ECE661: Computer Vision (Fall 2014)

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1 Introduction

In this assignment my own version of Harris corner detector will be implemented. The correspondences of interest points between two images (the same object with views from different angles) would be established based on SSD (Sum of Squared Differences) and NCC (Normalized Cross Correlation) method. Then, we will check the quality of Harris corner detector by applying the SIFT operator to the same sets of images.

Based on our experiment, it can be concluded although Harris corner detector can detect those obvious corners easily and accurately, it is not a good method when the features are not strict corners/more robust features.

It has been found in the experiment that the NCC based SIFT works better than anything else regarding those robust features. Please refer to figure 26 and figure 39 for great output from NCC based SIFT matching.

2 Harris Corner Detector

2.1 Overview of Harris Corner Detection

Before SIFT and SURF operators were invented, Harris Corner Detector was widely used in digital image interest points detection. The idea of Harris corner detection is based on that the characterization of a corner pixel should be invariant to rotations of images.

Although scale was not introduced when Harris corner detector was first introduced, we now can implement the Harris corner detector with variable scale.

2.2 Harris Corner Detector Implementation

1. We first need to calculate the gradient along x and y directions in the image. However, since we need to make our Harris corner detector scalable, we can not use Sobel Operator as Sobel Operator can not take care of scales properly. Instead, Haar Filter was implemented to replace Sobel Operator by finding the d_x and d_y . Below we will give an example of Haar filter with $\sigma = 1.2$.

Haar Filter for
$$\frac{\partial}{\partial x}$$
, with $\sigma = 1.2$:
$$\begin{bmatrix}
-1 & -1 & -1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 1 & 1
\end{bmatrix}$$

Please note the above form of Haar filter is based on the expansion of Haar wavelet at basic form. We have to make sure that the forms are scaled up to an M by M operator where M is the smallest even integer greater than $4 \times \sigma$. (Similarly we can easily prove that while $\sigma = 1.2$, M = 6. And when $\sigma = 1.4$, M = 8)

 $\sigma = 1.4$.

Furthermore, in order to minimize the noise in the pictures, before gradient calculation was performed, we also need to filter our images with Gaussian smoothing filter.

2. After the d_x and d_y was obtained, we then create a neighbourhood window of size $5\sigma \times 5\sigma$. Note that the σ should be consistent of that used in the first part when we are filtering the image using Haar filter. The C matrix could then be constructed:

$$C = \begin{bmatrix} \sum d_x^2 & \sum d_x d_y \\ \sum d_x d_y & \sum d_y^2 \end{bmatrix}$$

- 3. While C in the previous step is a 2×2 matrix, we will first check the rank of $C_{i,j}$ at pixel location (i,j). As long as $rank(C) \neq 2$, we will remove the pixel locations from our candidates list of corners/interest points. The computation efficiency could be improved significantly if we can first eliminate majority of candidates points.
- 4. For the remaining corner candidates, we than need to determine the corner strength. Define

Conrner Response =
$$\lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$

While k is defined as a constant: 0.04, λ_1 and λ_2 is the eigenvalues of matrix C. Obviously if C is not rank 2 matrix the candidate point would not worth

investigating. In order to simplify the calculation,

$$\det(C) = \lambda_1 \lambda_2$$

$$trace(C) = \lambda_1 + \lambda_2$$

therefore, SVD of matrix C would then not be required.

- 5. After corner response at each candidates pixel has been calculated, we then set up a threshold to filter out those points whose corner responses are not strong enough. However, in practical we will notice that even after threshold, at certain regions there would still be too many corner candidates. In order to solve the problem, we will perform non-maxima suppression to extract only those points with local maxima values.
- 6. Now all the Harris corner detection technique has been performed and we have certain amount of interest points. Save the interest points extracted from each images separately for corner correspondence estimation.

3 Establishing Correspondences Between Image Pairs for Harris Corner Detector

3.1 SSD: Sum of Squared Differences

In order to use SSD to establish the correspondences between interest points of an image pair, we first need to define a window $(M+1) \times (M+1)$. For the Harris corner detector, let $f_1(i,j)$ denote the pixel values in image 1 within the $(M+1) \times (M+1)$ window, and let $f_2(i,j)$ denote the pixel values in image 2 within the $(M+1) \times (M+1)$ window. Pairwise SSD is defined as:

$$SSD = \sum_{i} \sum_{j} |f_1(i,j) - f_2(i,j)|^2$$

3.2 NCC: Normalized Cross Correlation

Similarly as SSD, In order to use NCC to establish the correspondences between interest points of an image pair, we first need to define a window $(M+1) \times (M+1)$. For the Harris corner detector, let $f_1(i,j)$ denote the pixel values in image 1 within the $(M+1) \times (M+1)$ window, and let $f_2(i,j)$ denote the pixel values in image 2 within the $(M+1) \times (M+1)$ window. Pairwise NCC is defined as:

$$NCC = \frac{\sum_{i} \sum_{j} (f_1(i,j) - \mu_1)(f_2(i,j) - \mu_2)}{\sqrt{\left[\sum_{i} \sum_{j} (f_1(i,j) - \mu_1)^2\right]\left[\sum_{i} \sum_{j} (f_2(i,j) - \mu_2)^2\right]}}$$

while μ_1 is the mean of window $f_1(i,j)$ and μ_2 is the mean of window $f_2(i,j)$.

3.3 False Matching Elimination

In general, a lot of pairs were matched incorrectly if we do not have any systematic way to avoid/reduce false matching.

SSD Case: As SSD is defined as the sum of squared errors, an ideal match would obviously have SSD = 0. However, based on our practical experiment we know that is almost impossible. Hence we use the following method to reduce/avoid false matching.

- 1. If SSD value of a certain pair is smaller than 5 × (the absolute minima values of SSD across all SSD matrix, we proceed, otherwise will dump the point. This step will actually dump a lot of good candidates.
- 2. If the $\frac{minimum\ of\ SSD}{second\ minimum\ of\ SSD}$ is smaller than a certain ratio (denoted as Rssd), we will establish correspondence between this specific pair. Otherwise we will again dump the point as candidate.

NCC Case: As NCC is defined as the normalized cross correlation, an ideal match would obviously have NCC = 1. However, based on our practical experiment we know that is almost impossible. Hence we use the following method to reduce/avoid false matching.

- 1. If NCC value of a certain pair is smaller than $0.9 \times$ (the absolute maxima values of NCC across all SSD matrix, we proceed, otherwise will dump the point. **This step will actually dump a lot of good candidates**.
- 2. If the $\frac{maxima\ of\ NCC}{second\ maxima\ of\ NCC}$ is larger than a certain ratio (denoted as Rncc), we will establish correspondence between this specific pair. Otherwise we will again dump the point as candidate.
- 3. Of course, if NCC value is negative, which mean two pixel is anti-correlated, they can not be a pair.

3.4 Parameters Table For Harris Corner Detector

Image	W_{Haar}	W_{SSD}	W_{NCC}	TH_{SSD}	TH_{NCC}	R_{SSD}	R_{NCC}
pic1.jpg	$5\sigma \times 5\sigma$	$10\sigma \times 10\sigma$	$10\sigma \times 10\sigma$	$\leq 40 \times SSD_{Minima}$	$\geq 0.3 \times NCC_{Maxima}$	0.85	1.1
pic2.jpg	$5\sigma \times 5\sigma$	$10\sigma \times 10\sigma$	$10\sigma \times 10\sigma$	$\leq 40 \times SSD_{Minima}$	$\geq 0.3 \times NCC_{Maxima}$	0.85	1.1
pic6.jpg	$5\sigma \times 5\sigma$	$10\sigma \times 10\sigma$	$10\sigma \times 10\sigma$	$\leq 40 \times SSD_{Minima}$	$\geq 0.3 \times NCC_{Maxima}$	0.8	1.01
pic7.jpg	$5\sigma \times 5\sigma$	$10\sigma \times 10\sigma$	$10\sigma \times 10\sigma$	$\leq 40 \times SSD_{Minima}$	$\geq 0.3 \times NCC_{Maxima}$	0.8	1.01
my1.jpg	$5\sigma \times 5\sigma$	$10\sigma \times 10\sigma$	$10\sigma \times 10\sigma$	$\leq 40 \times SSD_{Minima}$	$\geq 0.3 \times NCC_{Maxima}$	0.85	1.01
my2.jpg	$5\sigma \times 5\sigma$	$10\sigma \times 10\sigma$	$10\sigma \times 10\sigma$	$\leq 40 \times SSD_{Minima}$	$\geq 0.3 \times NCC_{Maxima}$	0.85	1.01

Note that the threshold values for corner responses is defined as:

$$(CR1_{Maxima} + CR2_{Maxima})/20$$

 $(CR1_{Maxima})$ is the Maxima for Corner Response of Image 1

 $(CR2_{Maxima})$ is the Maxima for Corner Response of Image 2

4 SIFT Algorithm: Scale Invariant Feature Transform

For SIFT algorithm, we first need to find all the local extrema from the DoG pyramid. Note that extrema include both maxima and minima. To be more detailed, each point in the DoG pyramid should be compared to:

- 1. 8 points in the 3 by 3 neighbourhood at the same scale
- 2. 9 points in the 3 by 3 neighbourhood at the next scale
- 3. 9 points in the 3 by 3 neighbourhood at the previous scale

Usually, those points in original image that the grey levels change rapidly in several directions are likely to be the DoG extrema.

In order to locate the extrema in the sub-pixel accuracy, we need to estimate the second-order derivatives of $D(x, y, \sigma)$ at the sampling points in the DoG pyramid. First, find the Taylor series expansion of $D(x, y, \sigma)$ in the vicinity of $\vec{x_0} = (x_0, y_0, \sigma_0)^T$:

$$D(\vec{x}) \approx D(\vec{x_0}) + J^T(\vec{x_0})\vec{x} + \frac{1}{2}\vec{x}^T H(\vec{x_0})\vec{x}$$

where \vec{x} is a incremental of $\vec{x_0}$.

Easily, J is the gradient vector estimated at \vec{x}_0 :

$$J(\vec{x}_0) = (\frac{\partial D}{\partial x}, \frac{\partial D}{\partial y}, \frac{\partial D}{\partial \sigma})^T|_{\vec{x}_0}$$

And Hessian matrix is:

$$H(\vec{x}_0) = \begin{bmatrix} \frac{\partial^2 D}{\partial x^2} & \frac{\partial^2 D}{\partial x \partial y} & \frac{\partial^2 D}{\partial x \partial \sigma} \\ \frac{\partial^2 D}{\partial y \partial x} & \frac{\partial^2 D}{\partial y^2} & \frac{\partial^2 D}{\partial y \partial \sigma} \\ \frac{\partial^2 D}{\partial \sigma \partial x} & \frac{\partial^2 D}{\partial \sigma \partial y} & \frac{\partial^2 D}{\partial \sigma^2} \end{bmatrix}$$

For the true locations of extrema:

$$\vec{x} = -H^{-1}(\vec{x}_0)J(\vec{x}_0)$$

As the extrema points are found, we need to threshold out those extremas who are week. For example we can set a hard cut off at:

$$D(\vec{x}) \ge 0.03$$

to be qualified for an extrema candidate.

After the candidates of the local extrema are found, we then need to establish the dominant local orientation for each candidate point found in previous step. To find the local

dominant orientation we need to calculate the gradient vector of the Gaussian-smoothed image $f(x, y, \sigma)$ at the scale σ of the extrema. It magnitude is defined as:

$$m(x,y) = \sqrt{|f(x+1,y,\sigma) - f(x,y,\sigma)|^2 + |f(x,y+1,\sigma) - f(x,y,\sigma)|^2}$$

While the orientation is:

$$\theta(x,y) = \arctan \frac{f(x+1,y,\sigma) - f(x,y,\sigma)}{f(x,y+1,\sigma) - f(x,y,\sigma)}$$

Finally, we divide the 16 by 16 neighbourhood of point into 4 by 4 cells (each cell with 4 by 4 points and totally we have 16 cells). Now, for each of the cell, an 8-bin orientation histogram is calcualted from the gradient-magnitude-weighted values of $\theta(x,y)$ at 16 pixels. That is, total of $8 \times 16 = 128$. Hence, for each interest point, we will have 128-element descriptor.

In next section, we will explain how to establish correspondences based on features extracted by SIFT operator

5 Establishing Correspondences Between Image Pairs for SIFT

5.1 SSD: Sum of Squared Differences

In order to use SSD to establish the correspondences between interest points of an image pair yield by SIFT, we need:

$$SSD = \sum_{i} \sum_{j} |f_1(i,j) - f_2(i,j)|^2$$

While $f_1(i,j)$ is the 128-elements descriptor obtained at each interest points location.

5.2 NCC: Normalized Cross Correlation

Similarly as SSD, In order to use NCC to establish the correspondences between interest points of an image pair, we need:

$$NCC = \frac{\sum_{i} \sum_{j} (f_1(i,j) - \mu_1)(f_2(i,j) - \mu_2)}{\sqrt{\left[\sum_{i} \sum_{j} (f_1(i,j) - \mu_1)^2\right] \left[\sum_{i} \sum_{j} (f_2(i,j) - \mu_2)^2\right]}}$$

While $f_1(i,j)$ is the 128-elements descriptor obtained at each interest points location.

5.3 False Matching Elimination

In general, a lot of pairs were matched incorrectly if we do not have any systematic way to avoid/reduce false matching.

SSD Case: As SSD is defined as the sum of squared errors, an ideal match would obviously have SSD = 0. However, based on our practical experiment we know that is almost impossible. Hence we use the following method to reduce/avoid false matching.

- 1. If SSD value of a certain pair is smaller than $5 \times$ (the absolute minima values of SSD across all SSD matrix, we proceed, otherwise will dump the point. This step will actually dump a lot of good candidates.
- 2. If the $\frac{minimum\ of\ SSD}{second\ minimum\ of\ SSD}$ is smaller than a certain ratio (denoted as Rssd), we will establish correspondence between this specific pair. Otherwise we will again dump the point as candidate.

Euclidean Distance Case: Euclidean distance case is almost identical as SSD, the only difference is $Euclidean\ Distance = \sqrt{SSD}$

NCC Case: As NCC is defined as the normalized cross correlation, an ideal match would obviously have NCC = 1. However, based on our practical experiment we know that is almost impossible. Hence we use the following method to reduce/avoid false matching.

- 1. If NCC value of a certain pair is smaller than $0.9 \times$ (the absolute maxima values of NCC across all SSD matrix, we proceed, otherwise will dump the point. **This step will actually dump a lot of good candidates**.
- 2. If the $\frac{maxima\ of\ NCC}{second\ maxima\ of\ NCC}$ is larger than a certain ratio (denoted as Rncc), we will establish correspondence between this specific pair. Otherwise we will again dump the point as candidate.
- 3. Of course, if NCC value is negative, which mean two pixel is anti-correlated, they can not be a pair.

5.4 Parameters Table For SIFT

Image	$R_{Euclidean}$	R_{SSD}	R_{NCC}
pic1.jpg	$\leq 5 \times Euclidean_{Minima}$	$\leq 5 \times SSD_{Minima}$	$\geq 0.9 \times NCC_{Maxima}$
pic2.jpg	$\leq 5 \times Euclidean_{Minima}$	$\leq 5 \times SSD_{Minima}$	$\geq 0.9 \times NCC_{Maxima}$
pic6.jpg	$\leq 5 \times Euclidean_{Minima}$	$\leq 5 \times SSD_{Minima}$	$\geq 0.9 \times NCC_{Maxima}$
pic7.jpg	$\leq 5 \times Euclidean_{Minima}$	$\leq 5 \times SSD_{Minima}$	$\geq 0.9 \times NCC_{Maxima}$
my1.jpg	$\leq 5 \times Euclidean_{Minima}$	$\leq 5 \times SSD_{Minima}$	$\geq 0.9 \times NCC_{Maxima}$
my2.jpg	$\leq 5 \times Euclidean_{Minima}$	$\leq 5 \times SSD_{Minima}$	$\geq 0.9 \times NCC_{Maxima}$

Based on our empirical data, dynamic threshold method works great for SIFT.

6 Dynamic Threshold for Euclidean Distance, SSD and NCC

It has already proven useful and convenient in this experiment using dynamic threshold. The idea of dynamic threshold is to avoid manually change each threshold value for every single experiment. Because the threshold values would vary hugely based on the image quality, illumination, feature descriptors strength, corner response, etc.

For example, in order to qualify for a match SSD value should be at least smaller than 5 times the smallest SSD value. Or, similarly, in order to qualify for a match NCC value should be at least larger than 0.9 times the largest NCC value.

Great Result: NCC for SIFT

From the experiment, we have actually concluded that when the images/features are robust, NCC based on SIFT features can still work great. For more details please refer to each of the conclusion/discussion session in next section and figure 26, figure 39.

7 Results: Very Important Conclusions At End of Each Subsections

7.1 Set 1: Harris Operator/SIFT Comparison



Figure 1. Set1: pic1.jpg



Figure 2. Set1: pic2.jpg

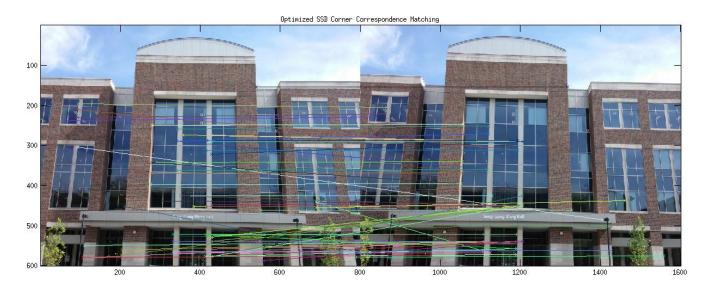


Figure 3. Harris: The SSD matching with $\sigma=0.6$

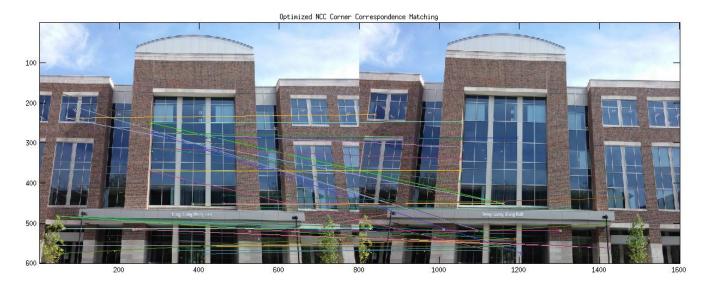


Figure 4. Harris: The NCC matching with $\sigma=0.6$

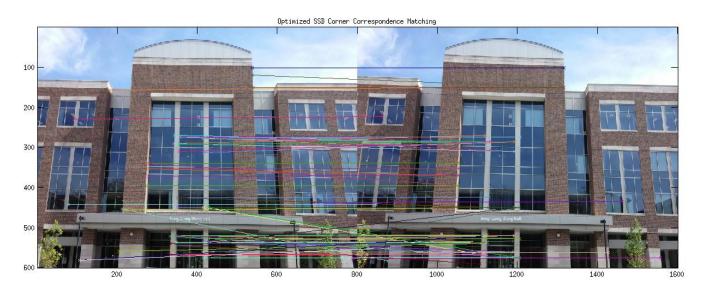


Figure 5. Harris: The SSD matching with $\sigma=1$

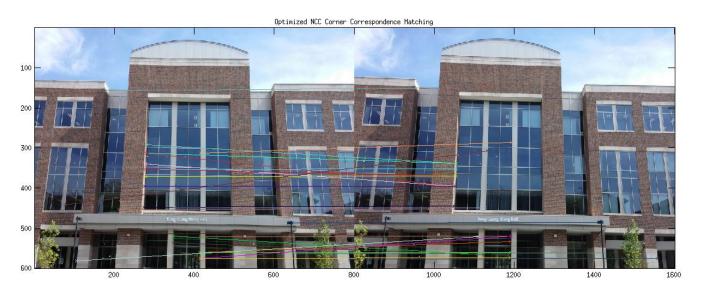


Figure 6. Harris: The NCC matching with $\sigma=1$

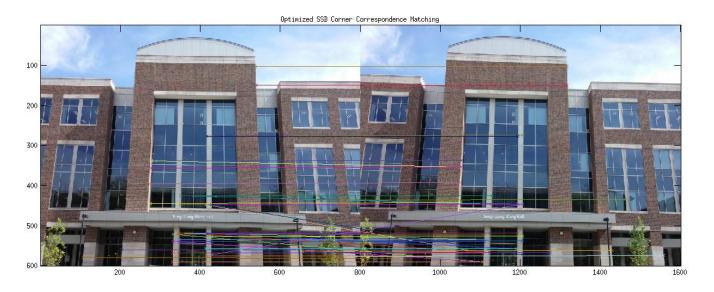


Figure 7. Harris: The SSD matching with $\sigma=1.4$

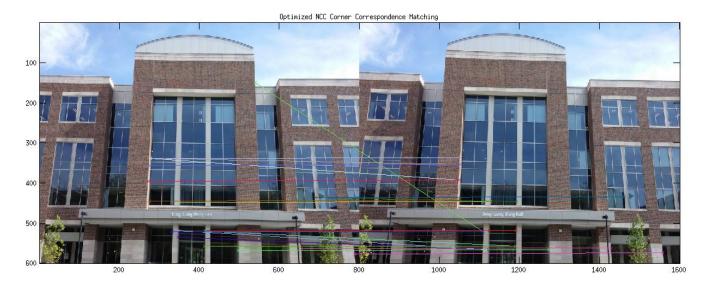


Figure 8. Harris: The NCC matching with $\sigma=1.4$

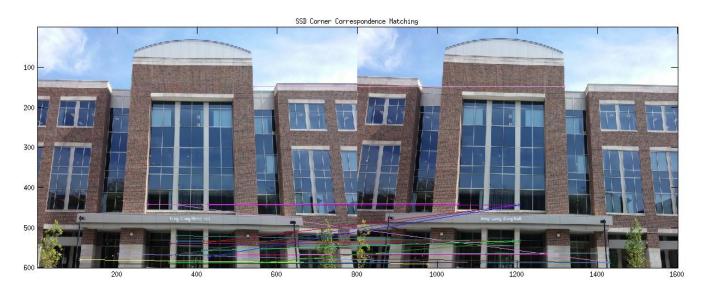


Figure 9. Harris: The SSD matching with $\sigma = 2.2$

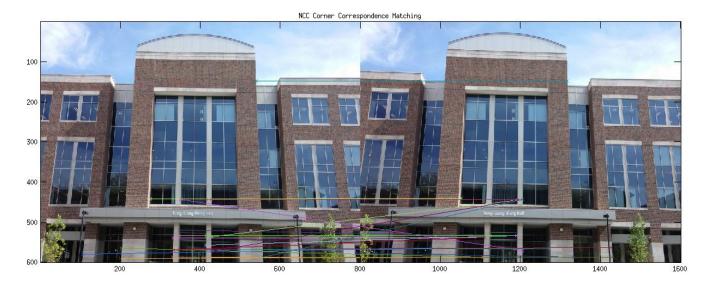


Figure 10. Harris: The NCC matching with $\sigma=2.2$

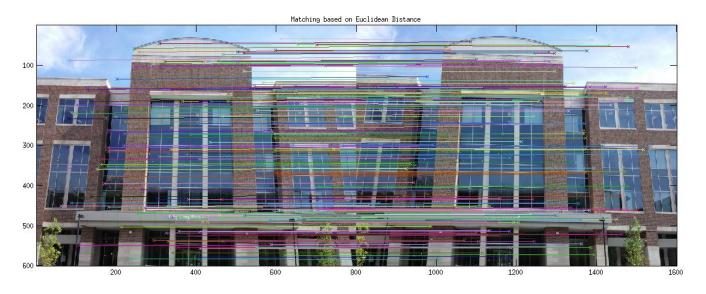


Figure 11. SIFT: Interest Points Matching Based on Euclidean Distance

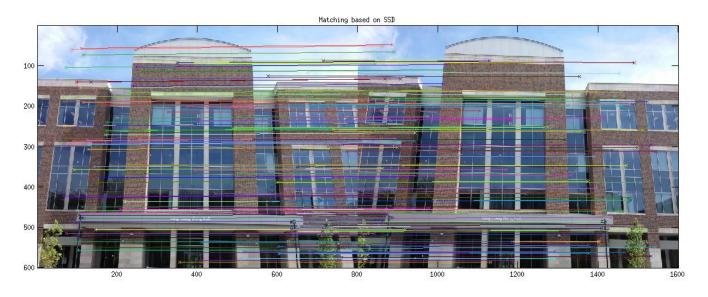


Figure 12. SIFT: Interest Points Matching Based on SSD

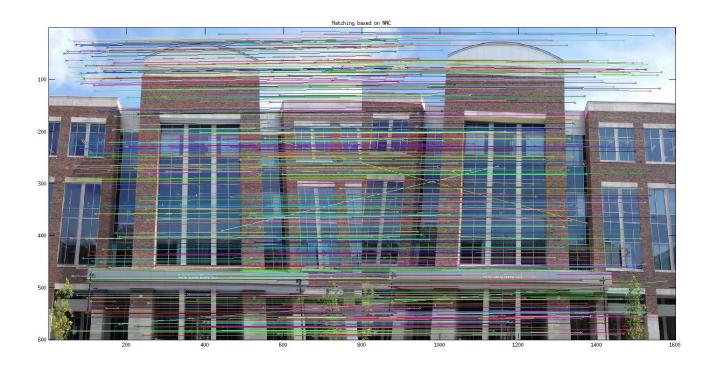


Figure 13. SIFT: Interest Points Matching Based on NCC

Conclusion for Set 1:

For this set of images as the view angle (also the lighting conditions, color saturation, etc) didn't change that much, Harris corner detector works pretty well. For the correspondences established based on SSD and NCC, except for a very few mismatch the overall correct matching rate is very high.

It can also be concluded that larger the σ , less sensitive the Harris Corner detector is (less interest points is not necessarily bad). Those points in the lower part of the image could always be detected by Harris Corner detector. As the Harris Corner detector already work pretty well, SIFT operator would not improve our result that much. (of course we will easily have a lot more interest points).

7.2 Set 2: Harris Operator/SIFT Comparison



Figure 14. Set2: pic6.jpg



Figure 15. Set1: pic7.jpg



Figure 16. Harris: The SSD matching with $\sigma=0.6$

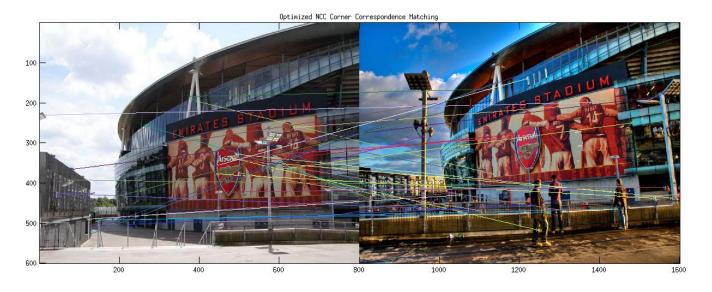


Figure 17. Harris: The NCC matching with $\sigma=0.6$

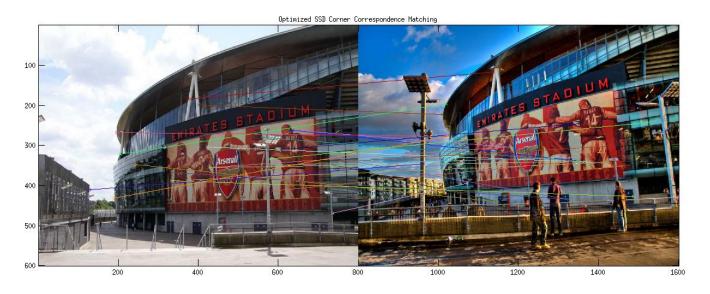


Figure 18. Harris: The SSD matching with $\sigma=1$



Figure 19. Harris: The NCC matching with $\sigma=1$



Figure 20. Harris: The SSD matching with $\sigma = 1.4$



Figure 21. Harris: The NCC matching with $\sigma=1.4$



Figure 22. Harris: The SSD matching with $\sigma = 2.2$



Figure 23. Harris: The NCC matching with $\sigma=2.2$

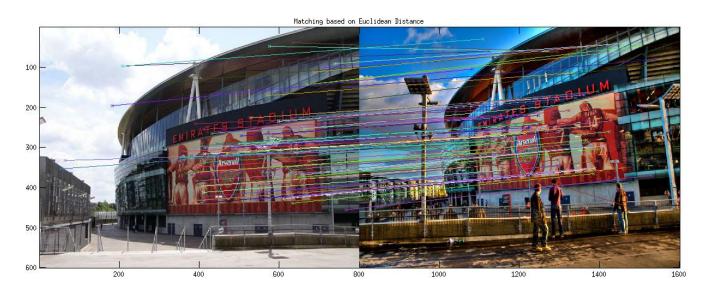


Figure 24. SIFT: Interest Points Matching Based on Euclidean Distance

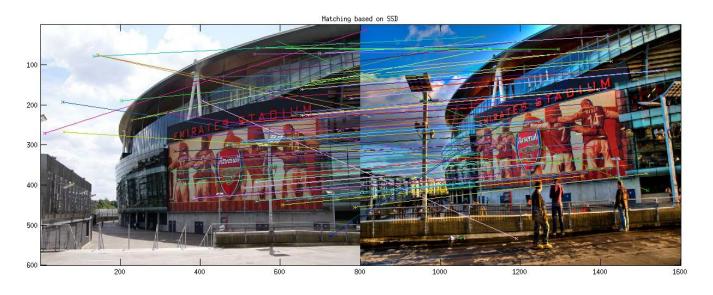


Figure 25. SIFT: Interest Points Matching Based on SSD

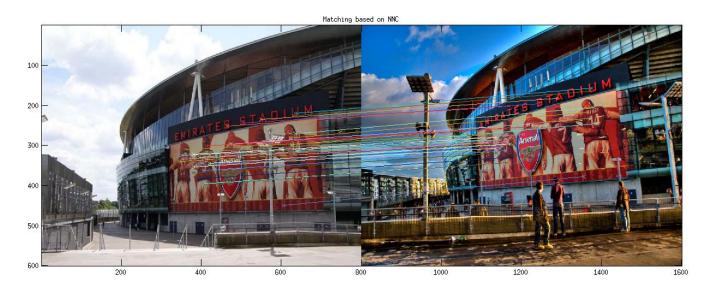


Figure 26. SIFT: Interest Points Matching Based on NCC

Conclusion for Set 2:

For this set of images as the view angle (**ESPECIALLY** the lighting conditions, color saturation, etc) changed **significantly**, Harris corner detector did not perform as well as in the previous set. For the correspondences established based on SSD and NCC, the matching rate decreased significantly.

Although larger the σ , less sensitive the Harris Corner detector is (less interest points detected is not necessarily bad). Those points detected with larger σ actually tend to be more accurately matched across the images.

As the Harris Corner detector yield bad results for this pair, SIFT operator actually works a lot better! The results based on Euclidean and SSD are great, but the result based on NCC is greater! With NCC, SIFT actually succeeded in matching the Arsenal Player Poster, while there are not so many significant corners in there (by human visual).

Based on the result from this part, we have concluded when the features are more robust, SIFT with NCC would improve our matching rate significantly.

7.3 My Own Set: Harris Operator/SIFT Comparison



Figure 27. My Set: my1.jpg



Figure 28. My Set: my2.jpg

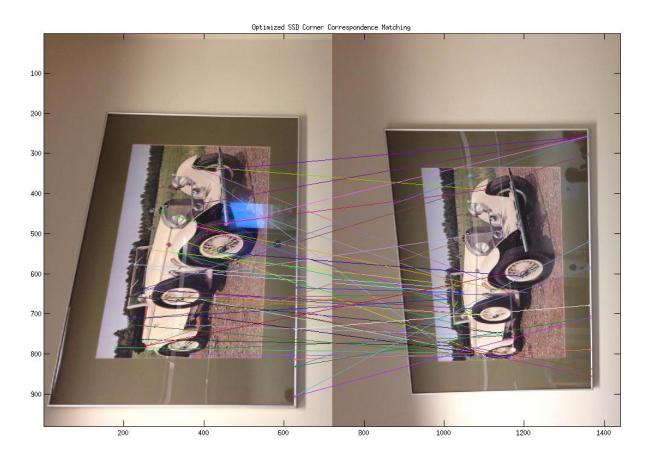


Figure 29. Harris: The SSD matching with $\sigma=0.6$

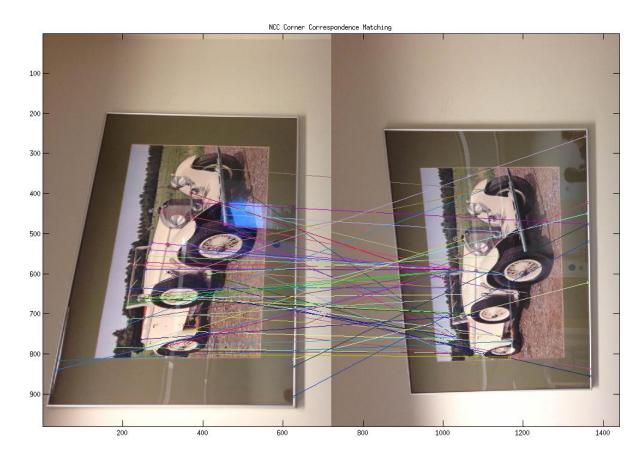


Figure 30. Harris: The NCC matching with $\sigma=0.6$

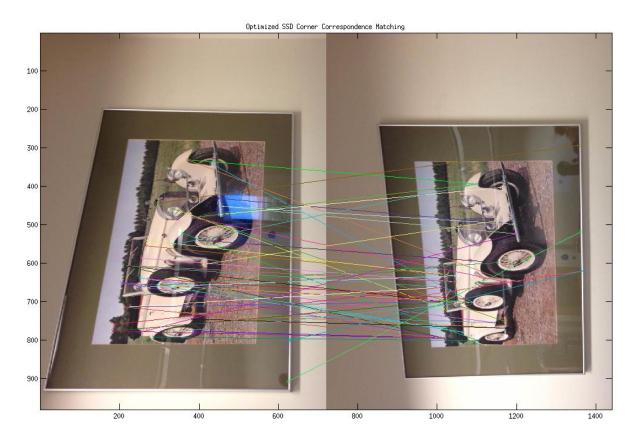


Figure 31. Harris: The SSD matching with $\sigma=1$

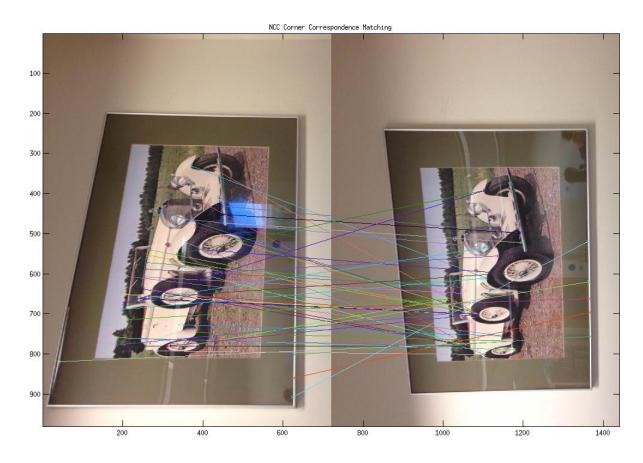


Figure 32. Harris: The NCC matching with $\sigma=1$

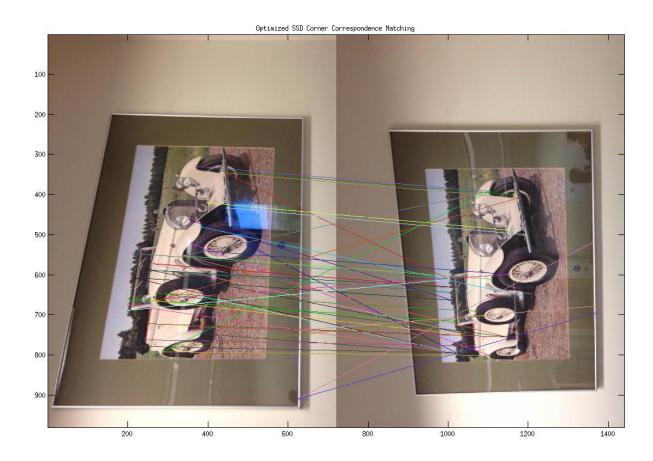


Figure 33. Harris: The SSD matching with $\sigma=1.4$

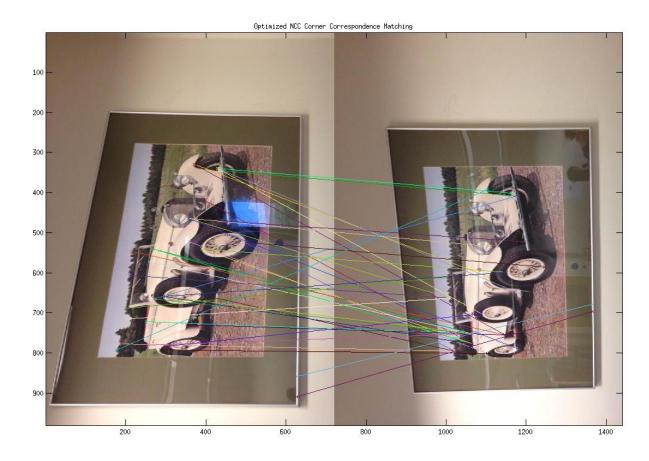


Figure 34. Harris: The NCC matching with $\sigma = 1.4$

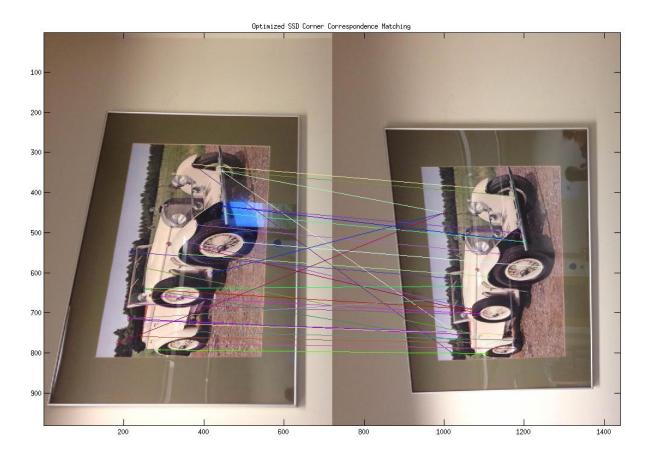


Figure 35. Harris: The SSD matching with $\sigma=2.2$

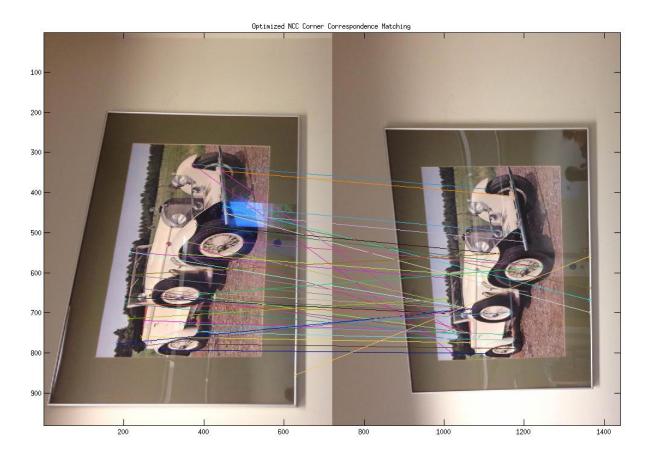


Figure 36. Harris: The NCC matching with $\sigma=2.2$

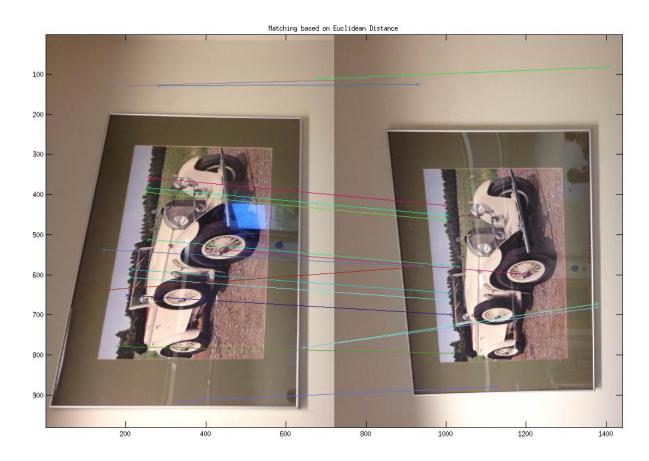


Figure 37. SIFT: Interest Points Matching Based on Euclidean Distance

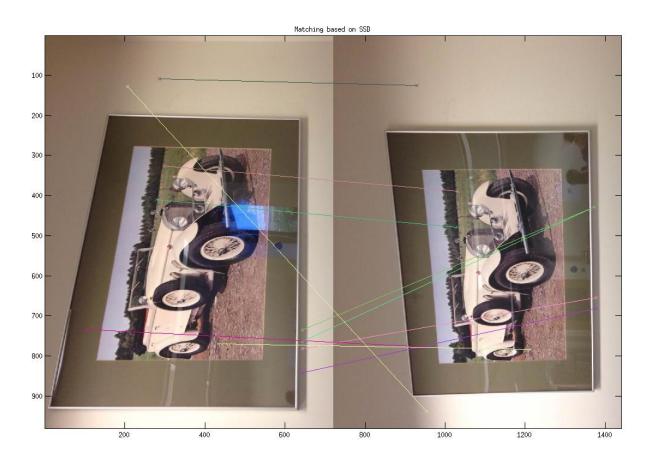


Figure 38. SIFT: Interest Points Matching Based on SSD

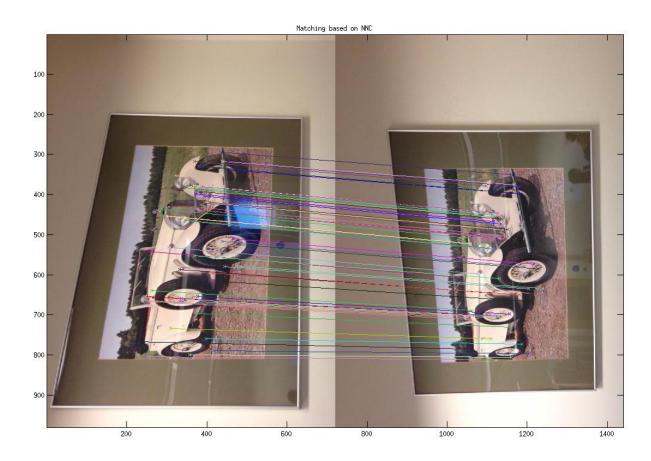


Figure 39. SIFT: Interest Points Matching Based on NCC

Conclusion for Set 1:

Although for this set of images the view angle changed slightly as in Set 1, Harris corner detector did not perform as well as in the previous set. For the correspondences established based on SSD and NCC, the matching rate decreased significantly compared to those in set 1.

Challenge: If we look closely into this pair, we will find there is a huge trouble. Unlike Set 1, a lot of features in the images are very similar. For example: The Frames, The Car White Paint, The Tyres, Those Trees. Even by human visual if we only look into the small details we can not distinguish one object from another. However, SIFT has proven to be more accurate/sensitive than human visual in this case.

Again, similar as to those in Set 2 larger the σ , less sensitive the Harris Corner detector is (less interest points detected is not necessarily bad). Those points detected with larger σ actually tend to be more accurately matched across the images.

As the Harris Corner detector did not yield ideal results for this pair, again, SIFT operator actually works better! The results based on Euclidean and SSD are great (there are only a few points because the threshold and other limiting conditions are strict in order to avoid

mismatch as much as possible), but the result based on NCC is greater! With NCC, SIFT actually succeeded in matching the Old Car in the images, while there are not so many significant corners in there (by human visual).

Based on the result from this part, once more we have concluded when the features are more robust, SIFT with NCC would improve our matching rate significantly.

7.4 Intermediate Results: Gradient, Corners

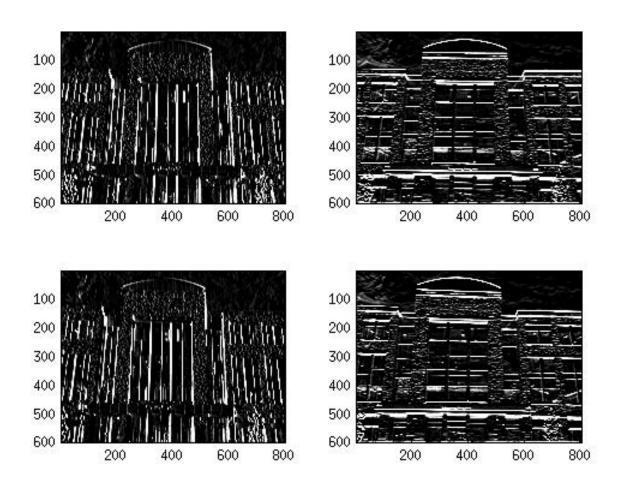


Fig 40. The x-gradient (left) and y-gradient (right) for pic1.jpg (upper) and pic2.jpg (lower) using $\sigma = 1$

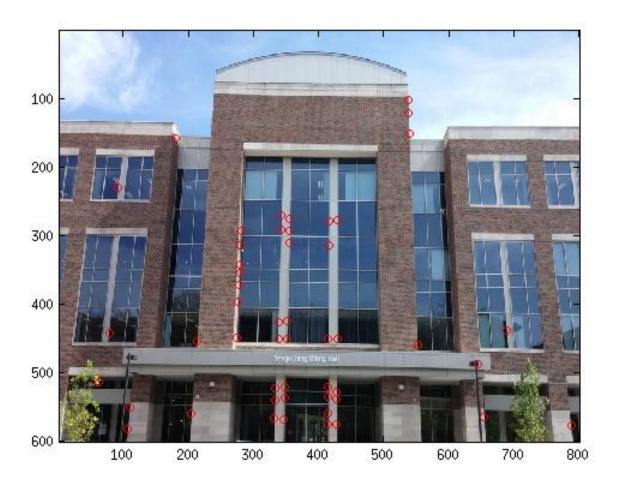


Fig 41. The corner points detected on pic1.jpg using $\sigma=1$

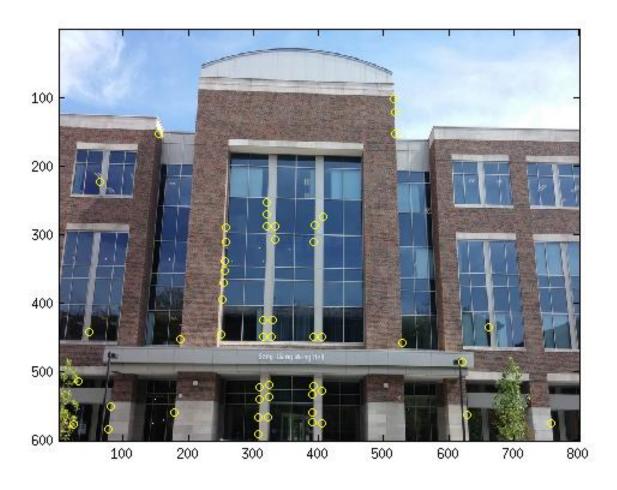


Fig 42. The corner points detected on pic2.jpg using $\sigma = 1$

7.5 Appendix A: Harris Corner Detection Matlab Script

```
pic1_gray = double(pic1_gray);
 pic2_gray = double(pic2_gray);
 18
 % Set up some coefficient, Rssd = ratio for SSD, Rncc = ratio for NCC
 Rssd = 0.85
22 Rncc = 1.01
 k = 0.04;
23
24
25
 I1 = rgb2gray(pic1);
 I2 = rgb2gray(pic2);
27
 size_{I} = size_{I}
29
 size_I2 = size(I2);
size_{II} = size_{II}:
 size_I2 = size(I2);
33
 I1x = zeros(size_{I1}(1), size_{I1}(2));
 I1y = zeros(size_I1(1), size_I1(2));
35
 I2x = zeros(size_I2(1), size_I2(2));
I2y = zeros(size_I2(1), size_I2(2));
38 I1 = double(I1);
 I2 = double(I2);
39
40
 haar_size = round(round((4*scale+1))/2)*2;
41
42
 43
 % Smooth the image a bit before processing to make sure those noise would
44
 % not be detected as corners (to improve computational efficiency)
 smooth_filter = fspecial('gaussian', 5*scale, scale);
48 I1 = imfilter(I1, smooth_filter);
 I2 = imfilter(I2, smooth_filter);
50
 % Applying 'Haar' Filter
\text{54} Hx(1:haar_size,1:haar_size/2) = -1;
55 Hx(1:haar_size,haar_size/2+1:haar_size) = 1;
56 Hy(1:haar_size/2,1:haar_size) = 1;
57 Hy (haar_size/2+1:haar_size, 1:haar_size) = -1;
58
59 I1x = imfilter(I1, Hx);
60 Ily = imfilter(I1, Hy);
12x = imfilter(I2, Hx);
 I2y = imfilter(I2, Hy);
63
 % This part no longer useful. The part is for the initial implementation of
    Sobel filter
```

```
% This part is for sobel operator
69
70
71 \% \text{ for } i = 2:1: \text{size-I1}(1) - 1;
    for j = 2:1:size_{I1}(2)-1;
72
          I1x(i,j) = I1(i-1,j+1) + 2*I1(i, j+1) + I1(i+1, j+1) - I1(i-1,j-1)
73
  읒
              -2*I1(i,j-1) - I1(i+1,j-1);
74
          I1y(i,j) = I1(i-1,j-1) + 2*I1(i-1,j) + I1(i-1,j+1) - I1(i+1,j-1)
75
             -2*I1(i+1, j) - I1(i+1, j+1);
76
  응
  응
       end
77
  % end
78
  응
79
  % for i = 2:1:size_{I2}(1)-1;
80
       for j = 2:1:size_{I2}(2)-1;
          I2x(i,j) = I2(i-1,j+1) + 2*I2(i, j+1) + I2(i+1, j+1) -I2(i-1,j-1)
82
  응
             -2*I2(i,j-1) - I2(i+1,j-1);
83
          I2y(i,j) = I2(i-1,j-1) + 2*I2(i-1,j) + I2(i-1,j+1) - I2(i+1,j-1) -
84
              2*I2(i+1, j) - I2(i+1, j+1);
85
      end
86
  % end
88
  89
  91
  % Plot the gradient as intermediate result to make sure Haar filter was
     correctly implemented
  figure
95 subplot (2, 2, 1)
96 image(I1x)
97 colormap(gray(256))
98 subplot (2,2,2)
  image(I1y)
100 colormap(gray(256))
101 subplot (2, 2, 3)
102 image(I2x)
103 colormap (gray (256))
104 subplot (2, 2, 4)
105 image (I2y)
  colormap(gray(256))
106
107
108 tic
  disp('Compute the C matrix for Image 1...')
  % Calculate the C matrix for image 1
for i =1:1:size_I1(1)
113
     for j = 1:1:size_{I1}(2)
         C_Matrix_{II} = [0,0;0,0];
115
         for m = -(5*scale-1)/2:1:(5*scale-1)/2
```

```
117
              for n = -(5*scale-1)/2:1:(5*scale-1)/2
                  if (i+m>0) && (i+m < size_{-}I1(1)) && (j+n>0) && (j+n < size_{-}I1
118
                  C_Matrix_{II}(1,1) = C_Matrix_{II}(1,1) + I1x(i+m,j+n)*I1x(i+m,j+n)
119
                     );
                  C_{matrix_{II}}(2,2) = C_{matrix_{II}}(2,2) + I1x(i+m,j+n)*I1x(i+m,j+n)
120
                     );
                  C_{matrix_{II}}(1,2) = C_{matrix_{II}}(1,2) + I1x(i+m,j+n)*I1y(i+m,j+n)
121
                     );
                  C_Matrix_{I1}(2,1) = C_Matrix_{I1}(2,1) + I1x(i+m,j+n)*I1y(i+m,j+n)
122
                     );
                  else
123
                  end
124
                  end
125
          end
126
          C_{I1}\{i,j\} = C_{Matrix_{I1}};
127
       end
128
   end
129
   toc
130
   disp('Check the rank of C matrix for Image 1...')
131
   % Check the rank of C matrix. If rank not eugal to 2 then dump the points
   % If rank(C) = 2 then save the points for further processing
134
   135
   for i =1:1:size_I1(1)
136
       for j = 1:1:size_I1(2)
137
          C_Matrix_{II} = C_{II}\{i, j\};
138
          if (rank(C_Matrix_I1) == 2)
139
              I\_corner\_I1(i,j) = 1;
140
                plot(j,i,'b*');
141
          else
142
              I\_corner\_I1(i,j) = 0;
143
          end
144
      end
145
   end
146
   toc
147
   disp('Evaluating the corner strength for Image 2...')
148
149
   Corner_strength_I1 = zeros(size_I1(1), size_I1(2));
150
   151
   % Estimate the corner strength at the remaining cadidate locations for image 1
152
   for i =1:1:size_I1(1)
154
      for j = 1:1:size_I1(2)
155
          if (I_corner_I1(i,j) == 1)
156
              %[U,S,V] = svd(C_I1\{i,j\});
                                        %%%No need to use SVD
157
              Corner_strength_{I1}(i,j) = S(1,1)*S(2,2) - k*(S(1,1)+S(2,2))^2;
158
                 %%%No need to use SVD
              Corner\_strength\_H\_I1(i,j) = det(C\_I1\{i,j\}) - k*(trace(C\_I1\{i,j\}))
159
                 ^2; %%%This is better way to calculate corner strength
          end
160
       end
161
   end
162
```

163

```
toc
  disp('Compute the C matrix for Image 2...')
  % Calculate the C matrix for image 2
  168
169
  for i =1:1:size_I2(1)
      for j = 1:1:size_I2(2)
170
         C_Matrix_{I2} = [0,0;0,0];
171
         for m = -(5*scale-1)/2:1:(5*scale-1)/2
172
            for n = -(5*scale-1)/2:1:(5*scale-1)/2
173
                if (i+m>0) && (i+m < size_12(1)) && (j+n>0) && (j+n < size_12(1))
174
                   (2))
                C_Matrix_I2(1,1) = C_Matrix_I2(1,1) + I2x(i+m,j+n)*I2x(i+m,j+n)
175
                C_{matrix_{12}(2,2)} = C_{matrix_{12}(2,2)} + I2x(i+m,j+n)*I2x(i+m,j+n)
176
                  );
                C_{Matrix_{12}(1,2)} = C_{Matrix_{12}(1,2)} + I2x(i+m,j+n)*I2y(i+m,j+n)
177
                C_{Matrix_{I2}(2,1)} = C_{Matrix_{I2}(2,1)} + I2x(i+m,j+n)*I2y(i+m,j+n)
178
                  );
                else
179
180
                end
                end
181
182
         C_{I2}\{i,j\} = C_{Matrix_{I2}};
183
      end
184
  end
185
186
187
  disp('Check the rank of C matrix for Image 2...')
188
  % Check the rank of C matrix. If rank not eugal to 2 then dump the points
190
  % If rank(C) = 2 then save the points for further processing
  192
  for i =1:1:size_I2(1)
193
      for j = 1:1:size_I2(2)
194
         C_Matrix_{I2} = C_{I2}\{i, j\};
195
         if (rank(C_Matrix_I2) == 2)
196
            I_corner_I2(i,j) = 1;
197
198
         else
            I\_corner\_I2(i,j) = 0;
199
200
         end
      end
201
  end
202
203
  Corner_strength_I2 = zeros(size_I2(1), size_I2(2));
204
205
206
  toc
207
  disp('Evaluating the corner strength for Image 2...')
  % Estimate the corner strength at the remaining cadidate locations for image 2
  211
212
```

```
for i =1:1:size_I2(1)
      for j = 1:1:size_I2(2)
214
          if (I_corner_I2(i,j) == 1)
215
             Corner\_strength\_H\_I2(i,j) = det(C\_I2\{i,j\}) - k*(trace(C\_I2\{i,j\}))
216
                ^2;
217
          end
218
      end
  end
219
220
  % Set up a dynamic threshold, thus if a candidate has a corner strength lower
     t.han
  % the threshold, it would be filtered out (to improve computational efficiency
223
   224
225
  threshold = (max(max(Corner_strength_H_I1)) + max(max(Corner_strength_H_I2)))
226
     /20;
227
  228
  % Counting the corners detected in both image and plot those corners
  % This is intermediate results and will not appear on homework report
  232 figure
233 image (I1)
234 colormap (gray (256))
235 hold on;
236 Actual_Corner_I1 = zeros(size_I1(1), size_I1(2));
237
  toc
238
  disp('Thresholding the corner candidates for Image 1...')
239
240
241
  cnt\_cor1 = 0;
242
   for i = 11:1:size_{I1}(1)-10
      for j = 11:1:size_{I}(2)-10
244
          if (Corner_strength_H_I1(i,j) > threshold) && ...
245
          (Corner_strength_H_I1(i,j) == max(max(Corner_strength_H_I1(i-10:1:i
246
            +10, j-10:1:j+10))))
      Actual_Corner_I1(i, j) = 1;
247
      plot(j,i,'rx');
248
      cnt_cor1 = cnt_cor1 + 1;
249
      corner_loc1(cnt_cor1,1:2) = [i;j];
250
         else
251
         end
252
      end
253
  end
254
255
  figure
256
257 image (I2)
258 colormap (gray (256))
  hold on;
260 Actual_Corner_I2 = zeros(size_I2(1), size_I2(2));
261
```

```
262
  toc
  disp('Thresholding the corner candidates for Image 2...')
263
  cnt\_cor2 = 0;
  for i = 11:1:size_I2(1)-10
265
      for j = 11:1:size_I2(2)-10
266
         if (Corner_strength_H_I2(i,j) > threshold) && ...
267
         (Corner_strength_H_I2(i,j) == max(max(Corner_strength_H_I2(i-10:1:i
268
            +10, j-10:1:j+10))))
      Actual_Corner_I2(i, j) = 1;
269
      plot(j,i,'bx');
270
      cnt\_cor2 = cnt\_cor2 + 1;
271
      corner_loc2(cnt_cor2,1:2) = [i;j];
272
273
         end
274
      end
275
276
  end
277
  corner_count1 = sum(sum(Actual_Corner_I1))
  corner_count2 = sum(sum(Actual_Corner_I2))
279
280
  window_size = scale * 20;
281
282
283
     Optimized
                             SSD
284
     285
  % Set a window so the the SSD of each candidate could be found
287
  289
  for m = 1:1:cnt_cor1
290
      i1 = corner_loc1(m,1);
291
      j1 = corner_loc1(m, 2);
292
      for n = 1:1:cnt_cor2
293
         i2 = corner_loc2(n, 1);
294
         j2 = corner_loc2(n, 2);
295
         SSD_Win = pic1_gray(i1-window_size/2:1:i1+window_size/2,j1-window_size
296
            /2:1:j1+window_size/2) - \dots
            pic2_gray(i2-window_size/2:1:i2+window_size/2,j2-window_size/2:1:
297
               j2+window_size/2);
         SSD(m,n) = sumsqr(SSD_Win);
298
      end
299
  end
300
301
  302
  % Set a new image prepared for displaying the matched interest points
  304
  new_image(1:(max(size_I1(1),size_I2(1))),1:size_I1(2)+size_I2(2),1:3) = ...
      zeros(max(size_I1(1), size_I2(1)), size_I1(2)+size_I2(2),3);
306
  new_image(1:size_I1(1),1:size_I1(2),:) = pic1;
  new_image(1:size_I2(1),1+size_I1(2):size_I2(2)+size_I1(2),:) = pic2;
  new_image = uint8(new_image);
```

```
figure
  image(new_image)
  truesize
313 hold on;
314
315
  316
  % If an interest point with SSD:
  % 1) Smaller than a threshold
  % 2) Minima
  % 3) Minima/Second Minama < Rssd (ratio)
320
  % Correspondence Established
  323
  for m = 1:1:cnt_cor1
324
      i1 = corner_loc1(m, 1);
325
      j1 = corner_loc1(m, 2);
326
      for n = 1:1:cnt_cor2
327
          if (SSD(m,n) = min(SSD(m,:))) & (SSD(m,n) < 40* min(min(SSD(:,:))))
328
             local_minima = SSD(m,n);
329
             SSD(m,n) = max(SSD(m,:));
330
             if (local_minima/min(SSD(m,:)) < Rssd)</pre>
331
             i2 = corner_loc2(n, 1);
332
             j2 = corner_loc2(n, 2);
333
             rand\_color = rand(1, 3);
334
             plot([j1;size_I1(2)+j2],[i1;i2],'-x','Color',rand_color(1,:));
335
             n = cnt_cor2;
336
             else
337
             end
338
         else
339
340
         end
341
      end
342
  end
343
  title('Optimized SSD Corner Correspondence Matching')
345
346
347
  348
349
  350
  % Set a window so the the NCC of each candidate could be found
351
  352
  for m = 1:1:cnt_cor1
353
      i1 = corner\_loc1(m, 1);
354
      j1 = corner_loc1(m, 2);
355
      for n = 1:1:cnt_cor2
356
         i2 = corner_loc2(n, 1);
357
         j2 = corner_loc2(n, 2);
358
359
         f1_m1 = pic1_gray(i1-window_size/2:1:i1+window_size/2,j1-window_size
360
            /2:1:j1+window_size/2) - ...
            mean (mean (pic1_gray (i1-window_size/2:1:i1+window_size/2, j1-
361
               window_size/2:1:j1+window_size/2)));
```

```
362
          f2_m2 = pic2_gray(i2-window_size/2:1:i2+window_size/2,j2-window_size
363
             /2:1:j2+window_size/2) - \dots
             mean(mean(pic2-gray(i2-window_size/2:1:i2+window_size/2,j2-
364
                window_size/2:1:j2+window_size/2)));
365
          NNC(m,n) = sum(sum(f1_m1.*f2_m2))/((sumsqr(f1_m1)*sumsqr(f2_m2))^(1/2)
366
             );
367
368
      end
369
   end
370
   371
   % Set a new image prepared for displaying the matched interest points
   new_image(1:(max(size_II(1),size_I2(1))),1:size_II(2)+size_I2(2),1:3) = ...
      zeros(max(size_I1(1), size_I2(1)), size_I1(2)+size_I2(2),3);
375
  new_image(1:size_I1(1),1:size_I1(2),:) = pic1;
376
  new_image(1:size_I2(1),1+size_I1(2):size_I2(2)+size_I1(2),:) = pic2;
377
  new_image = uint8(new_image);
  figure
379
  image(new_image)
  truesize
  hold on;
382
383
   384
  % If an interest point with SSD:
  % 1) Larger than a threshold
386
  % 2) Maxima
  % 3) Maxima/Second Maxima > Rncc (ratio)
  % Correspondence Established
  390
391
   for m = 1:1:cnt_cor1
392
      i1 = corner_loc1(m, 1);
393
      j1 = corner_loc1(m, 2);
394
      for n = 1:1:cnt_cor2
395
          if (NNC(m,n) == max(NNC(m,:))) && (NNC(m,n) > 0.3*max(max(NNC(:,:))))
396
             local_maxima = NNC(m,n);
397
             NNC(m, n) = min(NNC(m, :));
398
             if (local_maxima/max(NNC(m,:)) > Rncc)
399
             i2 = corner_loc2(n, 1);
400
             j2 = corner_loc2(n, 2);
401
             rand\_color = rand(1, 3);
402
             plot([j1;size_I1(2)+j2],[i1;i2],'-x','Color',rand_color(1,:));
403
             n = cnt\_cor2;
404
             else
405
406
             end
          else
407
          end
408
409
      end
410
411 end
  title ('Optimized NCC Corner Correspondence Matching')
```

7.6 Appendix B: SIFT Matlab Script

```
% Read the test images, and seek user input for a scale sigma
4 close all; clear all; clc
5 pic1 = imread('your_image_name_1.jpg'); % your_image_name_1 = the image1 want
    to be processed
6 pic2 = imread('your_image_name_2.jpg'); % your_image_name_2 = the image2 want
    to be processed
8 % Set up some coefficient, Rssd = ratio for SSD, Rncc = ratio for NCC
9 % Reuc = ratio for Euclidean Distance
11 Reuc = 0.7;
12 \text{ Rssd} = 0.7;
13 Rncc = 1.4;
15 % Change the RGB images into gray scale
17 pic1_gray = rgb2gray(pic1);
18 pic2_gray = rgb2gray(pic2);
19 pic1_gray = double(pic1_gray);
20 pic2_gray = double(pic2_gray);
22 % Perform SIFT feature extration and extract both locations and descriptors
23 % for each candidates interest point
25 [I1, sift_vec_I1] = vl_sift(im2single(rgb2gray(pic1)));
26 [I2, sift_vec_I2] = vl_sift(im2single(rgb2gray(pic2)));
27 size_image_I1 = size(pic1);
28 \text{ size_I1} = \text{size}(I1);
29 \text{ size}_{-}I2 = \text{size}(I2);
30 points_size_I1 = size_I1(2);
31 points_size_I2 = size_I2(2);
 34 % Round up those sub-pixel returned from SIFt operator
 for i = 1:1:points_size_I1
36
    corner_loc_I1(1,i) = round(I1(2,i));
    corner_loc_I1(2,i) = round(I1(1,i));
38
 end
39
40
 for i = 1:1:points_size_I2
    corner_loc_I2(1,i) = round(I2(2,i));
42
    corner_loc_I2(2,i) = round(I2(1,i));
43
 end
44
45
46
 disp('Calculating Euclidean Distance Matrix')
```

```
% Calculating the Euclidean Distance of vectors returned from SIFt operator
  for m = 1:1:points_size_I1
51
     i1 = corner\_loc\_I1(1,m);
52
     j1 = corner\_loc\_I1(2,m);
53
     for n = 1:1:points_size_I2
54
         i2 = corner\_loc\_I2(1,n);
55
         j2 = corner_loc_I2(2,n);
56
         sift_diff = double(sift_vec_I1(:,m)) - double(sift_vec_I2(:,n));
57
         sift_mat(m,n) = sumsqr(sift_diff);
     end
59
  end
  disp ('Matching Interest Points Based on Euclidean Distance Matrix')
61
  figure
63 image(cat(2, pic1, pic2));
64 truesize
65 hold on:
 title ('Matching based on Euclidean Distance')
 % Macthing images the Euclidean Distance of vectors returned from SIFt
     operator
  for m = 1:1:points_size_I1
     i1 = corner\_loc\_I1(1, m);
71
     j1 = corner_loc_I1(2, m);
72
     for n = 1:1:points_size_I2
73
         if (sift_mat(m,n) == min(sift_mat(m,:)) && (sift_mat(m,n)<5*min(min(</pre>
74
           sift_mat))))
            loc_minimum = sift_mat(m,n);
75
            sift_mat(m,n) = max(sift_mat(m,:));
76
            if (loc_minimum/min(sift_mat(m,:)) < Reuc)</pre>
77
            i2 = corner\_loc\_I2(1,n);
78
            j2 = corner_loc_I2(2,n);
79
            rand\_color = rand(1, 3);
80
            plot([j1;size_image_I1(2)+j2],[i1;i2],'-x','Color',rand_color(1,:)
               );
            n = points_size_I2;
82
            else
83
            end
84
         else
85
         end
86
     end
87
  end
88
  % Calculating the SSD from vectors returned from SIFt operator
  92 disp('Calculating SSD Matrix')
93 SSD = sift_mat.^2;
94 disp('Matching Interest Points Based on SSD Matrix')
95 figure
96 image(cat(2, pic1, pic2));
97 truesize
98 hold on;
99 title ('Matching based on SSD')
```

```
% Matching images based on the SSDf vectors returned from SIFt operator
  for m = 1:1:points_size_I1
103
      i1 = corner\_loc\_I1(1, m);
104
      j1 = corner_loc_I1(2, m);
105
      for n = 1:1:points_size_I2
106
         if (SSD(m,n) == min(SSD(m,:)) && (SSD(m,n) < 5*min(min(SSD))))
107
            loc_minimum = SSD(m,n);
108
            SSD(m,n) = max(SSD(m,:));
109
            if (loc_minimum/min(SSD(m,:)) < Rssd)</pre>
110
            i2 = corner_loc_I2(1, n);
111
            j2 = corner_loc_I2(2,n);
112
            rand\_color = rand(1, 3);
113
            plot([j1;size_image_I1(2)+j2],[i1;i2],'-x','Color',rand_color(1,:)
114
            n = points_size_I2;
115
            else
116
            end
117
         else
118
         end
119
120
      end
  end
121
122
  disp('Calculating the NCC matrix')
123
  124
  % Calculating the NCC from vectors returned from SIFt operator
  126
  for m = 1:1:points_size_I1
127
      i1 = corner_loc_I1(1, m);
128
      j1 = corner_loc_I1(2, m);
129
      for n = 1:1:points_size_I2
130
         i2 = corner\_loc\_I2(1,n);
131
         j2 = corner_loc_I2(2,n);
132
         f1m1 = double(sift_vec_I1(:,m)) - double(mean(sift_vec_I1(:,m)));
133
         f2m2 = double(sift_vec_I2(:,n)) - double(mean(sift_vec_I2(:,n)));
134
         NCC(m,n) = sum(f1m1.*f2m2)/((sumsqr(f1m1)*(sumsqr(f2m2)))^(1/2));
135
136
137
      end
  end
138
139
  figure
140
  image(cat(2, pic1, pic2));
141
142 truesize
143 hold on;
  title('Matching based on NNC')
  disp('Matching Interest Points Based on NCC Matrix')
  % Matching images based on the NCC from vectors returned from SIFt operator
147
  for m = 1:1:points_size_I1
149
      i1 = corner\_loc\_I1(1, m);
150
      j1 = corner_loc_I1(2, m);
151
152
      m
```

```
for n = 1:1:points_size_I2
153
            if (NCC(m,n) = max(NCC(m,:))) && (NCC(m,n) > max(max(NCC))*(0.9))
154
                local_maxima = max(NCC(m,n));
155
                NCC(m,n) = min(NCC(m,:));
156
                if (local_maxima/max(NCC(m,:)) > Rncc)
157
                i2 = corner_loc_I2(1,n);
158
                j2 = corner_loc_I2(2,n);
159
                rand\_color = rand(1, 3);
160
                plot([j1;size_image_I1(2)+j2],[i1;i2],'-x','Color',rand_color(1,:)
161
                n = points_size_I2;
162
163
                else
                end
164
            else
165
            end
166
167
        end
168 end
```