr>
</tbody>
</table>

Figure 1 Flow diagram for indirect refrigeration cooling system

2.2 Operating Conditions and Measurements. The temperature of HFE 7100 at the heat exchanger outlet in the primary loop is accurately maintained to within ±0.5°C by automatic feedback control of the secondary loop vapor compression chiller. Tests were performed with a constant test module outlet pressure of 1.138 bars and two different inlet temperatures, −30°C and 0°C. Lower temperatures were possible but avoided because of frost formation on the test section’s transparent cover plate below −30°C. The experiments were conducted at different flow rates. For each test, heat flux was increased gradually to generate a boiling curve, which, for some tests, continued to the critical heat flux (CHF) point. Table 3 provided ranges of all parameters of the study.

The test section’s instrumentation included pressure transducers and thermocouples for both the inlet and outlet plenums. Three type-T thermocouples were inserted in the copper block beneath...
the microchannels. Other measurements included electrical power input to the test section’s four cartridge heaters using a wattmeter and mass flow rate using a Coriolis flow meter. All measurements were made simultaneously and processed by an HP3852 data acquisition system.

Because of heat loss, not all the measured electrical power input to the cartridge heater was transferred to the coolant flowing through the test section. To estimate the heat loss, a finite element model of the entire test section and surrounding insulation was constructed in which local convective heat transfer coefficients were first determined from the measured electrical power input and copper block temperatures. Heat loss was then calculated from the model and heat transfer coefficient values were updated after deducting heat loss from the power input. Several iterations were used until the solution converged. Heat loss was estimated at 14–20% of the electrical power input for single-phase conditions and 6–14% for double-phase conditions. The heat flux data presented in this study were all corrected for this heat loss. Additional details concerning the heat loss calculation are available in a previous paper by the authors [27].

Uncertainties in the temperature measurements were ±0.5°C for inlet fluid temperature control and ±0.3°C for thermocouple readings. Accuracies of other measurement instruments were as follows: ±0.5% for the pressure transducers, ±0.1% for the Coriolis flow meter, and ±0.1% for the wattmeter.

3 Developing Homogeneous Layer Model

3.1 Model Description and Assumptions. Subcooled flow boiling is initiated when bubbles begin to nucleate in a mostly subcooled liquid flow and extends to the axial location where thermodynamic equilibrium quality reaches zero. The subcooled flow boiling region consists of two subregions, a highly subcooled subregion and a developing subcooled subregion. Vapor void in the highly subcooled subregion is a wall effect since strong condensation effects along the bubble interface suppress bubble growth and prevent coalescence between bubbles. The developing subcooled subregion is initiated at the point of bubble detachment off the heating wall. Here, bubbles are able to grow and depart into the subcooled liquid stream.

Prior studies by the other authors shed some light on the unique nature of subcooled boiling in microchannels [27]. Unlike flow in long macrochannels, the transition from single-phase liquid flow or highly subcooled flow to developing subcooled boiling in short microchannels results occurs abruptly after a relatively mild increase in heat flux. Furthermore, because of both small channel size and heat flux controlled boundary condition, the subcooled boiling region engulfs the entire length of the microchannel at once. The authors’ past flow visualization studies show that subcooled flow boiling inside the microchannels is dominated by bubbly flow and, to a lesser extent, slug flow. These studies also showed that vapor flow is initiated by bubbles that nucleate upstream, remaining close to the wall. The vapor void fraction increases gradually, culminating in bubbly flow that traverses the entire microchannel cross section downstream. This vapor development pattern constitutes the main rationale for the present model.

Figure 3 shows a schematic representation of the developing homogeneous layer model (DHLM) adopted in this study. This model is constructed specifically for developing subcooled flow boiling in microchannels and should be extended to neither the single-phase region nor the saturated boiling region. This model considers conditions where developed subcooled boiling engulfs

<table>
<thead>
<tr>
<th>Table 2 Test section dimensions</th>
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<tbody>
<tr>
<td>$W_{th}$ ($\mu$m)</td>
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<td>------------------</td>
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<tr>
<td>TS No. 1</td>
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<td>TS No. 2</td>
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<td>TS No. 3</td>
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<td>TS No. 4</td>
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<table>
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<tr>
<th>Table 3 Experimental operating conditions</th>
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<tr>
<td>$T_{in}$ (°C)</td>
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<td>------------------</td>
</tr>
<tr>
<td>TS No. 1</td>
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<td>TS No. 2</td>
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<td>TS No. 3</td>
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<td>TS No. 4</td>
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the entire length of the microchannels. Thus, it precludes an upstream single-phase region or downstream saturated boiling region along the heated wall. Extension of the developing subcooled boiling layer both upstream and downstream of the heated length as shown in Fig. 3(a) will be discussed later.

DHLM assumes that flow consists of two layers, a subcooled liquid core and a homogeneous two-phase flow layer; the latter represents the bubbly region of the flow. The homogeneous layer thickens along the flow direction due to wall heating. Key assumptions of the model are as follows.

1. The flow is one-dimensional, meaning flow velocity in each layer is constant at any axial location.
2. The flow is steady.
3. The liquid layer flow is laminar and incompressible. Also, because of the large density and relatively strong momentum of liquid relative to the two-phase layer, as well as the relatively short length of the microchannel, the liquid velocity is assumed constant along the flow direction.
4. The two-phase layer behaves as a homogeneous two-phase mixture whose properties are weighted averages of those of liquid and vapor.
5. There is no thermal equilibrium between the two layers. The two-phase layer is assumed to maintain saturated temperature, while the liquid layer is subcooled. In the flow direction, heat supplied from the wall increases vapor quality in the homogeneous layer and liquid temperature. Heat is also transferred between the two layers.
6. Pressure is uniform across the flow area of the microchannel at every axial location.
7. As shown in Fig. 3(b), an idealized flat interface is assumed between the homogeneous two-phase layer and the liquid layer to determine mean thickness of the two-phase layer from the model. As shown in Fig. 3(c), the actual interface is distorted by greater void toward the sidewalls and lesser void near the center of the microchannel because of heating from the sidewalls.

### 3.2 Control Volume Conservation Equations

Figure 4(a) shows mass flow terms for a microchannel control volume of length Δz along the flow direction. Mass conservations for the liquid layer and the homogeneous layer can be expressed, respectively, as

\[
\frac{d(\rho_f U_f A_y)}{dz} + K_I = 0 \tag{1}
\]

and

\[
\frac{d(G_H A_H)}{dz} - K_I = 0 \tag{2}
\]

where \(G_H = \rho_H U_H\) is the mass velocity of the homogeneous layer, and \(K_I\) is the rate of mass transfer from the liquid layer to the homogeneous layer per unit axial distance. Since liquid density is assumed constant, it can be taken out of the derivative in Eq. (1).

Figure 4(b) shows energy conservation for the homogeneous layer alone. Heat supplied from the wall is consumed in two different ways. A portion of this heat increases the temperature of the liquid mass transferred between the liquid layer and the homogeneous layers from \(T_f\) to \(T_{sat}\). The remaining portion increases the latent heat of the homogeneous layer. Therefore, energy conservation for the homogeneous layer can be expressed as

\[
q''(W_{ch} + W_w) = \frac{d}{dz}(G_H A_H h_H) + K_{T_f,sat}(T_{sat} - T_f) \tag{3}
\]

Because of the subcooled boiling state of the flow, the enthalpy \(h_H\) of the homogeneous layer requires careful assessment. This issue will be discussed in Sec. 3.3.

Figure 5 shows the momentum changes and forces acting on the control volume. Momentum conservation for the liquid layer and the homogeneous layer can be expressed, respectively, as

\[
\frac{d(\rho_f U_f^2 A_y)}{dz} + K_{U_f} = - A_f \frac{dP}{dz} - P \frac{dA_f}{dz} - \tau_{f,s}(W_{ch} + 2(H_{ch} - \delta)) - \tau_{W_{ch}} \tag{4}
\]
Combining Eqs. (4) and (5) eliminates both the interfacial momentum and interfacial friction terms, resulting in the following equation:

\[
\frac{d\left(\rho_f U_f^2 A_f\right)}{dz} - K_f U_f = -A_f \frac{dP}{dz} - \left(\tau_{f,n} W_{ch} + 2 \delta\right) + \tau_{f,h} W_{ch} \\
\frac{d\left(\rho_f U_f^2 A_f\right)}{dz} + \frac{d\left(\rho_f U_f^2 A_f\right)}{dz} = -A \frac{dP}{dz} - \left(\tau_{f,n} W_{ch} + 2(H_{ch} - \delta)\right) \\
+ \tau_{f,h} W_{ch} + 2 \delta) 
\]  

Combining Eqs. (4) and (5) eliminates both the interfacial momentum and interfacial friction terms, resulting in the following equation:

\[
\frac{d\left(\rho_f U_f^2 A_f\right)}{dz} - K_f U_f = -A_f \frac{dP}{dz} - \left(\tau_{f,n} W_{ch} + 2 \delta\right) + \tau_{f,h} W_{ch} \\
\frac{d\left(\rho_f U_f^2 A_f\right)}{dz} + \frac{d\left(\rho_f U_f^2 A_f\right)}{dz} = -A \frac{dP}{dz} - \left(\tau_{f,n} W_{ch} + 2(H_{ch} - \delta)\right) \\
+ \tau_{f,h} W_{ch} + 2 \delta) 
\]  

Equations (1)–(3) and (6) are the main governing equations for DHLM.
Table 4  Additional relations of DHLM

<table>
<thead>
<tr>
<th>Equations</th>
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<tr>
<td>Two-phase density</td>
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<td>Two-phase viscosity (McAdams model)</td>
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<td></td>
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<tr>
<td>Liquid layer wall shear [28]</td>
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<tr>
<td>Homogeneous layer wall shear</td>
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3.3 Solution Scheme. Referring to Fig. 3(b) the flow areas for the liquid layer and the homogeneous layer can be expressed, respectively, as

\[ A_f = W_{ch}(H_{ch} - \delta) \] (7)

and

\[ A_H = W_{ch}\delta \] (8)

Therefore, the area derivatives with respect to \( z \) can be expressed as

\[ \frac{dA_f}{dz} = -W_{ch}\frac{d\delta}{dz} \] (9)

and

\[ \frac{dA_H}{dz} = W_{ch}\frac{d\delta}{dz} \] (10)

Based on the assumptions of DHLM, the liquid layer preserves its velocity along the microchannel. Therefore,

\[ U_f = \frac{G}{\rho_f} \] (11)

and

\[ \frac{dU_f}{dz} = 0 \] (12)

where \( G \) is the mass velocity of the incoming subcooled nonboiling liquid. Since both \( \rho_f \) and \( U_f \) are constant along the flow direction, Eq. (1) yields

\[ \frac{dA_f}{dz} = -p_f U_f \] (13)

Introducing the above relation in Eq. (9) gives

\[ \frac{d\delta}{dz} = \frac{K_f}{\rho_f U_f - G_H} \] (14)

Combining Eq. (14) with Eqs. (2), (8), and (10) gives

\[ \frac{d\delta}{dz} = \frac{dG_H}{dz} \] (15)

One of the key parameters in DHLM is the enthalpy \( h_H \) of the homogeneous two-phase layer. Because vapor is generated along the microchannel in a subcooled boiling region, thermodynamic equilibrium quality cannot be used to determine the enthalpy of the two-phase layer. Therefore, the enthalpy of this layer is defined as

\[ h_H = h_f + \chi_H h_{fg} \] (16)

where \( \chi_H \) is an apparent flow quality. Two different formulations
were recommended for apparent quality in subcooled boiling. Levy’s [20] formulation was attempted but produced unrealistic predictions for microchannel flow. A second formulation by Kroeger and Zuber [22] was used here.

\[ x'_H(z) = \frac{c_p H^3 \Delta T_{sub, in}(Z - T^*)}{h_{fl} + c_p H \Delta T_{sub, in}(1 - T^*)} \]  \hspace{1cm} (17)

where

\[ Z^* = \frac{z + z_{bd}}{z_{sat} + z_{bd}} \]  \hspace{1cm} (18)

and

\[ T^* = \tanh(Z^*) \]  \hspace{1cm} (19)

Now, two-phase flow apparent quality can be predetermined over the entire test section domain without calculation of other governing equation variables. Quality gradient is also required later but it can be easily obtained since quality values are set by Eq. (17).

Combining Eqs. (10) and (14)–(17) with energy conservation equation (3) yields

\[ \frac{dG_H}{dz} = \frac{(p_j U_f - G_H)}{GW_ch[H + c_p(T_{sat} - T_f)]} \left[ q'(W_{ch} + W_w) - W_ch \delta G_H h_{fl} \right] \frac{dx'_H}{dz} \]  \hspace{1cm} (20)

Another equation for homogeneous layer thickness \( \delta \) can be determined by substituting Eq. (20) into Eq. (15).

\[ \frac{d\delta}{dz} = \frac{q'(W_{ch} + W_w) - W_ch \delta G_H h_{fl} \frac{dx'_H}{dz}}{GW_ch[H + c_p(T_{sat} - T_f)]} \]  \hspace{1cm} (21)

Equations (20) and (21) are coupled ordinary differential equations. Referring to Fig. 2(a), these equations must satisfy the following conditions:

\[ \delta = \begin{cases} 0 & \text{at } z = -z_{bd} \\ H_{ch} & \text{at } z = z_{sat} \end{cases} \]  \hspace{1cm} (22)

and

\[ G_H = \begin{cases} 0 & \text{at } z = -z_{bd} \\ G & \text{at } z = z_{sat} \end{cases} \]  \hspace{1cm} (23)

To meet those boundary conditions, the homogeneous two-phase layer is assumed to evolve from zero thickness at an equivalent distance \(-z_{bd}\) associated with preheating the flow at the same heat flux supplied at the microchannel. The homogeneous layer is also assumed to engulf the entire cross section at another equivalent distance \(z_{sat}\) extending beyond the microchannel exit and where the flow is subjected to the same heat flux supplied at the microchannel.

Finally, Eqs. (9) and (10) are used along with \( U_f = G_H / \rho_H \) to simplify the momentum equation (6) to the following explicit form:

\[ \frac{dP}{dz} = -\frac{1}{W_{ch} H_{ch}} \left[ -p_j U_f^2 W_{ch} \frac{d\delta}{dz} + \rho_H U_f^2 W_{ch} \frac{d\delta}{dz} + 2\rho_H U_f W_{ch} \frac{dU_f}{dz} \right] + U_f^2 W_{ch} \frac{dp_H}{dz} \frac{d\delta}{dz} + \left( \tau_{wall}[W_{ch} + 2(H_{ch} - \delta)] + \tau_{wall}[W_{ch} + 2 \delta] \right) \]  \hspace{1cm} (24)

Notice that the solutions of Eqs. (20) and (21) are not dependent on Eq. (24). Therefore Eqs. (20) and (21) can be solved first for \( \delta \) and \( G_H \). Then the variations in \( \delta, U_f, \delta/dz, \) and \( dU_f/dz \) with \( z \) are substituted into Eq. (24) to solve separately for pressure drop. Two-phase density and its derivative are predetermined from the apparent quality calculations.

Several additional parameters are required to solve Eqs. (20), (21), and (24). They include mixture density and mixture viscosity of the homogeneous two-phase layer and friction factor relations for the liquid layer and the homogeneous layer specific to rectangular microchannels. These relations are summarized in Table 4. A more comprehensive discussion of homogeneous flow viscosity models is provided in a previous study by the authors [2].

There are also other parameters that define the domain of the subcooled boiling region. These parameters must be determined in advance. Referring to Fig. 3(a), the location of saturated boiling...
which is also the downstream edge of DHLM domain, can be determined with the aid of a simple heat balance.

$$z_{sat} = \frac{GW_{ch}H_{ch}}{q'} - \frac{T_{in}}{H_{water}}$$

A key difficulty lies in the determination of $z_{bd}$. Two approaches are available from the literature for estimating the location of bubble departure. Both involve estimating flow subcooling $\Delta T_{sub,bd}$ at the departure point from which $z_{bd}$ is calculated from the simple energy balance.

$$z_{bd} = z_{sat} - \Delta T_{sub,bd} = \frac{GW_{ch}H_{ch}}{q'} - \frac{T_{sub,bd}}{H_{water}}$$

Relations recommended by Bowring (Collier and Thome [29]) and Saha and Zuber [21] for $\Delta T_{sub,bd}$ for macrochannels produced poor predictions for the conditions of the present study. The bubble departure point was predicted with both relations to occur in the microchannel at much higher heat fluxes than actually measured.

An alternative relation for $\Delta T_{sub,bd}$ is suggested here that follows the same general form as the Saha and Zuber model but ensures the occurrence of bubble departure within the microchannel as captured experimentally in the authors’ prior flow visualization studies [27].

$$\Delta T_{sub,bd} = \frac{q'C_G}{h_{bd}}$$

where $h_{bd}$ is the heat transfer coefficient measured when bubble departure was first observed inside the microchannel, and $C_G$ is a dimensionless function developed specifically for rectangular mi-
microchannels and correlated with aspect ratio (AR) \( \beta \) from data for the four different test sections used in the present study.

\[
C_d = 0.632\beta + 0.014
\]

(43)

The key challenge in using Eqs. (27) and (28) is that the value of \( h_{bd} \) used to develop these correlations was measured experimentally for a condition that produced bubble departure inside the microchannel. However, as illustrated in Fig. 3(a), the value of \( h_{bd} \) sought in the DHLM is that of a pseudoheat transfer coefficient that yields bubble departure upstream of the microchannel. This value was determined with an iterative procedure described next.

Figure 6 outlines the entire solution procedure for DHLM. The model is initiated by guessing microchannel inlet pressure \( P_{in} \) at \( z=0 \), then \( h_{bd} \). Different values of \( h_{bd} \) produced different variations in \( \delta \) with \( z \). The value of \( h_{bd} \) was updated until the model yielded \( \delta = H \) at \( z=z_{out} \). Then the model used this last value of \( h_{bd} \) to determine the variation in \( P \) with \( z \). Since the present experiments were performed with a fixed outlet plenum pressure \( P_{out} = 1.138 \) bars, the predicted microchannel exit pressure \( P_L \) at \( z = L \) was corrected for pressure recovery due to expansion from the microchannels to the outlet plenum in order to determine \( P_{out} \). If this value of \( P_{out} \) matched the measured pressure, the solution was deemed convergent. If, on the other hand, the pressure values did not agree, a new value of \( P_{in} \) was entered and the entire iteration repeated. Overall, pressure differences due to recovery were quite small and had virtually no impact on the pressure drop calculation. Details concerning two-phase pressure recovery relations are available in a paper by Qu and Mudawar [24].

4 Model Results

4.1 Profiles of Main Variables. Figure 7 shows predictions of the axial variations in the DHLM main variables: velocity of homogenous two-phase layer \( U_L \), thickness of homogeneous layer \( \delta \), and pressure \( P \) for each of the four test sections. Notice that the profiles include the pseudo-upstream and downstream extensions of the subcooled boiling layer; the extent of the actual microchannel is indicated for each case. The results are based on the experimentally measured heat flux, which varies slightly between the four cases. Because of the relatively high heat flux, these conditions correspond to the upper range of the nucleate boiling region, which is indicated by the microchannel length corresponding to the relatively high void fraction region of the model domain.

Pressure drop across the microchannel is the difference between \( P_{in} \) predicted by DHLM at \( z=0 \) minus the measured outlet pressure \( P_{out} \) corrected for pressure recovery. Equation (24) shows that pressure drop is inversely proportional to the microchannel cross-sectional area and is strongly influenced by \( U_L \). Figure 7 shows decreasing hydraulic diameter, which yields a substantial increase in \( U_L \), increases pressure drop.

4.2 Validation of Model Predictions. Figure 8 compares pressure drop predictions of DHLM with measured pressure drop for the four test sections, two different flow rates, and two inlet temperatures. Most cases are predicted quite accurately with the exception of the largest hydraulic diameter at the higher inlet temperature. This shows the model is well suited to small rectangular microchannels and may begin to lose accuracy with large microchannels. The model’s accuracy is apparently compromised when the homogenous two-phase layer assumption begins to fall apart as the two-phase layer undergoes transition from bubbly to slug flow at higher inlet temperatures (i.e., lower subcoolings), especially at high heat fluxes. It should be noted that DHLM provided excellent predictions of pressure drop for all three smaller hydraulic diameters and its predictive accuracy increased at low heat fluxes. Relatively poor predictions for the largest hydraulic diameter may be related to strong entrance effects, especially at high Reynolds numbers, causing the assumption of constant mean liquid layer velocity in the model to be violated. Another reason for the deviation, which is mainly encountered at high fluxes, concerns both the aforementioned transition form bubbly to slug flow, and the flow instabilities (even flow reversal) at conditions approaching CHF as described in Ref. [27].

Figure 9 compares the model predictions with the entire pressure drop database of this study. Except for the large diameter data...
corresponding to the higher inlet temperature, good predictions are achieved for most of the 191 data points, evidenced by a mean absolute error (MAE) of 14.9%.

5 Conclusions

This study examined subcooled boiling pressure drop in a microchannel heat sink. A new control volume model is proposed in which the subcooled flow is described as consisting of two layers: a homogeneous two-phase layer near the heated wall and a subcooled bulk liquid layer. Mass, momentum, and energy conservation equations are combined to predict pressure drop for thermodynamic equilibrium qualities below zero. Achieving closure in the construction of this developing homogeneous layer model required incorporating a relation for apparent quality across the two-phase layer as well as a new criterion for bubble departure. This model enables the determination of axial variations in two-phase layer thickness and velocity as well as pressure drop and is specifically tailored to rectangular microchannels. The model shows good predictions of pressure drop data for different mass velocities and subcoolings for four different microchannel sizes.

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Nomenclature

- $A$ = cross-sectional area
- $C_G$ = dimensionless constant for rectangular microchannels
- $c_p$ = specific heat
- $D_h$ = hydraulic diameter of microchannel
- $f$ = friction factor
- $G$ = mass velocity
- $h$ = heat transfer coefficient; enthalpy
- $H_{ch}$ = height of microchannel
- $h_v$ = latent heat of vaporization
- $K$ = friction constant for fully developed flow
- $K_I$ = interfacial mass flux
- $L$ = length of microchannel
- $m$ = mass flow rate of heat sink
- $N$ = number of microchannels in heat sink
- $P$ = pressure
- $\Delta P$ = pressure drop
- $q''$ = heat flux through heat sink base area
- $Re$ = Reynolds number
- $Re_{ch}$ = Reynolds number based on microchannel hydraulic diameter
- $T$ = temperature
- $T^*$ = dimensionless temperature
- $TS$ = test section
- $U$ = mean layer velocity
- $W_{ch}$ = width of microchannel
- $W_w$ = width of solid wall separating microchannels
- $x^*$ = apparent quality
- $Y^*$ = dimensionless thickness of homogeneous two-phase layer
- $z$ = streamwise coordinate
- $Z^*$ = dimensionless axial distance
- $-\delta_{hy}$ = dimensionless length

Greek Symbols

- $\beta$ = aspect ratio
- $\delta =$ thickness of homogeneous two-phase layer
- $\mu$ = viscosity
- $\rho$ = density
- $\tau$ = shear stress

Subscripts

- $bd$ = bubble departure from wall
- $ch$ = microchannel
- $exp$ = experimental
- $f$ = liquid layer; liquid
- $g$ = saturated vapor
- $H$ = homogeneous layer
- $i$ = interface between homogeneous two-phase layer and liquid layer
- $in$ = microchannel inlet
- $L$ = microchannel length
- $out$ = microchannel outlet
- $pred$ = predicted
- $sat$ = saturated
- $sub$ = subcooling
- $w$ = microchannel walls

References


