

ECE66100 Homework #1

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Question 1

The origin in \mathbb{R}^2 is given by $\begin{bmatrix} 0 & 0 \end{bmatrix}^T$. The corresponding Homogeneous Coordinates (HC) representation is given by $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. We know that $\forall k \in \mathbb{R} (k \neq 0), k \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ corresponds to the same physical point. Therefore, the HC of the origin in physical \mathbb{R}^2 can be written as $\begin{bmatrix} 0 & 0 & k \end{bmatrix}^T$, where $k \in \mathbb{R}$ and $k \neq 0$.

Question 2

No, not all points at infinity in the physical plane \mathbb{R}^2 are the same.

Consider two infinity points \mathbf{p}_1 and \mathbf{p}_2 , where \mathbf{p}_1 is the infinity point generated by following the line $y = x, x \rightarrow \infty$; and \mathbf{p}_2 is the infinity point generated by following the line $y = 2x, x \rightarrow \infty$, respectively. Intuitively, \mathbf{p}_1 and \mathbf{p}_2 are on different locations of the \mathbb{R}^2 plane, because they are approaching infinity from different directions.

Given any point (x_1, y_1) on the line $y = x$, the corresponding HC representation is given by $\begin{bmatrix} 1 & 1 & 1/x_1 \end{bmatrix}^T$. Given any point (x_2, y_2) on the line $y = 2x$, the corresponding HC representation is given by $\begin{bmatrix} 1 & 2 & 1/x_2 \end{bmatrix}^T$. When we set $x_1 \rightarrow \infty$ and $x_2 \rightarrow \infty$, the infinity points of $y = x$ and $y = 2x$ are $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$ and $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T$, respectively. It can be seen that they are different infinity points with distinct HC representation.

In general, an infinity point is of the form $\begin{bmatrix} u & v & 0 \end{bmatrix}^T$, where the (u, v) pair determines the direction that the point approaches infinity.

Question 3

Let $\mathbf{l} = [l_1 \ l_2 \ l_3]^T$, $\mathbf{m} = [m_1 \ m_2 \ m_3]^T$, it can be seen that

$$\mathbf{lm}^T = \begin{bmatrix} l_1 m_1 & l_1 m_2 & l_1 m_3 \\ l_2 m_1 & l_2 m_2 & l_2 m_3 \\ l_3 m_1 & l_3 m_2 & l_3 m_3 \end{bmatrix}, \quad (1)$$

$$\mathbf{ml}^T = \begin{bmatrix} l_1 m_1 & l_2 m_1 & l_3 m_1 \\ l_1 m_2 & l_2 m_2 & l_3 m_2 \\ l_1 m_3 & l_2 m_3 & l_3 m_3 \end{bmatrix}. \quad (2)$$

Clearly, all three columns of \mathbf{lm}^T and \mathbf{ml}^T are linearly dependent. That is to say, $\text{rank}(\mathbf{lm}^T) = \text{rank}(\mathbf{ml}^T) = 1$.

Definition 1. Let U and V be two vector spaces, the sum of two the two vector spaces, denoted by $U + V$, is the set given by

$$\{\mathbf{u} + \mathbf{v} \mid \mathbf{u} \in U, \mathbf{v} \in V\}.$$

Theorem 1. Let U and V be two vector spaces, then $\dim(U + V) \leq \dim U + \dim V$.

Proof. Let $\dim U = u$, $\dim V = v$. We can further let $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_u\}$ and $B_2 = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_v\}$ to be the basis of U and V , respectively.

From Definition 1, we know that an arbitrary vector $\mathbf{q} \in U + V$ can be written in the form of $\mathbf{q} = \mathbf{u} + \mathbf{v}$, where $\mathbf{u} \in U$, $\mathbf{v} \in V$. With the basis of U and V provided above, we can further write

$$\mathbf{u} = k_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 + \dots + k_u \mathbf{u}_u, \quad (3)$$

$$\mathbf{v} = s_1 \mathbf{v}_1 + s_2 \mathbf{v}_2 + \dots + s_v \mathbf{v}_v, \quad (4)$$

where k_1, \dots, k_u and s_1, \dots, s_v are scalars. Since $\mathbf{q} = \mathbf{u} + \mathbf{v}$, we know $\mathbf{q} \in \text{span}(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_u, \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_v)$. It can be seen that

$$\dim(U + W) \leq \dim \text{span}(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_u, \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_v) \leq u + v = \dim(U) + \dim(V). \quad (5)$$

□

Theorem 2. Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices, then $\text{rank}(\mathbf{A} + \mathbf{B}) \leq \text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B})$.

Proof. Denote the column space of a matrix \mathbf{M} by $C(\mathbf{M})$. By definition, we know that $\text{rank}(\mathbf{M}) = \dim C(\mathbf{M})$. If we write \mathbf{A} and \mathbf{B} in terms of column vectors, that is $\mathbf{A} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \dots \ \mathbf{a}_n]^T$ and $\mathbf{B} = [\mathbf{b}_1 \ \mathbf{b}_2 \ \dots \ \mathbf{b}_n]^T$, then we can express the column spaces of \mathbf{A} and \mathbf{B} as

$$C(\mathbf{A}) = \text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n), \quad (6)$$

$$C(\mathbf{B}) = \text{span}(\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n). \quad (7)$$

It is obvious that

$$C(\mathbf{A} + \mathbf{B}) = \text{span}(\mathbf{a}_1 + \mathbf{b}_1, \mathbf{a}_2 + \mathbf{b}_2, \dots, \mathbf{a}_n + \mathbf{b}_n). \quad (8)$$

$\forall \mathbf{p} \in C(\mathbf{A} + \mathbf{B})$, we have

$$\mathbf{p} = k_1(\mathbf{a}_1 + \mathbf{b}_1) + k_2(\mathbf{a}_2 + \mathbf{b}_2) + \dots + k_n(\mathbf{a}_n + \mathbf{b}_n) \quad (9)$$

$$= (k_1\mathbf{a}_1 + k_2\mathbf{a}_2 + \dots + k_n\mathbf{a}_n) + (k_1\mathbf{b}_1 + k_2\mathbf{b}_2 + \dots + k_n\mathbf{b}_n), \quad (10)$$

where k_1, \dots, k_n are scalars. It can be seen that $\mathbf{p} \in \text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) + \text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)$. We can derive

$$\text{rank}(\mathbf{A} + \mathbf{B}) = \dim C(\mathbf{A} + \mathbf{B}) \quad (11)$$

$$\leq \dim [\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) + \text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)] \quad (12)$$

$$\leq \dim [\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)] + \dim [\text{span}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n)] \quad (13)$$

$$= \dim C(\mathbf{A}) + \dim C(\mathbf{B}) = \text{rank } \mathbf{A} + \text{rank } \mathbf{B}. \quad (14)$$

□

Using Theorem 2, it can be seen that

$$\text{rank } \mathbf{C} = \text{rank}(\mathbf{lm}^T + \mathbf{lm}^T) \leq \text{rank}(\mathbf{lm}^T) + \text{rank}(\mathbf{lm}^T) = 2. \quad (15)$$

Question 4

We know that a conic can be written in the form of

$$\mathbf{C} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}. \quad (16)$$

For an HC point $\mathbf{x}_1 = [x_1 \ y_1 \ 1]^T$, it can be seen that

$$\mathbf{x}_1^T \mathbf{C} \mathbf{x}_1 = ax_1^2 + bx_1y_1 + cy_1^2 + dx_1 + ey_1 + f = 0. \quad (17)$$

We know that $\forall k \in \mathbb{R} (k \neq 0)$, $k\mathbf{C}$ and \mathbf{C} represent the same conic. Therefore, the following expression should also hold true:

$$\mathbf{x}_1^T (k\mathbf{C}) \mathbf{x}_1 = kax_1^2 + kbx_1y_1 + kcy_1^2 + kdx_1 + key_1 + kf = 0. \quad (18)$$

In a nontrivial conic, at least one of the coefficients among a, b, c, d, e, f will be nonzero. Without loss of generality, we can assume a is nonzero, and let $k = \frac{1}{a}$. Now Eq. (18) can be written as

$$x_1^2 + \frac{b}{a}x_1y_1 + \frac{c}{a}y_1^2 + \frac{d}{a}x_1 + \frac{e}{a}y_1 + \frac{f}{a} = 0. \quad (19)$$

If we let $b/a = b'$, $c/a = c'$, $d/a = d'$, $e/a = e'$, $f/a = f'$, then this expression becomes

$$x_1^2 + b'x_1y_1 + c'y_1^2 + d'x_1 + e'y_1 + f' = 0. \quad (20)$$

This is an equation with 5 unknowns, which means we need 5 points $\mathbf{x}_1, \dots, \mathbf{x}_5$ to solve for the coefficients of the conic. That is to say, a conic is defined with 5 points.

Question 5

(1) The HC representation of l_1 is given by

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}. \quad (21)$$

The HC representation of l_2 is given by

$$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}. \quad (22)$$

The intersection between l_1 and l_2 is

$$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix}. \quad (23)$$

That is, the intersection is given by $(1, 2)$.

(2) The HC representation of line l_3 is given by

$$\begin{bmatrix} 7 \\ -8 \\ 1 \end{bmatrix} \times \begin{bmatrix} -7 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -16 \\ -14 \\ 0 \end{bmatrix}. \quad (24)$$

Since the third element of l_1 and l_3 are both zero, we know that the two points pass through the origin. Therefore, they must intersect at the origin. In this case, we only need to compute l_1 and l_3 , which consists of two steps.

Question 6

The HC representation of \mathbf{l}_1 is given by

$$\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -2 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 2 \\ -32 \end{bmatrix}. \quad (25)$$

The HC representation of \mathbf{l}_2 is given by

$$\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 14 \\ 1 \end{bmatrix} = \begin{bmatrix} -16 \\ 4 \\ 8 \end{bmatrix}. \quad (26)$$

The intersection between \mathbf{l}_1 and \mathbf{l}_2 is given by

$$\begin{bmatrix} -8 \\ 2 \\ 32 \end{bmatrix} \times \begin{bmatrix} -16 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 144 \\ 576 \\ 0 \end{bmatrix}. \quad (27)$$

Since this is an infinity point, we know that \mathbf{l}_1 and \mathbf{l}_2 are parallel.

Question 7

$x = 1$ can be written as $1x + 0y - 1 = 0$, which means its corresponding HC representation is $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$. $y = -1$ can be written as $0x + 1y + 1 = 0$, which means its corresponding HC representation is $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T$. The intersection between them is given by

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}. \quad (28)$$

That is, the intersection is at $(1, -1)$.

Question 8

The equation of the ellipse is

$$\frac{(x-2)^2}{\left(\frac{1}{2}\right)^2} + (y-3)^2 = 1. \quad (29)$$

This simplifies to

$$4x^2 + y^2 - 16x - 6y + 24 = 0. \quad (30)$$

This conic can be written as

$$\mathbf{C} = \begin{bmatrix} 4 & 0 & -8 \\ 0 & 1 & -3 \\ -8 & -3 & 24 \end{bmatrix} \quad (31)$$

Since \mathbf{p} is the origin, the HC representation is given by $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Therefore, the polar line is given by $\mathbf{l} = \mathbf{C}\mathbf{p} = \begin{bmatrix} -8 & -3 & 24 \end{bmatrix}^T$.

The HC representation of the x axis is given by $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$. Therefore, the intersection between \mathbf{l} and the x axis is

$$\begin{bmatrix} -8 \\ -3 \\ 24 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ -8 \end{bmatrix}. \quad (32)$$

That is to say, the intersection between the polar line and the x axis is $(3, 0)$.

The HC representation of the y axis is given by $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$. Therefore, the intersection between \mathbf{l} and the y axis is

$$\begin{bmatrix} -8 \\ -3 \\ 24 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 24 \\ 3 \end{bmatrix}. \quad (33)$$

That is to say, the intersection between the polar line and the y axis is $(0, 8)$.

A. Source Code

A.1. Source Listing of ./alanhm.cls

```
1 \NeedsTeXFormat{LaTeX2e}
2
3 \gdef\AlanHMDate{2022/08/22}
4 \gdef\AlanHMVersion{0.0.1}
5
6 \ProvidesExplClass{alanhm}{\AlanHMDate}{\AlanHMVersion}{Alan-Xiang's-Homework-Template}
7
8
9 \LoadClass[12pt]{scrartcl}
10
11 \RequirePackage{fontspec}
12 \RequirePackage{scrlayer-scrpage}
13 \RequirePackage[bold-style=TeX]{unicode-math}
14 \RequirePackage{booktabs}
15 \RequirePackage{microtype}
16 \RequirePackage{enumitem}
17 \RequirePackage{amsthm}
18 \RequirePackage{caption,float}
19 \RequirePackage{xcolor}
20 \RequirePackage{etoolbox}
21 \RequirePackage{csquotes}
22 \RequirePackage[nicefrac]{}
23 \RequirePackage[skins,breakable,listings,minted]{tcolorbox}
24 \RequirePackage{metalogo}
25 \RequirePackage[acronym]{glossaries}
26 \RequirePackage[english]{babel}
27 \RequirePackage[colorlinks]{hyperref}
28 \RequirePackage[capitalise]{cleveref}
29
30
31 \setmainfont{ETbb-Regular}[
32   Extension=.otf,
33   BoldFont=ETbb-Bold,
34   ItalicFont=ETbb-Italic,
35   BoldItalicFont=ETbb-BoldItalic
36 ]
37
38 \setsansfont{texgyreadventor-regular}[
39   Extension=.otf,
40   BoldFont=texgyreadventor-bold,
41   ItalicFont=texgyreadventor-italic,
42   BoldItalicFont=texgyreadventor-bolditalic,
43   Scale=0.8
44 ]
45
46 \newfontfamily{\FONTqtfloraline}{QTFloraline}[
47   Extension=.otf,
48   BoldFont=QTFloraline-Bold,
49   Scale=0.8
50 ]
51
52 \setmonofont{NewCMMono10-Regular}[
53   Extension=.otf,
54   BoldFont=NewCMMono10-Bold,
55   ItalicFont=NewCMMono10-Italic,
56   BoldItalicFont=NewCMMono10-BoldOblique,
57   NFSSFamilY=NewCMMonoFamily,
58   Scale=0.85
59 ]
60
61 \setmathfont{Asana-Math}[
62   Extension=.otf,
63   bold-style=TeX
64 ]
65
66 % make bold math fonts look heavier
67 \setmathfont{Asana-Math}[
68   Extension=.otf,
69   bold-style=TeX,
70   FakeBold=1.6,
71   range={bfup,bfit}
72 ]
73
74 \newfontfamily{\FONTfiramono}{FiraMono-Regular}[
75   Extension=.otf,
76   BoldFont=FiraMono-Bold,
77   ItalicFont=FiraMono-Oblique,
78   BoldItalicFont=FiraMono-BoldOblique,
79   NFSSFamilY=FiraMonoFamily,
80   RawFeature={fallback=stdfallback},
81   Scale=0.75
82 ]
83
84 % set paper size and margin
85 \KOMAoption{paper}{letter}
86 \KOMAoption{DIV}{12}
87 % set title font
88 \addtokomafont{disposition}{\FONTqtfloraline}
89
90 \tl_new:N \l_ahm_now_time_tl
91 \cs_gset:Npn \ahm_get_time: {
92   \sys_get_shell:nnN {date---iso-8601=ns} {\cctab_select:N \c_document_cctab} \l_ahm_now_time_tl
93   \tl_use:N \l_ahm_now_time_tl
94 }
95
96 \tl_new:N \l_ahm_xetex_info_tl
97 \cs_gset:Npn \ahm_get_xetex_info: {
98   \sys_get_shell:nnN {bash--c~'xelatex---version|head--n-1'} {\cctab_select:N \c_document_cctab} \l_ahm_xetex_info_tl
99   \tl_use:N \l_ahm_xetex_info_tl
100 }
101
102 \tl_new:N \l_ahm_system_info_tl
103 \tl_new:N \l_ahm_system_info_tmpa_tl
104 \cs_gset:Npn \ahm_get_system_info: {
105   \tl_clear:N \l_ahm_system_info_tl
106   \sys_get_shell:nnN {bash--c~'echo-$USER@$HOSTNAME'} {\cctab_select:N \c_code_cctab} \l_ahm_system_info_tmpa_tl
107   \tl_put_right:N \l_ahm_system_info_tl \l_ahm_system_info_tmpa_tl
108   \tl_put_right:N \l_ahm_system_info_tl {}
109   \sys_get_shell:nnN {lsb_release--d--s} {\cctab_select:N \c_document_cctab} \l_ahm_system_info_tmpa_tl
110   \tl_put_right:N \l_ahm_system_info_tl \l_ahm_system_info_tmpa_tl
111   \sys_get_shell:nnN {uname--r} {\cctab_select:N \c_code_cctab} \l_ahm_system_info_tmpa_tl
112   \tl_put_right:N \l_ahm_system_info_tl {
113     \ (\l_ahm_system_info_tmpa_tl)
114   }
115   \tl_use:N \l_ahm_system_info_tl
```

```

116 }
117
118 % page margin metadata
119 \DeclareNewLayer[
120   background,
121   bottommargin,
122   align=c,
123   mode=picture,
124   everypage,
125   contents={%
126     \putC{\ahm_generate_bottom:}
127   }
128 ]{bottom}
129
130 %\definecolor{button-primary}{HTML}{007bff}
131 %\definecolor{button-primary}{HTML}{007bff}
132 %\definecolor{button-primary}{rgb}{gray!50}
133 \colorlet{button-primary}{gray!50}
134
135 % hook for going back to the first page
136 \AtBeginDocument{
137   \hypertarget{@ahm@begin@doc}{}
138 }
139
140
141 \newtcbbox{\AHMPrimaryButton}{
142   colback=button-primary,
143   colframe=white,
144   fontupper=\sffamily\scriptsize\color{white},
145   boxsep=0ex,
146   left=0.8ex,
147   right=0.8ex,
148   top=0.8ex,
149   bottom=0.8ex
150 }
151
152 \newcommand{\AHMBackToTopButton}{
153   \hyperlink{@ahm@begin@doc}{\AHMPrimaryButton{BACK~TO~TOP}}
154 }
155
156 % construct button nav bar
157 \seq_new:N \g_ahm_bottom_nav_seq
158 \tl_new:N \l_ahm_bottom_nav_tmpa_tl
159
160
161 \DeclareNewLayer[
162   background,
163   bottommargin,
164   align=c,
165   mode=picture,
166   everypage,
167   contents={%
168     \putC{
169       \parbox{\paperwidth}{
170         \centering
171         \vspace*{4cm}
172         \seq_gclear:N \g_ahm_bottom_nav_seq
173         \seq_gput_left:Nn \g_ahm_bottom_nav_seq {\mbox{\AHMBackToTopButton}}
174         \clist_map_variable:NNn \g_ahm_questions_aux_clist \l_ahm_bottom_nav_tmpa_tl {
175           \seq_gput_right:Nx \g_ahm_bottom_nav_seq {
176             \exp_not:N \hyperlink {@ahm@question@} \exp_not:N \l_ahm_bottom_nav_tmpa_tl}{
177               \exp_not:N \mbox{
178                 \exp_not:N \AHMPrimaryButton{Q \exp_not:N \l_ahm_bottom_nav_tmpa_tl}
179             }
180           }
181         }
182       }
183       \seq_use:Nn \g_ahm_bottom_nav_seq {\hspace*{1pt}}
184     }
185   }
186 }
187 ]{bottom-link}
188
189
190 \AddLayersAtEndOfPageStyle{scrheadings}{bottom}
191 \AddLayersAtEndOfPageStyle{scrheadings}{bottom-link}
192 % make sure there is bottom note to the title page
193 \renewcommand\titlepagestyle{scrheadings}
194 \pagestyle{scrheadings}
195
196
197 \tl_new:N \l_ahm_gb_tmpa_tl
198 \tl_new:N \l_ahm_gb_tmpb_tl
199 \cs_gset:Npn \ahm_generate_bottom: {
200   \parbox{\paperwidth}{
201     \centering
202     \scriptsize
203     \sffamily
204     \vspace*{6em}
205     \color{gray!50}
206     \int_compare:nTF {\the page = 1} {
207       Ziyue~`Alan"~Xiang's~Homework~Template~(alanhm.cls);~VERSION=\AlanHMVersion;~TEMPLATE~DATE=\AlanHMDate\\
208       \XeLaTeX\ signature:~\ahm_get_xetex_info:\\
209       User~and~computer~system~signature:~\ahm_get_system_info:\\
210       Page~compile~timestamp:~\ahm_get_time:\\
211     }{
212       Page~compile~timestamp:~\ahm_get_time:
213     }
214     \par
215   }
216 }
217
218 % change the TOC style
219 % \let\oldtoc\tableofcontents
220 % \renewcommand{\tableofcontents}{
221 %   \bgroup
222 %   \normalfont\sffamily
223 %   \oldtoc
224 %   \egroup
225 % }
226 % \clist_map_inline:nn {section,subsection} {
227 %   \addtokomafont{#1entry}{\normalfont\sffamily}
228 % }
229
230
231 \setkomafont{sectionentry}{\normalfont\sffamily}
232 \setkomafont{sectionentrypagenumber}{\normalfont\sffamily}
233 \RedeclareSectionCommands[
234   tocentryformat=\normalfont\sffamily,
235   tocentrynumberformat=\normalfont\sffamily,
236   tocpagenumberformat=\normalfont\sffamily
237 ]{subsection,subsection}

```

```

238
239 % large appendix for source code
240 \newcommand{\largeappendix}{
241   \clearpage
242   \KOMAOptions{paper=a3,DIV=18}
243   \recalctypearea
244   \appendix
245   % \setkomafont{sectionentry}{\normalfont\sffamily}
246   % \setkomafont{sectionentrypagenumber}{\normalfont\sffamily}
247 }
248
249
250
251 % implementation of quick matrices
252 % note: nested usage of this command is not allowed!
253 \seq_new:N \l_ahm_qmat_tmpa_seq
254 \seq_new:N \l_ahm_qmat_tmpb_seq
255 \seq_new:N \l_ahm_qmat_tmpc_seq
256 \tl_new:N \l_ahm_qmat_tmpa_tl
257 % #1: matrix type
258 % #2: command name
259 \newcommand{\DeclareQuickMatrix}[2]{
260   \NewDocumentCommand{#2}{sm}{
261     \regex_split:nnN {;} {#2} \l_ahm_qmat_tmpa_seq
262     \seq_clear:N \l_ahm_qmat_tmpb_seq % store rows
263     \seq_map_variable:NNn \l_ahm_qmat_tmpa_seq \l_ahm_qmat_tmpa_tl {
264       \exp_args:NNV \seq_set_from_clist:Nn \l_ahm_qmat_tmpc_seq \l_ahm_qmat_tmpa_tl
265       \seq_put_right:Nx \l_ahm_qmat_tmpb_seq {
266         \seq_use:Nn \l_ahm_qmat_tmpc_seq {&}
267       }
268     }
269     \ensuremath{
270       \begin{#1matrix}
271         \seq_use:Nn \l_ahm_qmat_tmpb_seq {\}
272       \end{#1matrix}
273       \IfBooleanTF{#1}{^T}{}
274     }
275   }
276 }
277
278 \DeclareQuickMatrix{v}{\qvmatrix}
279 \DeclareQuickMatrix{p}{\qpvmatrix}
280 \DeclareQuickMatrix{b}{\qbvmatrix}
281
282
283 % short hands for common math commands
284 \KOMAOptions{sectionentrydots=true}
285 \newcommand{\DefineShortHand}[2]{
286   \cs_if_exist:NTF #1 {
287     \GenericError{}{\cs_to_str:N #1~already-exists}{}{}
288   }{
289     \cs_gset_eq:NN #1 #2
290   }
291 }
292
293
294 \DefineShortHand\bit\symbit
295 \DefineShortHand\bup\symbfup
296 \DefineShortHand\bm\symbfit
297 \DefineShortHand\mca\mathcal
298 \DefineShortHand\mrm\mathrm
299 \DefineShortHand\mbb\mathbb
300
301
302 % change enumitem settings
303 \setlist{
304   left=1em,
305   itemsep=0ex,
306 }
307
308
309
310 \clist_new:N \g_ahm_questions_clist
311 \clist_new:N \g_ahm_questions_aux_clist
312
313 % homework related commands
314 \newcommand{\question}[1]{
315   \clearpage
316   \addcontentsline{toc}{section}{Question-#1}
317   \hypertarget{@ahm@question@#1}{}
318   \section*{Question-#1}
319   \clist_gput_right:Nn \g_ahm_questions_clist {#1}
320 }
321
322
323
324 \tl_new:N \l_ahm_auxout_tmpa_tl
325 \AtEndDocument{
326   \iow_now:cn {@auxout} {\ExplSyntaxOn}
327   \iow_now:cx {@auxout} {
328     \exp_not:N \clist_gset:Nn
329     \exp_not:N \g_ahm_questions_aux_clist
330     {\exp_not:N \g_ahm_questions_clist}
331   }
332   \iow_now:cn {@auxout} {\ExplSyntaxOff}
333 }
334
335 % amsthm
336 \newtheorem{theorem}{Theorem}
337 \newtheorem{definition}{Definition}
338
339
340 \makeatletter
341 % metadata
342 \AtBeginDocument{
343   \hypersetup{
344     pdftitle={\@title},
345     pdfauthor={\@author}
346   }
347 }
348 \makeatother
349
350
351 % change line number style
352 \renewcommand{\theFancyVerbLine}{
353   \textcolor{gray!80}{
354     \fontsize{7}{7}\selectfont\ttfamily
355     %\oldstylenums{
356     \arabic{FancyVerbLine}
357     %}
358   }
359 }

```

```

360
361 % source code listings
362 \clist_new:N \l_ahm_src_listing_clist
363 \tl_new:N \l_ahm_src_listing_tmpa_tl
364 \tl_new:N \l_ahm_src_listing_tmpb_tl
365 \tl_new:N \l_ahm_src_listing_tmpc_tl
366 \tl_new:N \l_ahm_src_listing_tmpd_tl
367 \newcommand{\SourceListing}[1]{
368   \clist_set:Nn \l_ahm_src_listing_clist {#1}
369   \section{Source-Code}
370   \bool_do_until:nn {\clist_if_empty_p:N \l_ahm_src_listing_clist} {
371     \clist_pop:NN \l_ahm_src_listing_clist \l_ahm_src_listing_tmpa_tl % filename
372     \clist_pop:NN \l_ahm_src_listing_clist \l_ahm_src_listing_tmpb_tl % language
373     \str_set:NV \l_ahm_src_listing_tmpa_tl \l_ahm_src_listing_tmpa_tl
374     \str_set:NV \l_ahm_src_listing_tmpb_tl \l_ahm_src_listing_tmpb_tl
375
376     \tl_set:Nx \l_ahm_src_listing_tmpd_tl {
377       \exp_not:N \subsection[
378         Source-Listing-of~
379         \exp_not:N \texorpdfstring{
380           \exp_not:N \texttt {
381             \c_backslash_str detokenize{\exp_not:N \l_ahm_src_listing_tmpa_tl}
382           }
383         }{
384           \exp_not:N \l_ahm_src_listing_tmpa_tl
385         }
386       ]{
387         Source-Listing-of~\exp_not:N \texttt{\exp_not:N \l_ahm_src_listing_tmpa_tl}
388       }
389     }
390     \tl_use:N \l_ahm_src_listing_tmpd_tl
391     \exp_args:Nx
392     \tcbinputlisting{
393       enhanced,
394       listing~file=\exp_not:N \l_ahm_src_listing_tmpa_tl,
395       listing~engine=minted,
396       listing~only,
397       breakable,
398       left=2.6em,
399       minted~options={
400         autogobble,
401         fontfamily=FiraMonoFamily,
402         fontsize=\exp_not:N\fontsize{8}{8}\exp_not:N\selectfont,
403         obeytabs,
404         breaklines,
405         linenos,
406         numbersep=1em
407       }
408     }
409   }
410 }

```

A.2. Source Listing of 1_ZiyueXiang.tex

```

1 % !TeX TS-program = xelatex
2
3 \documentclass{../alanhm}
4 \usepackage{svg}
5
6 \title{ECE66100 Homework \#1}
7 \author{Ziyue ``Alan'' Xiang}
8 \date{\today}
9
10
11 % acronyms
12
13 \newacronym{hc}{HC}{Homogeneous Coordinates}
14 \newcommand{\rank}{\operatorname{rank}}
15 \newcommand{\algspan}{\operatorname{span}}
16
17 \begin{document}
18 \maketitle
19 \tableofcontents
20
21 \question{1}
22
23 The origin in  $\mathbb{R}^2$  is given by  $\mathbf{q}_{\mathbf{0},\mathbf{0}}$ .
24 The corresponding  $\mathbf{gls}_{\mathbf{hc}}$  representation is given by  $\mathbf{q}_{\mathbf{0},\mathbf{0},\mathbf{1}}$ .
25 We know that  $\forall k \in \mathbb{R} \setminus \{0\}$ ,  $\mathbf{q}_{\mathbf{0},\mathbf{0},\mathbf{1}}$  corresponds to the same physical point.
26 Therefore, the  $\mathbf{gls}_{\mathbf{hc}}$  of the origin in physical  $\mathbb{R}^2$  can be written as  $\mathbf{q}_{\mathbf{0},\mathbf{0},\mathbf{k}}$ , where  $k \in \mathbb{R}$  and  $k \neq 0$ .
27
28 \question{2}
29
30 No, not all points at infinity in the physical plane  $\mathbb{R}^2$  are the same.
31
32 Consider two infinity points  $\mathbf{p}_1$  and  $\mathbf{p}_2$ , where  $\mathbf{p}_1$  is the infinity point generated by following the line  $y=x$ ,  $x \rightarrow \infty$ ; and  $\mathbf{p}_2$  is the infinity
33  $\rightarrow$  point generated by following the line  $y=2x$ ,  $x \rightarrow \infty$ , respectively.
34 Intuitively,  $\mathbf{p}_1$  and  $\mathbf{p}_2$  are on different locations of the  $\mathbb{R}^2$  plane, because they are approaching infinity from different directions.
35
36 Given any point  $(x_1, y_1)$  on the line  $y=x$ , the corresponding  $\mathbf{gls}_{\mathbf{hc}}$  representation is given by  $\mathbf{q}_{\mathbf{1},\mathbf{1},\frac{1}{x_1}}$ .
37 Given any point  $(x_2, y_2)$  on the line  $y=2x$ , the corresponding  $\mathbf{gls}_{\mathbf{hc}}$  representation is given by  $\mathbf{q}_{\mathbf{1},\mathbf{2},\frac{1}{x_2}}$ . When we set  $x_1 \rightarrow \infty$  and  $x_2$ 
38  $\rightarrow \infty$ , the infinity points of  $y=x$  and  $y=2x$  are  $\mathbf{q}_{\mathbf{1},\mathbf{1},\mathbf{0}}$  and  $\mathbf{q}_{\mathbf{1},\mathbf{2},\mathbf{0}}$ , respectively. It can be seen that they are different infinity points with
39  $\rightarrow$  distinct  $\mathbf{gls}_{\mathbf{hc}}$  representation.
40
41 In general, an infinity point is of the form  $\mathbf{q}_{\mathbf{u},\mathbf{v},\mathbf{0}}$ , where the  $(u,v)$  pair determines the direction that the point approaches infinity.
42
43 \question{3}
44
45 Let  $\mathbf{m} = \mathbf{q}_{\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3}$ ,  $\mathbf{m} = \mathbf{q}_{\mathbf{m}_1,\mathbf{m}_2,\mathbf{m}_3}$ , it can be seen that
46 \begin{align}
47 \mathbf{m} \mathbf{m}^T &= \mathbf{left}[\begin{matrix} l_1 & m_1 & l_1 & m_2 & l_1 & m_3 \\ l_2 & m_1 & l_2 & m_2 & l_2 & m_3 \\ l_3 & m_1 & l_3 & m_2 & l_3 & m_3 \end{matrix} \end{align}
48 \mathbf{m} \mathbf{m}^T &= \mathbf{left}[\begin{matrix} l_1 & m_1 & l_2 & m_2 & l_3 & m_3 \\ l_2 & m_1 & l_2 & m_2 & l_3 & m_3 \\ l_3 & m_1 & l_3 & m_2 & l_3 & m_3 \end{matrix} \end{align}
49 \end{align}
50 Clearly, all three columns of  $\mathbf{m} \mathbf{m}^T$  and  $\mathbf{m} \mathbf{m}^T$  are linearly dependent. That is to say,  $\mathbf{rank}(\mathbf{m} \mathbf{m}^T) = \mathbf{rank}(\mathbf{m} \mathbf{m}^T) = 1$ .
51
52 \begin{definition} \label{def:q3-1}
53 Let  $U$  and  $V$  be two vector spaces, the sum of two the two vector spaces, denoted by  $U + V$ , is the set given by
54 \begin{align*}
55 U + V &= \{ \mathbf{u} + \mathbf{v} \mid \mathbf{u} \in U, \mathbf{v} \in V \}.
56 \end{align*}
57 \end{definition}
58
59 \begin{theorem} \label{thm:q3-1}
60 Let  $U$  and  $V$  be two vector spaces, then  $\dim(U+V) \leq \dim U + \dim V$ .
61 \end{theorem}

```

```

61 \begin{proof}
62 Let  $\dim U = u$ ,  $\dim V = v$ . We can further let
63  $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_u\}$  and
64  $B_2 = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_v\}$ 
65 to be the basis of  $U$  and  $V$ , respectively.
66
67 From \cref{def:q3-1}, we know that an arbitrary vector  $\mathbf{q} \in U+V$  can be written in the form of  $\mathbf{q} = \mathbf{u} + \mathbf{v}$ , where  $\mathbf{u} \in U, \mathbf{v} \in V$ .
68 With the basis of  $U$  and  $V$  provided above, we can further write
69 \begin{align}
70 \mathbf{u} &= k_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 + \dots + k_u \mathbf{u}_u, \\
71 \mathbf{v} &= s_1 \mathbf{v}_1 + s_2 \mathbf{v}_2 + \dots + s_v \mathbf{v}_v, \\
72 \end{align}
73 where  $k_1, \dots, k_u$  and  $s_1, \dots, s_v$  are scalars.
74 Since  $\mathbf{q} = \mathbf{u} + \mathbf{v}$ , we know  $\mathbf{q} \in \text{algspan}(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_u, \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_v)$ .
75 It can be seen that
76 \begin{align}
77 \dim(U + V) &\leq \dim \text{span} \text{algspan}(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_u, \mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_v) \leq u + v = \dim(U) + \dim(V). \\
78 \end{align}
79 \end{proof}
80
81 \begin{theorem}\label{thm:q3-2}
82 Let  $A$  and  $B$  be  $m \times n$  matrices, then  $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$ .
83 \end{theorem}
84
85 \begin{proof}
86 Denote the column space of a matrix  $M$  by  $C(M)$ . By definition, we know that  $\text{rank}(M) = \dim C(M)$ .
87 If we write  $A$  and  $B$  in terms of column vectors, that is  $A = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n]$  and  $B = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n]$ ,
88  $\rightarrow$  then we can express the column spaces of  $A$  and  $B$  as
89 \begin{align}
90 C(A) &= \text{algspan}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n), \\
91 C(B) &= \text{algspan}(\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n). \\
92 \end{align}
93 It is obvious that
94 \begin{align}
95 C(A + B) &= \text{algspan}(\mathbf{a}_1 + \mathbf{b}_1, \mathbf{a}_2 + \mathbf{b}_2, \dots, \mathbf{a}_n + \mathbf{b}_n). \\
96 \end{align}
97  $\forall \mathbf{p} \in C(A + B)$ , we have
98 \begin{align}
99 \mathbf{p} &= k_1(\mathbf{a}_1 + \mathbf{b}_1) + k_2(\mathbf{a}_2 + \mathbf{b}_2) + \dots + k_n(\mathbf{a}_n + \mathbf{b}_n) \\
100 &= (k_1 \mathbf{a}_1 + k_2 \mathbf{a}_2 + \dots + k_n \mathbf{a}_n) + (k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2 + \dots + k_n \mathbf{b}_n), \\
101 \end{align}
102 where  $k_1, \dots, k_n$  are scalars.
103 It can be seen that  $\mathbf{p} \in \text{algspan}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) + \text{algspan}(\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n)$ .
104 We can derive
105 \begin{align}
106 \text{rank}(A + B) &= \dim C(A + B) \\
107 &\leq \dim \left[ \text{algspan}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) + \text{algspan}(\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n) \right] \\
108 &\leq \dim \left[ \text{algspan}(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n) \right] + \dim \left[ \text{algspan}(\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n) \right] \\
109 &= \dim C(A) + \dim C(B) = \text{rank}(A) + \text{rank}(B). \\
110 \end{align}
111 \end{proof}
112
113 Using \cref{thm:q3-2}, it can be seen that
114 \begin{align}
115 \text{rank}(C) &= \text{rank}(L^T + L) \leq \text{rank}(L^T) + \text{rank}(L) = 2. \\
116 \end{align}
117
118 \question{4}
119
120 We know that a conic can be written in the form of
121 \begin{align}
122 \mathbf{C} &= \\
123 \mathbf{C} &= \begin{bmatrix} a & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{bmatrix}. \\
124 \end{align}
125
126 For an \gls{hc} point  $\mathbf{x}_1 = [x_1, y_1, 1]^T$ , it can be seen that
127 \begin{align}
128 \mathbf{x}_1^T \mathbf{C} \mathbf{x}_1 &= a x_1^2 + b x_1 y_1 + c y_1^2 + d x_1 + e y_1 + f = 0. \\
129 \end{align}
130
131 We know that  $\forall k \in \mathbb{R} \setminus \{0\}$ ,  $k\mathbf{C}$  and  $\mathbf{C}$  represent the same conic. Therefore, the following expression should also hold true:
132 \begin{align}
133 \mathbf{x}_1^T (k\mathbf{C}) \mathbf{x}_1 &= k a x_1^2 + k b x_1 y_1 + k c y_1^2 + k d x_1 + k e y_1 + k f = 0. \\
134 \end{align}
135
136 In a nontrivial conic, at least one of the coefficients among  $a, b, c, d, e, f$  will be nonzero. Without loss of generality, we can assume  $a$  is nonzero, and let  $k = \frac{1}{a}$ . Now
137  $\rightarrow$  \cref{eqn:q4-2} can be written as
138 \begin{align}
139 \mathbf{x}_1^T \mathbf{C} \mathbf{x}_1 &= x_1^2 + \frac{b}{a} x_1 y_1 + \frac{c}{a} y_1^2 + \frac{d}{a} x_1 + \frac{e}{a} y_1 + \frac{f}{a} = 0. \\
140 \end{align}
141
142 If we let  $\frac{b}{a} = b', \frac{c}{a} = c', \frac{d}{a} = d', \frac{e}{a} = e', \frac{f}{a} = f'$ , then this expression becomes
143 \begin{align}
144 \mathbf{x}_1^T \mathbf{C} \mathbf{x}_1 &= x_1^2 + b' x_1 y_1 + c' y_1^2 + d' x_1 + e' y_1 + f' = 0. \\
145 \end{align}
146
147 This is an equation with 5 unknowns, which means we need 5 points  $\mathbf{x}_1, \dots, \mathbf{x}_5$  to solve for the coefficients of the conic. That is to say, a conic is defined with 5
148  $\rightarrow$  points.
149
150 \question{5}
151
152 \begin{enumerate}[label=(\arabic*)]
153 \item The \gls{hc} representation of  $L_1$  is given by
154 \begin{align}
155 \mathbf{C}_1 &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ -2 & 2 & 0 \end{bmatrix}. \\
156 \end{align}
157 The \gls{hc} representation of  $L_2$  is given by
158 \begin{align}
159 \mathbf{C}_2 &= \begin{bmatrix} 3 & 4 & 1 \\ 5 & 6 & 1 \\ -2 & 2 & -2 \end{bmatrix}. \\
160 \end{align}
161 The intersection between  $L_1$  and  $L_2$  is
162 \begin{align}
163 \mathbf{C}_1 - \mathbf{C}_2 &= \begin{bmatrix} -3 & -4 & 0 \\ -5 & -5 & 0 \\ 0 & -4 & 2 \end{bmatrix}. \\
164 \end{align}
165 That is, the intersection is given by  $\left(1, 2\right)$ .
166 \item The \gls{hc} representation of line  $L_3$  is given by
167 \begin{align}
168 \mathbf{C}_3 &= \begin{bmatrix} 7 & -8 & 1 \\ -7 & 8 & 1 \\ -16 & -14 & 0 \end{bmatrix}. \\
169 \end{align}
170 Since the third element of  $L_1$  and  $L_3$  are both zero, we know that the two points pass through the origin. Therefore, they must intersect at the origin. In this case,
171  $\rightarrow$  we only need to compute  $L_1$  and  $L_3$ , which consists of two steps.
172 \end{enumerate}
173
174 \question{6}
175
176 The \gls{hc} representation of  $L_1$  is given by
177 \begin{align}
178 \mathbf{C}_1 &= \begin{bmatrix} -4 & 0 & 1 \\ -2 & 8 & 1 \\ -8 & 2 & -32 \end{bmatrix}. \\
179 \end{align}

```

```

179 The \gls{hc} representation of  $\mathbf{l}_2$  is given by
180 \begin{align}
181 \quad \mathbf{qmat}\{0;-2;1\} \times \mathbf{qmat}\{4;14;1\} = \mathbf{qmat}\{-16;4;8\}.
182 \end{align}
183 The intersection between  $\mathbf{l}_1$  and  $\mathbf{l}_2$  is given by
184 \begin{align}
185 \quad \mathbf{qmat}\{-8;2;32\} \times \mathbf{qmat}\{-16;4;8\} = \mathbf{qmat}\{144;576;0\}.
186 \end{align}
187 Since this is an infinity point, we know that  $\mathbf{l}_1$  and  $\mathbf{l}_2$  are parallel.
188
189 \question{7}
190
191  $x=1$  can be written as  $1x + 0y - 1 = 0$ , which means its corresponding \gls{hc} representation is  $\mathbf{qmat}\{1,0,-1\}$ .  $y=-1$  can be written as  $0x+1y+1=0$ , which means its corresponding
192  $\leftrightarrow$  \gls{hc} representation is  $\mathbf{qmat}\{0,1,1\}$ . The intersection between them is given by
193 \begin{align}
194 \quad \mathbf{qmat}\{1;0;-1\} \times \mathbf{qmat}\{0;1;1\} = \mathbf{qmat}\{1;-1;1\}.
195 \end{align}
196 That is, the intersection is at  $(1,-1)$ .
197
198 \question{8}
199 The equation of the ellipse is
200 \begin{align}
201 \quad \frac{(x-2)^2}{\left(\frac{1}{2}\right)^2} + (y-3)^2 = 1.
202 \end{align}
203 This simplifies to
204 \begin{align}
205 \quad 4x^2 + y^2 - 16x - 6y + 24 = 0.
206 \end{align}
207 This conic can be written as
208 \begin{align}
209 \quad \mathbf{C} = \mathbf{qmat}\{
210 \quad 4, 0, -8;
211 \quad 0, 1, -3;
212 \quad -8, -3, 24.
213 \}
214 \end{align}
215 Since  $\mathbf{p}$  is the origin, the \gls{hc} representation is given by  $\mathbf{qmat}\{0,0,1\}$ . Therefore, the polar line is given by  $\mathbf{l} = \mathbf{C} \mathbf{p} = \mathbf{qmat}\{-8,-3,24\}$ .
216
217 The \gls{hc} representation of the  $x$  axis is given by  $\mathbf{qmat}\{0,1,0\}$ . Therefore, the intersection between  $\mathbf{l}$  and the  $x$  axis is
218 \begin{align}
219 \quad \mathbf{qmat}\{-8;-3;24\} \times \mathbf{qmat}\{0;1;0\} = \mathbf{qmat}\{-24;0;-8\}.
220 \end{align}
221 That is to say, the intersection between the polar line and the  $x$  axis is  $(3, 0)$ .
222
223 The \gls{hc} representation of the  $y$  axis is given by  $\mathbf{qmat}\{1,0,0\}$ . Therefore, the intersection between  $\mathbf{l}$  and the  $y$  axis is
224 \begin{align}
225 \quad \mathbf{qmat}\{-8;-3;24\} \times \mathbf{qmat}\{1;0;0\} = \mathbf{qmat}\{0;24;3\}.
226 \end{align}
227 That is to say, the intersection between the polar line and the  $y$  axis is  $(0, 8)$ .
228
229
230 \largeappendix
231 \SourceListing{
232 \quad ../alanhm.cls,latex,
233 \quad 1_ZiyueXiang.tex,latex
234 }
235
236 \end{document}

```