ECE 661 - Homework-1

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1 Problem 1

1.1 Problem Statement

What are all the points in the representational space R^3 that are the homogeneous coordinates of the origin in the physical space in R^2?

1.2 Solution

The points in the representational space R^3 that gives the homogeneous coordinates of origin in the physical space in R^2 are given by

$$\begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}$$

for all w $\in$ R $\neq 0$

Since the physical space of 2D is represented as

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

where as x= u/w and y = v/w. Also, If u=v=0 and w can be any non zero number, that represents the origin in 2D physical space.

2 Problem 2

2.1 Problem Statement

Are all the points at infinity in the physical space R^2 the same? Justify your answer.

2.2 Solution

No. The points at infinity can be represented as $\begin{bmatrix} u \\ v \\ w \end{bmatrix}$ where u,v can be any number that $\in$ R and w = 0. So all points at infinity are not same, but can be connected in a single straight line whose equation is $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

3 Problem 3

3.1 Problem Statement

Argue that the matrix rank of a degenerate conic can never exceed 2.

3.2 Solution

The degenerate conic equation is given by C= lm^T+ml^T. We know that rank of lm^T = ml^T = 1. since each matrix is made of 'generators'(outer product of two vectors).
Also we know that by vector spaces,

dim(A+B) = dim(A) + dim(B) - dim(A∩B) where A,B should be matrices formed using basis vectors.
here, A = lm^T;and B = ml^T;
dim(A∩B) = dimension/rank of null space of A∩B.
therefore, rank (A+B) = rank (A) +rank (B) - dim(N(A∩B))
rank (A+B) $\leq$ rank (A) +rank (B).
Since dim(N(A∩B)) = 0 only if A, B does not have any common null space.
rank (A+B) $\leq$ 2
rank (C) $\leq$ 2

4 Problem 4

4.1 Problem Statement

Derive in just three steps the intersection of two lines l_1 and l_2 with l_1 passing through the points (0,0) and (2,3), and with l_2 passing through the points (-3,3) and (-1,2). How many steps would take you if the second line pass through (-4,-5) and (4,5)?

4.2 Solution

The equation of the line l_1 = p1 X p2. where p1 and p2 are in representational form $\in R^3$

$$l_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} X \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}$$

lllrly,

$$l_2 = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix} X \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

The intersection point is given by l_1 X l_2

$$Intersectionpoint = \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} X \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -6 \\ -9 \\ -8 \end{bmatrix}$$

Intersection point = (6/8,9/8)
If the second line, l_2 passes through (-4,-5) and (4,5) then

$$l_2 = \begin{bmatrix} -4 \\ -5 \\ 1 \end{bmatrix} X \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}$$

The new l_2 and l_2 has 0 as their third element also the cross product of first two elements is non-zero. Hence the Intersection point is (0,0). So it will take only two steps to figure it out.

5 Problem 5

5.1 Problem Statement

Consider there are two lines. The first line passing through the points (0,0) and (2,-2).The second line passing through the points (-3,0) and (0,-3). Find the intersection between two lines. Comment on your answer.

5.2 Solution

The equation of the line l_1 = p1 X p2. where p1 and p2 are in representational form $\in R^3$

$$l_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} X \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

lllrly,

$$l_2 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} X \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1 \end{bmatrix}$$

The intersection point is **infinity**, since the ratio of -a/b i.e (the negated ratio of first two elements of the resultant vector) is same in both line equations, we can say that they form llel lines.

6 Problem 6

6.1 Problem Statement

As you know, when a point x is on a conic, the tangent to the conic is given by l=Cx. That raises the question of what Cx corresponds to when x is outside the conic. As you 'll see later in class, when x is outside the conic, Cx is the line that joins the two points of contact if you draw tangents to C from the point x. This line is referred to as *polar line*. Now consider for our conic a circle of radius 1 that is centered at the coordinates (-6,-6) and let x be the origin of the R^2 physical plane. Where does the polar line intersect the x and y axes in this case?

6.2 Solution

Given conic is a circle.
Hence the equation of the circle is given by $(x - x_1)^2 + (y - y_1)^2 = r^2$
$x_1 = -6; y_1 = -6; r = 1$
$(x + 6)^2 + (y + 6)^2 = 1$
$x^2 + 0xy + y^2 + 12x + 12y + 71 = 0$
therefore,

$$C = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 6 \\ 6 & 6 & 71 \end{bmatrix}$$

Now, equation of the x-axis is given by $l_2 = [0 \;\; 1 \;\; 0]^T$. Since x-axis is orthogonal to y-axis it is y=0 and the straight line (x-axis) to goes to infinity, so the third element is also 0.
lllrly the equation of y-axis is $l_3 = [1 \;\; 0 \;\; 0]^T$
We know that, l=Cx
so

$$l_1 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 6 \\ 6 & 6 & 71 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 71 \end{bmatrix}$$

The intersection point on x-axis is given by, $p_x = l_1 X l_2$

$$p_x = \begin{bmatrix} 6 \\ 6 \\ 71 \end{bmatrix} X \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -71 \\ 0 \\ 6 \end{bmatrix}$$

$p_x = (-71/6, 0)$
The intersection point on y-axis is given by, $p_y = l_1 X l_3$

$$p_y = \begin{bmatrix} 6 \\ 6 \\ 71 \end{bmatrix} X \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 71 \\ -6 \end{bmatrix}$$

$p_y = (0, -71/6)$