CV HW3

Jonathan Stoschek, jstosche@purdue.edu

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Point-to-point correspondences Logic

To calculate the required homography we first write down the matrix form of the equation to project a point.

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

This can be written as

$$x_1' = h_{11}x_1 + h_{12}x_2 + h_{13}x_3$$

$$x_2' = h_{21}x_1 + h_{22}x_2 + h_{23}x_3$$

$$x_3' = h_{31}x_1 + h_{32}x_2 + h_{33}x_3$$

We can divide by the formula for x_3' to get x' and y'

$$x' = \frac{x_1'}{x_3'} = \frac{h_{11}x_1 + h_{12}x_2 + h_{13}x_3}{h_{31}x_1 + h_{32}x_2 + h_{33}x_3}$$

$$y' = \frac{x_2'}{x_3'} = \frac{h_{21}x_1 + h_{22}x_2 + h_{23}x_3}{h_{31}x_1 + h_{32}x_2 + h_{33}x_3}$$

now divide by x_3 and set $h_{33} = 1$

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1}$$

We can write these to include all of the variables

$$xh_{11} + yh_{12} + 1h_{13} + 0h_{21} + 0h_{22} + 0_{23} - xx'h_{31} - yx'h_{32} - x' = 0$$

$$0h_{11} + 0h_{12} + 0h_{13} + xh_{21} + yh_{22} + 1h_{23} - xy'h_{31} - yy'h_{32} - y' = 0$$

Taking the 8 points from the corners of the picture frames and stacking them to get a matrix which we can solve using least squares method.

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & -y_2x_2' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & -y_2y_2' \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x_3' & -y_3x_3' \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y_3' & -y_3y_3' \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x_4' & -y_4x_4' \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y_4' & -y_4y_4' \end{bmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{pmatrix} = \begin{pmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \\ x_4' \\ y_4' \end{pmatrix}$$

The Two-Step Method Logic

The first part of the two-step method is to correct for the projective distortion. We use $l = x \times x$ for two pairs of points that form the two pair of lines that are supposed to be parallel. We then use $x = l \times l$ for the two sets of parallel lines to get both vanishing points. Finally we get the vanishing line by using $l = x \times x$ with the two vanishing points. The vanishing line $l = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}$ Finally to get the homography to get rid of the projective distortion we use the formula discussed in class.

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$$

The second, more difficult step is to get rid of the affine distortion. We do this by using the $cos\theta$ formula discussed in class. With m and l being lines and θ being the angle between them.

$$\cos\theta = \frac{l^T C^*_\infty m}{\sqrt{(l^T C^*_\infty l)(m^T C^*_\infty m)}}$$

Since we want the lines to be at 90 degrees, and $\cos 90 = 0$, we only need to look at the numerator. The numerator can be rewritten as

$$l'^T H C^*_\infty H^T m' = 0$$

$$\begin{pmatrix} l'_1 & l'_2 & l'_3 \end{pmatrix} \begin{bmatrix} AA^T & 0 \\ 0^T & 0 \end{bmatrix} \begin{pmatrix} m'_1 \\ m'_2 \\ m'_3 \end{pmatrix} = 0$$

We set $S = AA^T$ and get

$$\begin{pmatrix} l'_1 & l'_2 \end{pmatrix} \begin{bmatrix} s_{11} & s_{12} \\ s_{12} & 1 \end{bmatrix} \begin{pmatrix} m'_1 \\ m'_2 \end{pmatrix} = 0$$

We can rewrite this to get

$$s_{11} l'_1 m'_1 + s_{12}\left(l'_1 m'_2 + l'_2 m'_1\right) = -l'_2 m'_2$$

We can solve for $s_{11} s_{12}$ by using two sets of l and m lines. Once we get $s_{11} s_{12}$ we can construct $S = AA^T$. We can then use SVD to get A from AA^T. Finally we contract H

$$H = \begin{bmatrix} A & 0 \\ 0^T & 1 \end{bmatrix}$$

One-Step Method Logic

We again start with the $cos\theta$ formula

$$\cos\theta = \frac{l^T C^*_\infty m}{\sqrt{(l^T C^*_\infty l)(m^T C^*_\infty m)}}$$

This time we can rewrite the formula as

$$l'^T C^{*\prime}_\infty m'$$

we know that

$$C^{*\prime}_\infty = \begin{bmatrix} a & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & 1 \end{bmatrix}$$

So if we normalize l and m we can rewrite this as

$$a l'_1 m'_1 + \frac{b}{2}\left(l'_1 m'_2 + l'_2 m'_1\right) + c l'_2 m'_2 \frac{d}{2}\left(l'_1 + m'_1\right) + \frac{e}{2}\left(l'_2 + m'_2\right) = -1$$

With 5 pairs of l and m lines we can solve for a, b, c, d, e We can solve for

$$H = \begin{bmatrix} A & 0 \\ v^T & 1 \end{bmatrix}$$

by noting that

$$AA^T = \begin{bmatrix} a & \frac{b}{2} \\ \frac{b}{2} & c \end{bmatrix} \text{ and } Av = \begin{bmatrix} \frac{d}{2} \\ \frac{e}{2} \end{bmatrix}$$

We solve for A using the same SVD method as above.

Task 1-1

Figure 1: Task 1-1 original board Image

Figure 2: Task 1-1 Board Image with Points

Figure 3: Task 1-1 original corridor board Image

Figure 4: Task 1-1 original corridor Image with points

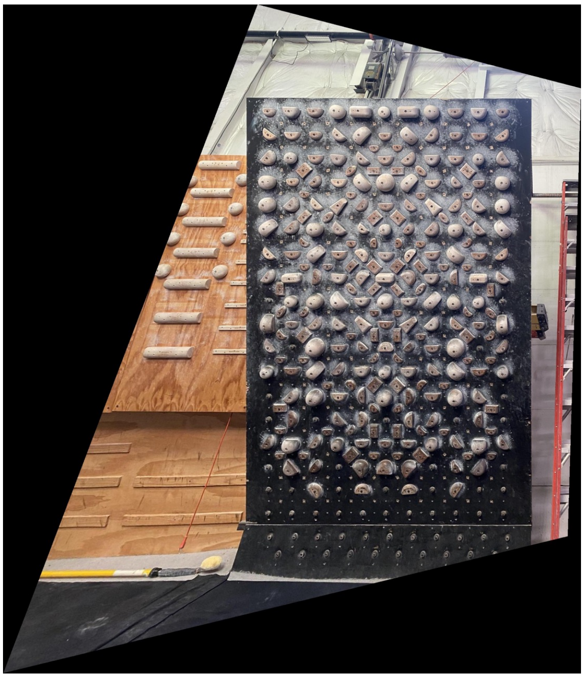

Figure 5: Task 1-1 Transformed Board Image

Figure 6: Task 1-1 Transformed Corridor Image

Task 1-2

Figure 7: Task 1-2 original board Image

Figure 8: Task 1-2 Board Image with Points

Figure 9: Task 1-2 original corridor board Image

Figure 10: Task 1-2 original corridor Image with points

Figure 11: Task 1-2 Step 1 Board Image

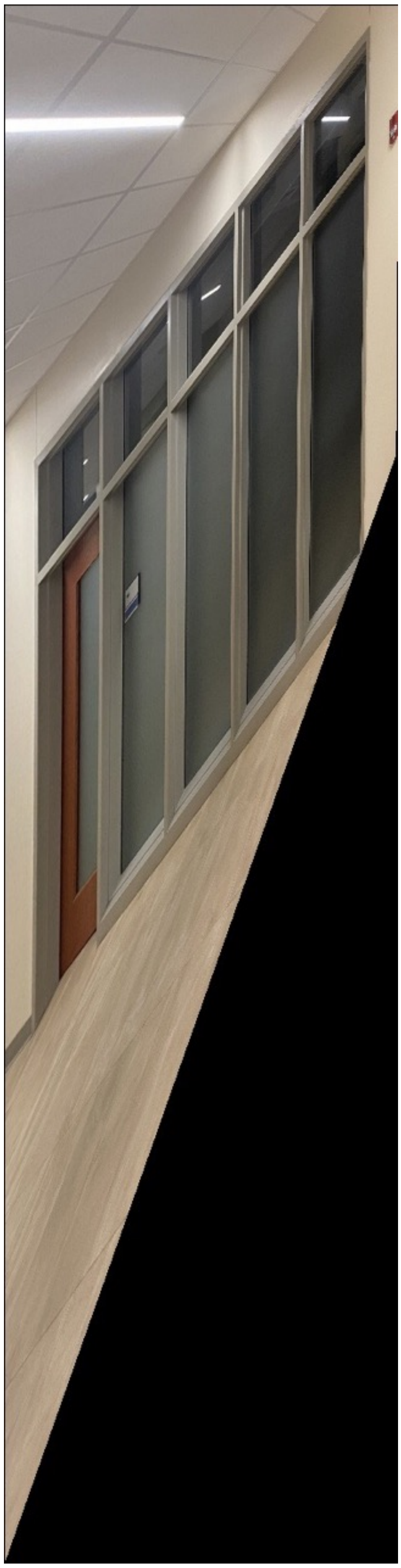

Figure 12: Task 1-2 Step 1 Corridor Image

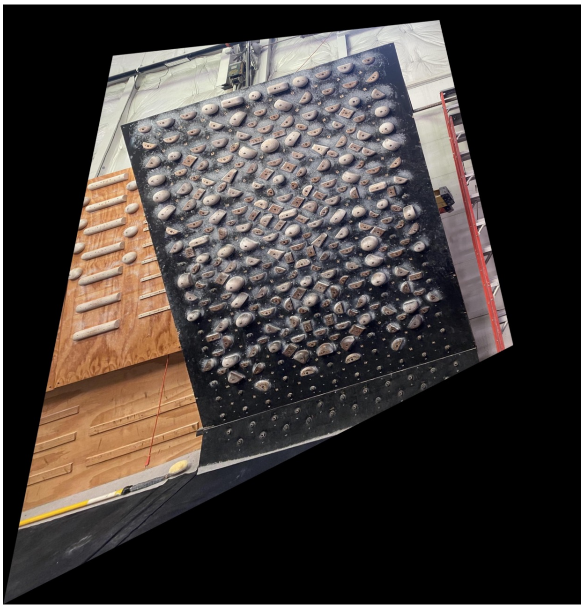

Figure 13: Task 1-2 Step 2 Board Image

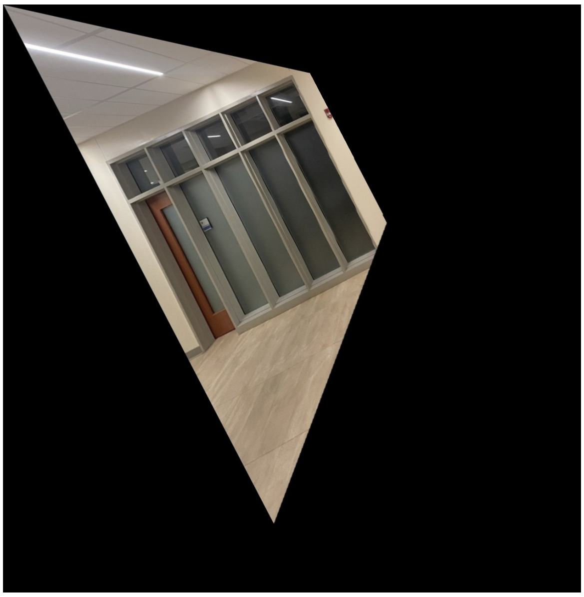

Figure 14: Task 1-2 Step 2 Corridor Image

Task 1-3

Figure 15: Task 1-3 original board Image

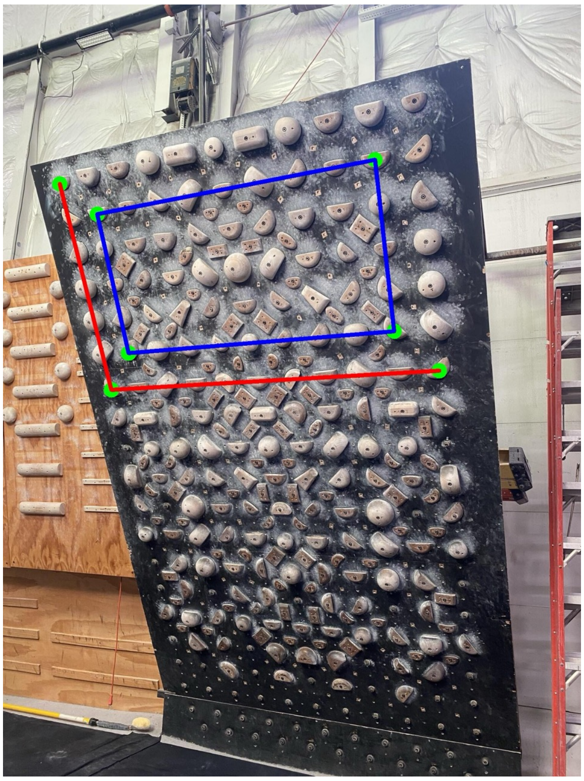

Figure 16: Task 1-3 Board Image with Points

Figure 17: Task 1-3 original corridor board Image

Figure 18: Task 1-3 original corridor Image with points

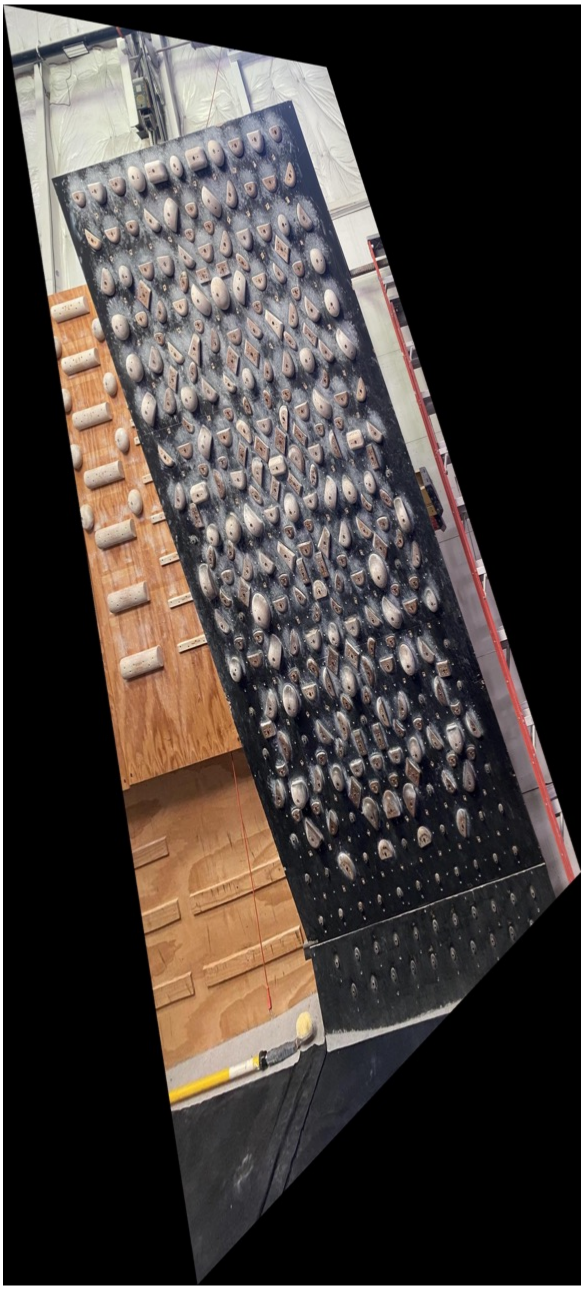

Figure 19: Task 1-3 corridor Image with affine correction

Figure 20: Task 1-3 corridor Image with affine correction

task 2-1

Figure 21: Task 2-1 original picture Image

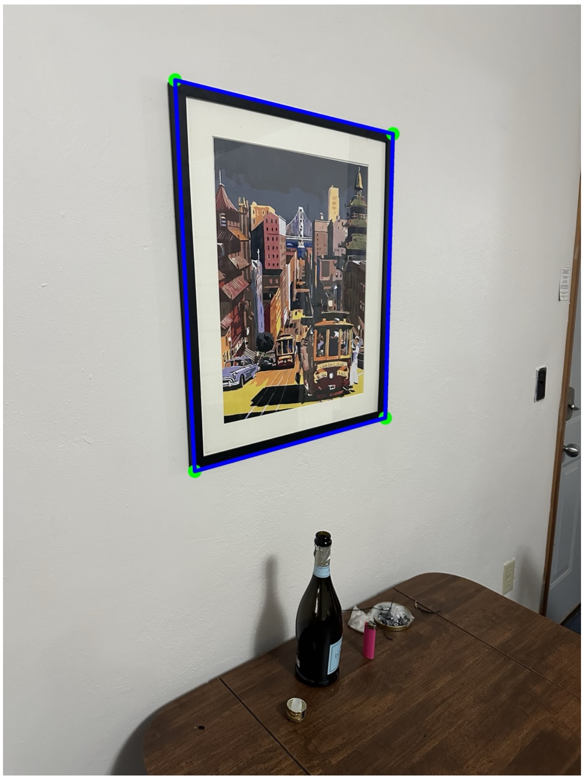

Figure 22: Task 2-1 original picture Image with Points

Figure 23: Task 2-1 original yuengling Image

Figure 24: Task 2-1 original yuengling Image with points

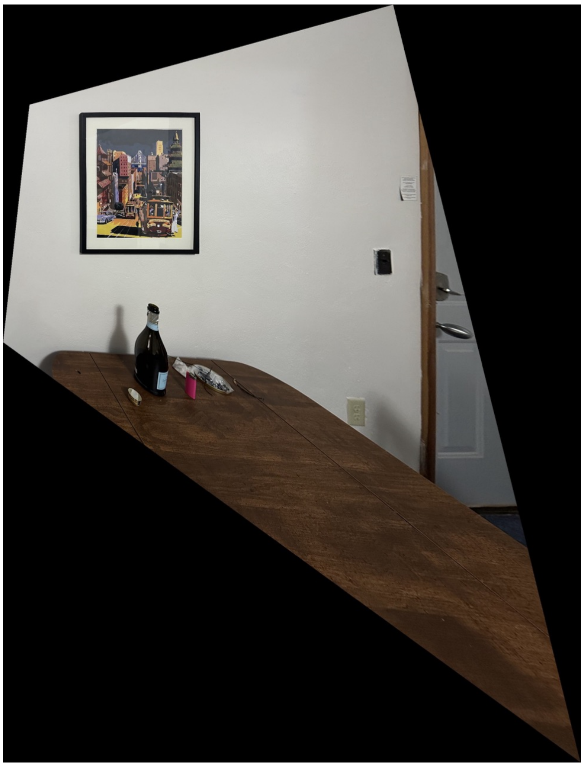

Figure 25: Task 1-1 Transformed picture Image

Figure 26: Task 1-1 Transformed yuengling Image

Task 2-2

Figure 27: Task 2-2 original picture Image

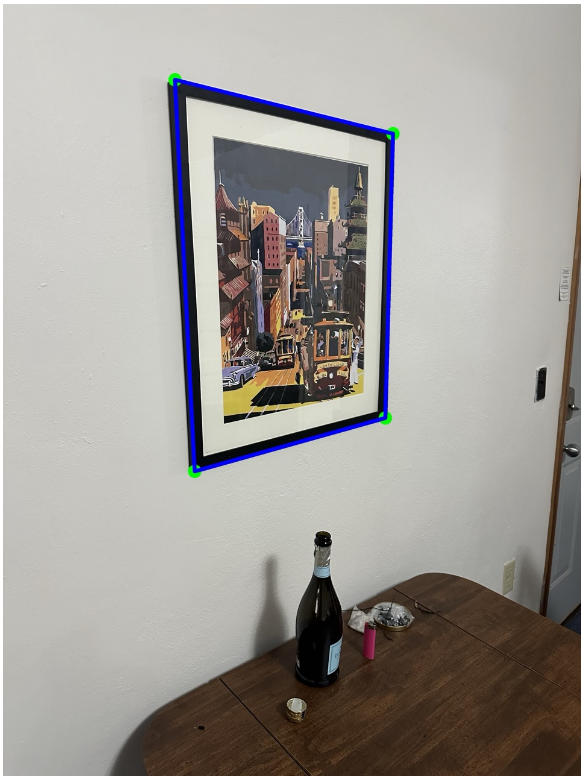

Figure 28: Task 2-1 original picture Image

Figure 29: Task 2-2 original yuengling Image

Figure 30: Task 2-1 original yuengling Image with points

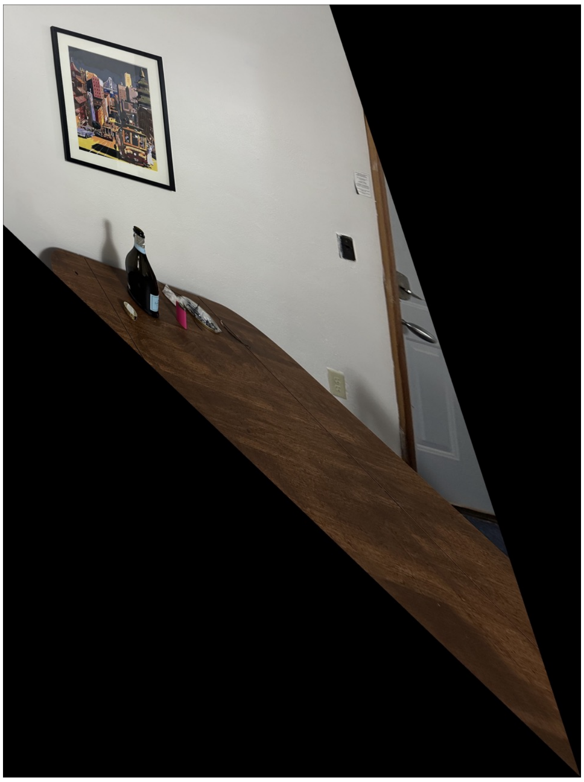

Figure 31: Task 2-2 Step 1 picture Image

Figure 32: Task 2-2 Step 1 yuengling Image

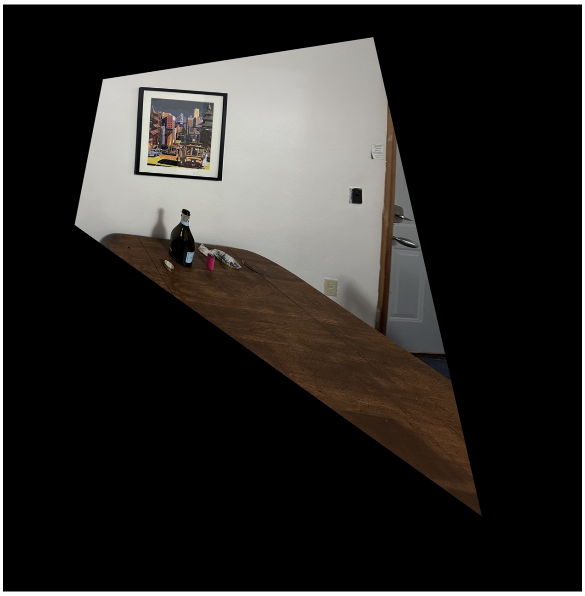

Figure 33: Task 2-2 Step 2 picture Image

Figure 34: Task 2-2 Step 2 yuengling Image

Task 2-3

Figure 35: Task 2-3 original picture Image

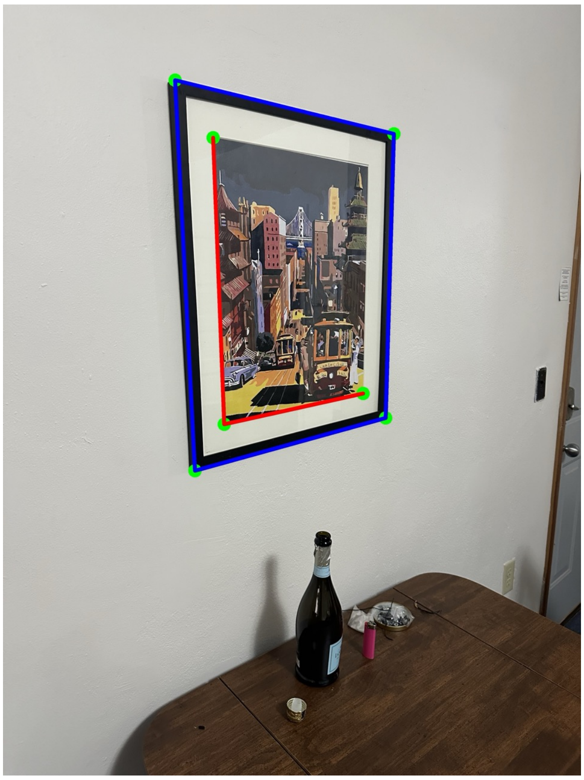

Figure 36: Task 2-3 original picture Image

Figure 37: Task 2-3 original yuengling Image

Figure 38: Task 2-3 original yuengling Image with points

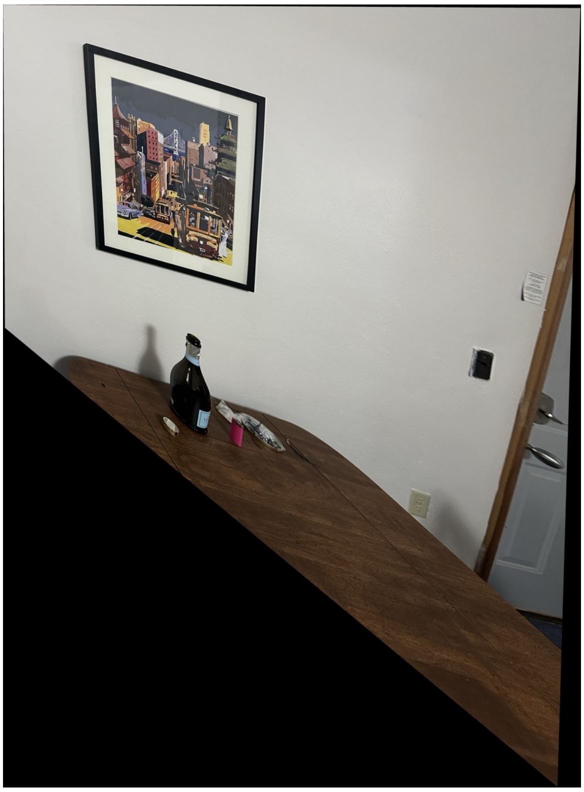

Figure 39: Task 2-3 Affine correction

Figure 40: Task 2-3 Affine correction