ECE661 Computer Vision - Fall 2014 - HW3

Fu-Chen Chen chen1623@purdue.edu

2.1.1 Remove projective distortion

To remove the projective distortion we have to take back the vanishing line in the image to infinity. First we choose 2 pairs of lines they are physically parallel, then each line pair should intersect on certain point. These points are the vanishing points. Let $L = [l1 \quad l2 \quad l3]^T$ is the vanishing line across these two vanishing points, then the homography:

$$H_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l1 & l2 & l3 \end{bmatrix}$$

could make the vanishing points back to the infinite. So apply this homography would remove the projective distortion.

The vanishing line of the two images of the same scene should be identical in the world space, but they are different in the image space which result in different homography. So we cannot correct the projective distortion in one image of a scene using the vanishing line from another image of the same scene.

2.1.2 Remove affine distortion

Consider two lines $L = [l1 \quad l2 \quad l3]^T$ and $M = [m1 \quad m2 \quad m3]^T$ with angle θ. Then we could know

$$\cos\theta = \frac{L^T C_\infty^* M}{\sqrt{(L^T C_\infty^* L)(M^T C_\infty^* M)}} \text{ where } C_\infty^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

By letting L and M be orthogonal and using the fact that $C^{*\prime} = HC^*H$, the above formulation become

$$L'^T H_a C_\infty^* H_a^T M' = 0 \text{ where } H_a = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix} \text{ is the homography that remove the affine distortion.}$$

Expanding the above formulation we get $[l1' \quad l2' \quad l3'] \begin{bmatrix} AA^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m1' \\ m2' \\ m3' \end{bmatrix} = 0$. Now we define

S= $AA^T = \begin{bmatrix} s11 & s12 \\ s12 & s22 \end{bmatrix}$, we can got

$$s11m1'l1' + s12(l1'm2'+l2'm1') + s22l2'm2' = 0$$

By the fact that we only need to know the ration of elements in homography, we could set s22=1 and apply two orthogonal line pairs to solve s11 and s12. Once we get S, since S= AA^T we could use the property of SVD that shows

$$A=UDU^T,\ S=AA^T = UDU^TUDU^T = UD^2U^T$$

to obtain A and the H_a, then we could use the homography $H_a^{-1}H_p$ to remove the projective and affine distortion.

2.2 Single step method

A general homography that remove projective and affine distortion could be express as

$$H = \begin{bmatrix} A & 0 \\ v^T & 1 \end{bmatrix}$$

using the fact that $C^{*\prime} = HC^*H$, we could present the dual conic in the image plane as

$$C_\infty^{*\prime} = HC_\infty^*H^T = \begin{bmatrix} AA^T & Av \\ v^TA^T & v^Tv \end{bmatrix} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

If there are two lines $L' = [l1' \quad l2' \quad l3']^T$ and $M' = [m1' \quad m2' \quad m3']^T$ that are orthogonal in the world plane, we have

$$L'^T C_\infty^{*\prime} M' = 0 = [l1' \quad l2' \quad l3'] \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} [m1' \quad m2' \quad m3']^T$$

Again we only care about the ratio of elements in conics, so we could set f=1 and solve a, b, c, d and e by 5 pairs of orthogonal lines and get $C_\infty^{*\prime}$. Now since $AA^T = S = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$ we can apply SVD on S again to get A the use $v^TA^T=[d/2 \quad e/2]$ to get v and build the homography H that could remove projective and affine distortion in a single step.

2.3 Compare two-step method and one-step method

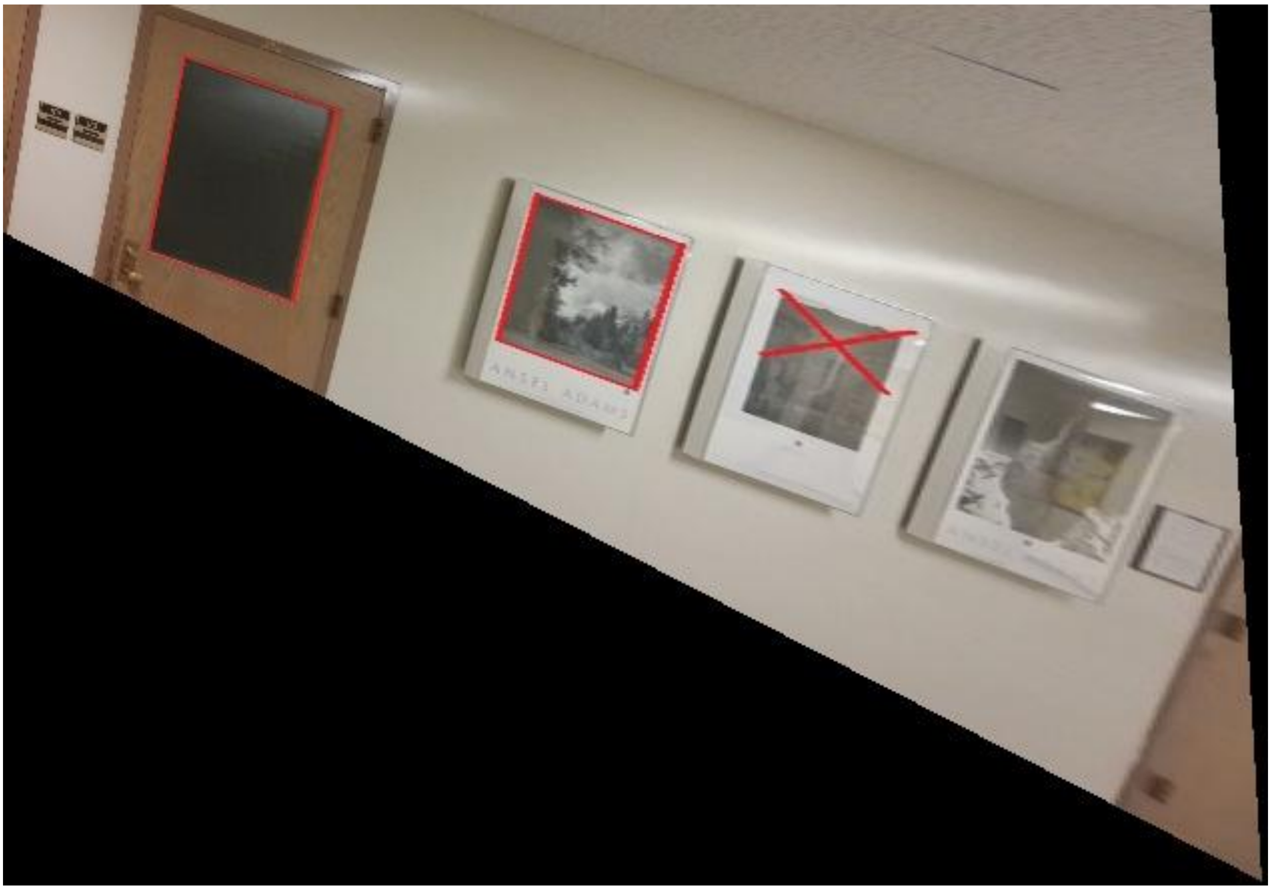
According to the image result, we could find that the two-step method seems to be more robust than one-step method. For example if I make a slight change of line, the result of one-step method often crash while the two-step method remain stable. Thus I conclude the one-step method is very vulnerable to noise. I think the reason is that it requires 5 pairs of orthogonal lines in world space, which means if any pair of lines are not orthogonal then the process of computing a, b, c, d, e would be problematic.

3. Result images (I had crop some images to reduce the image size)

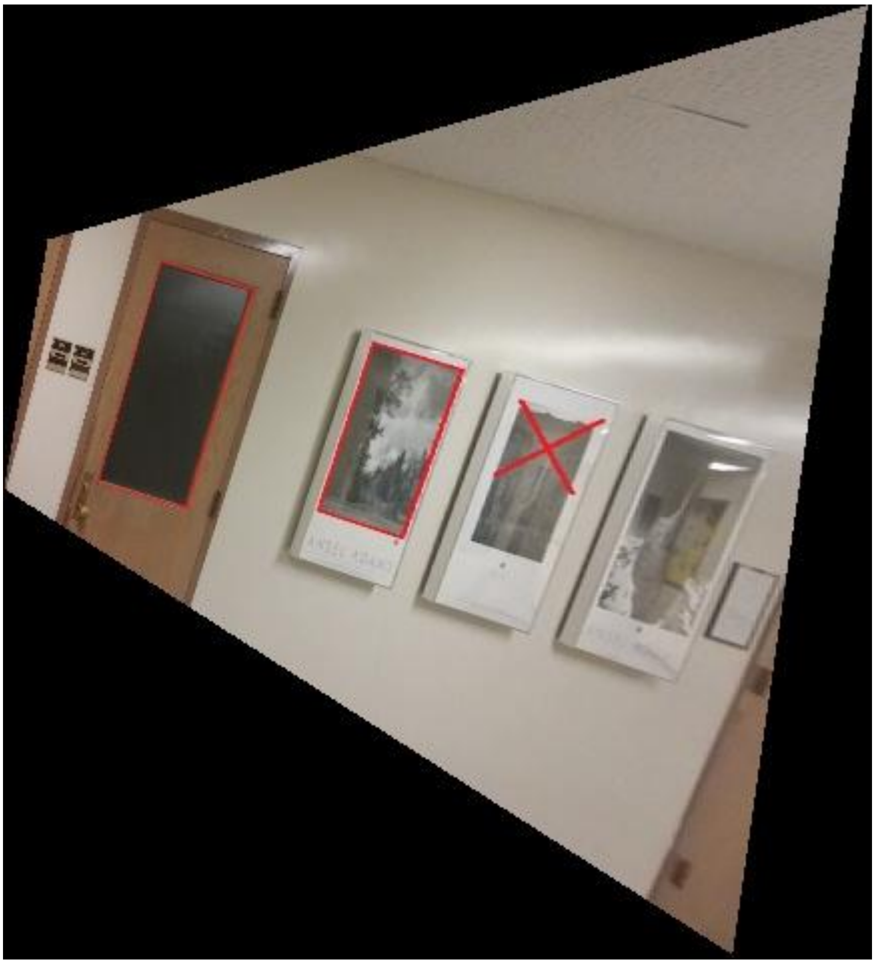
Set1-Img1

original

projective removed



affine removed



one-step method

Set1-Img2

original

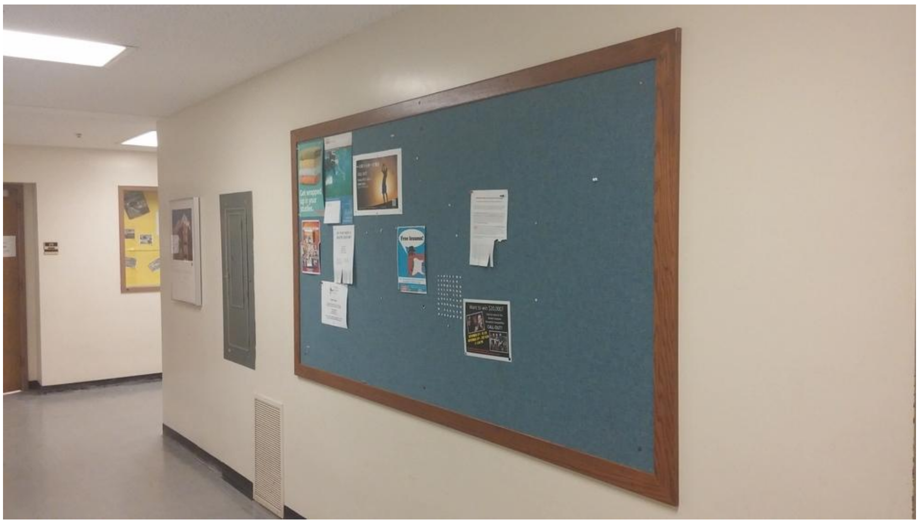
projective removed

affine removed

one-step method

Set2-Img1

original



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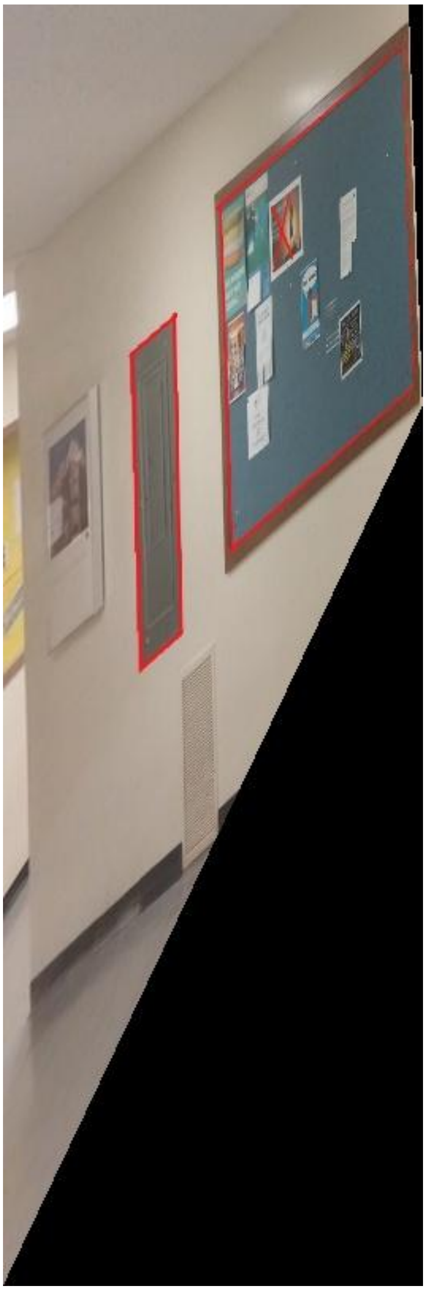

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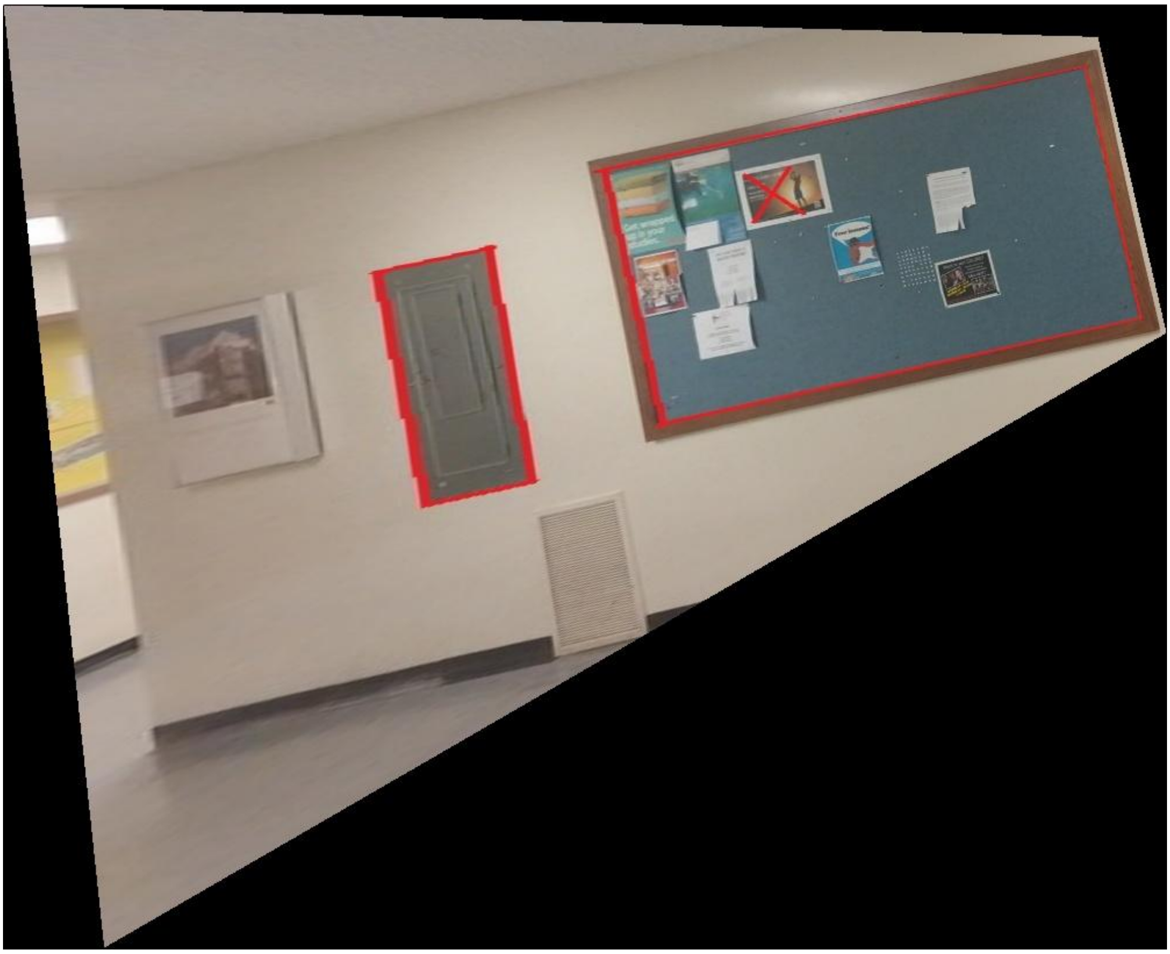
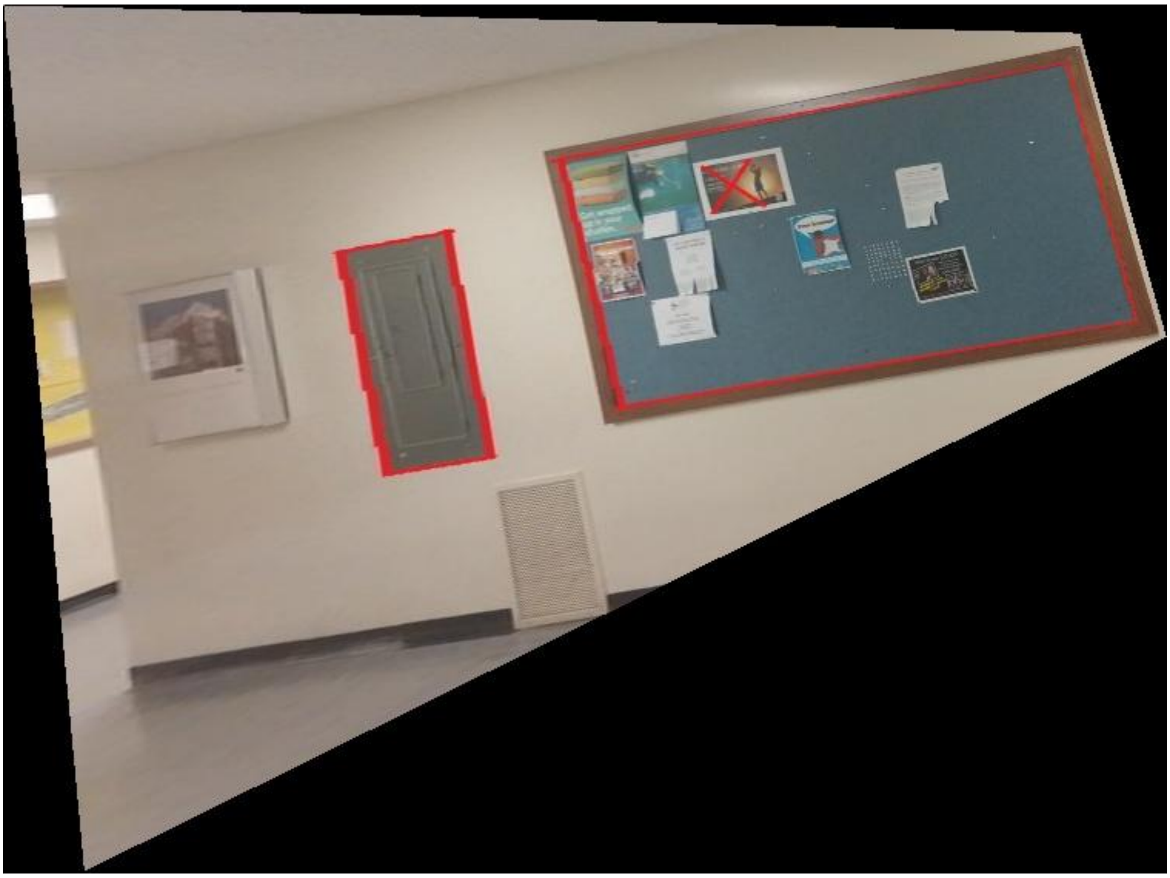


one-step method

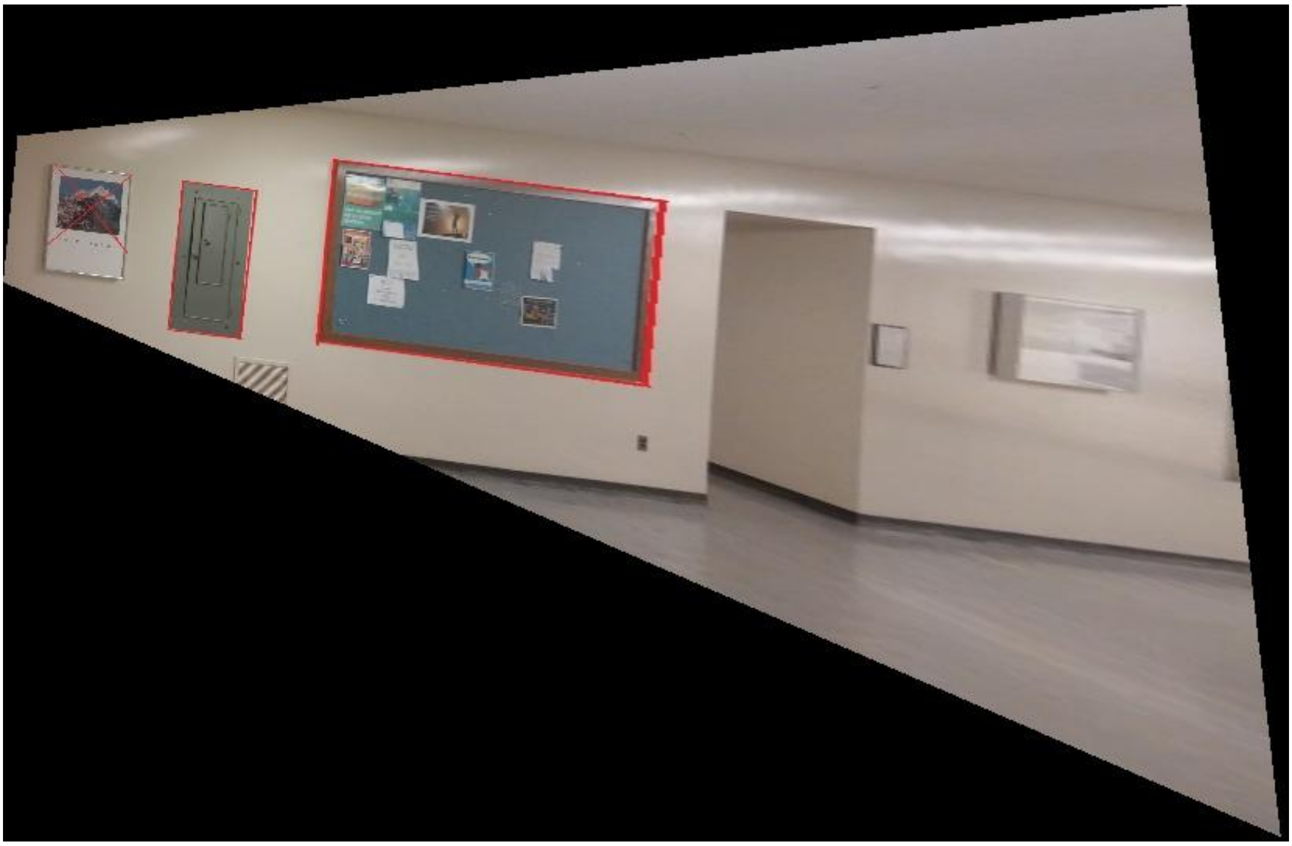


Set2-Img2

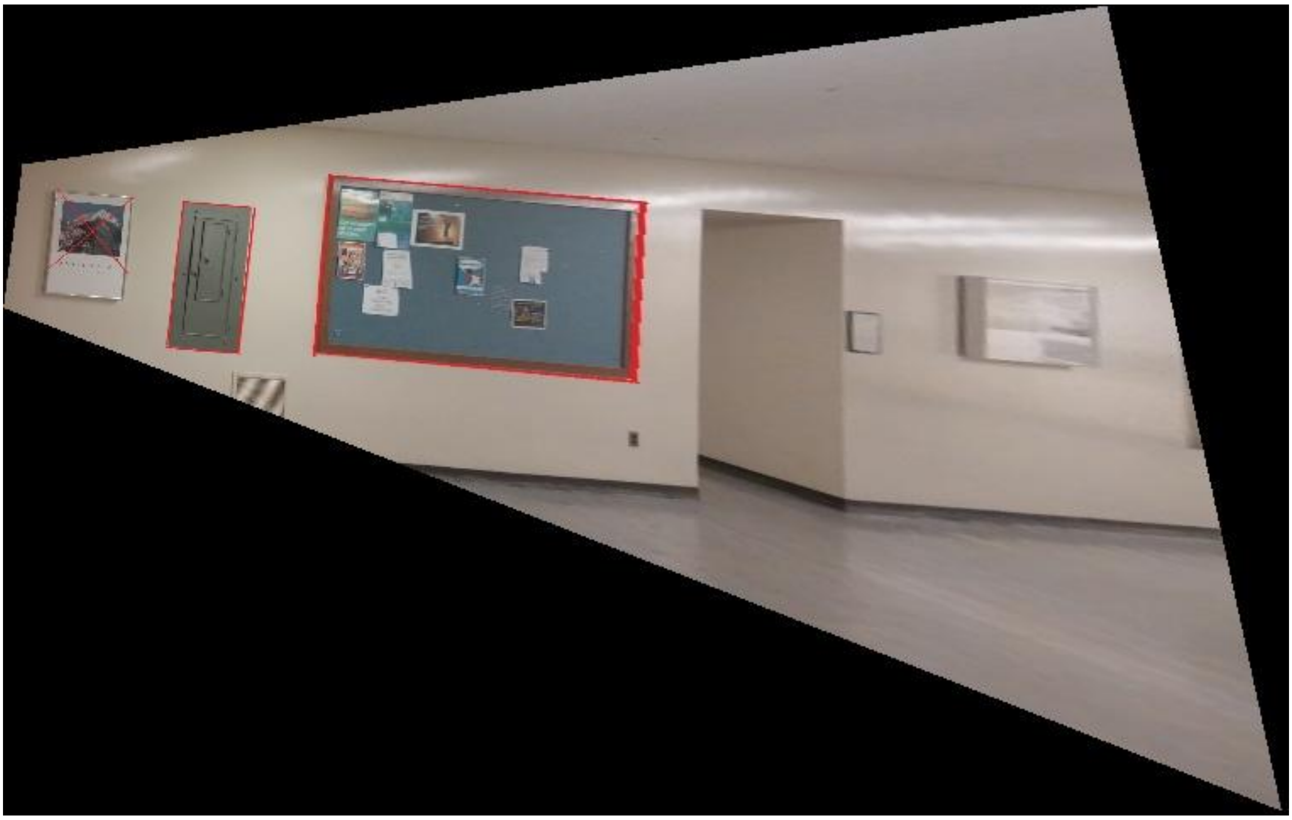
original

projective removed

affine removed



one-step method



Set3-Img1

original

projective removed

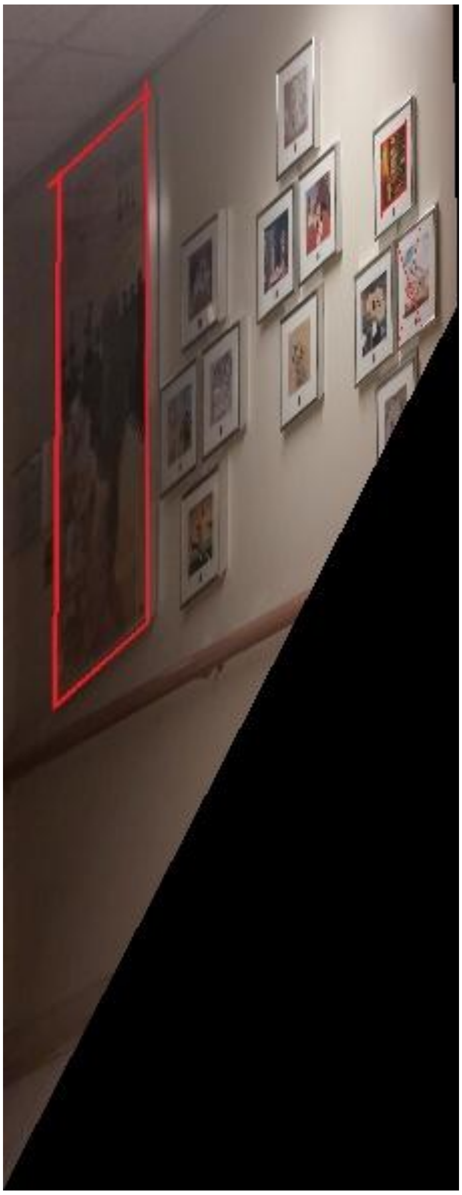
affine removed

one-step method

Set3-Img2

original

projective removed



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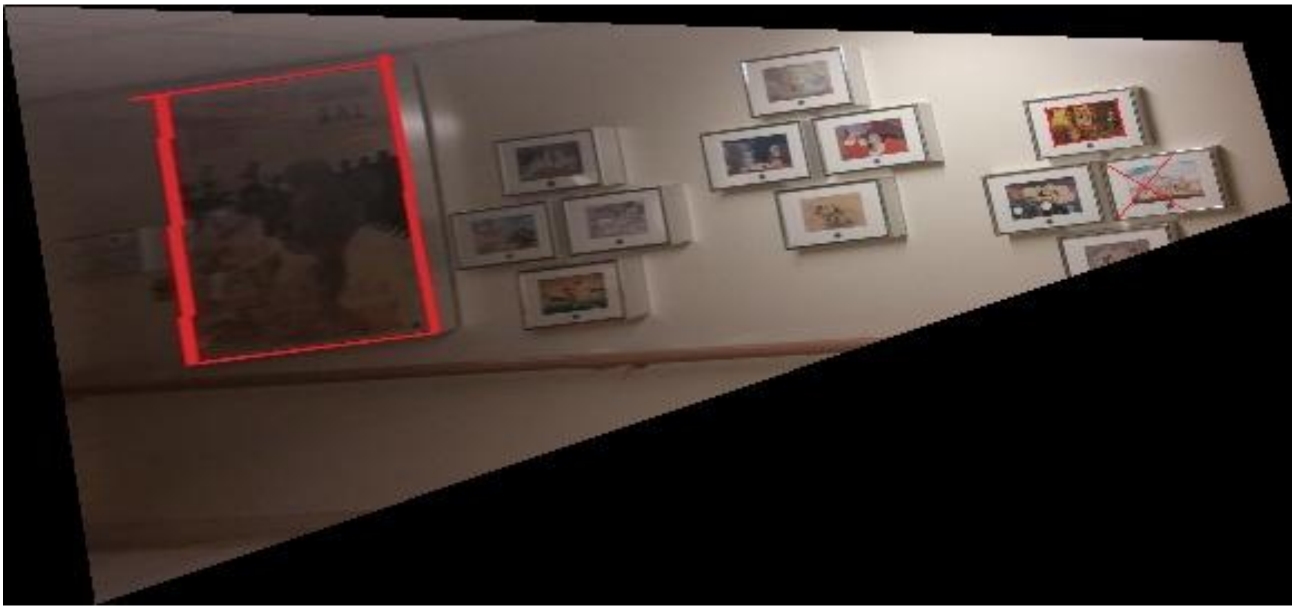
one-step method



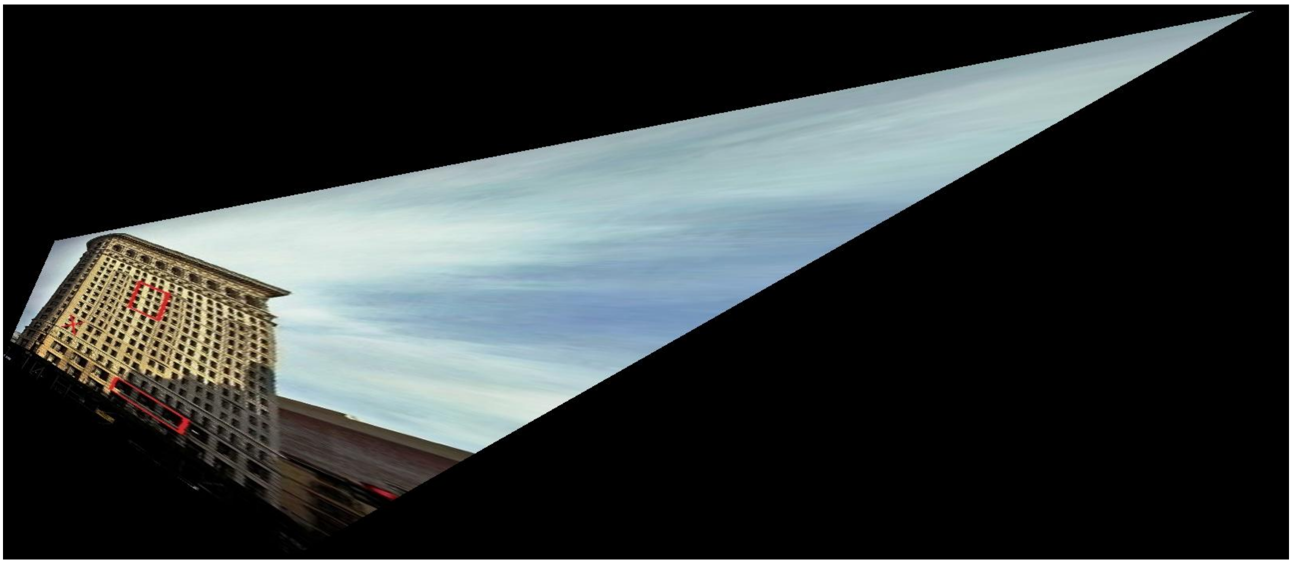
Set4-Img1

original

projective removed

affine removed

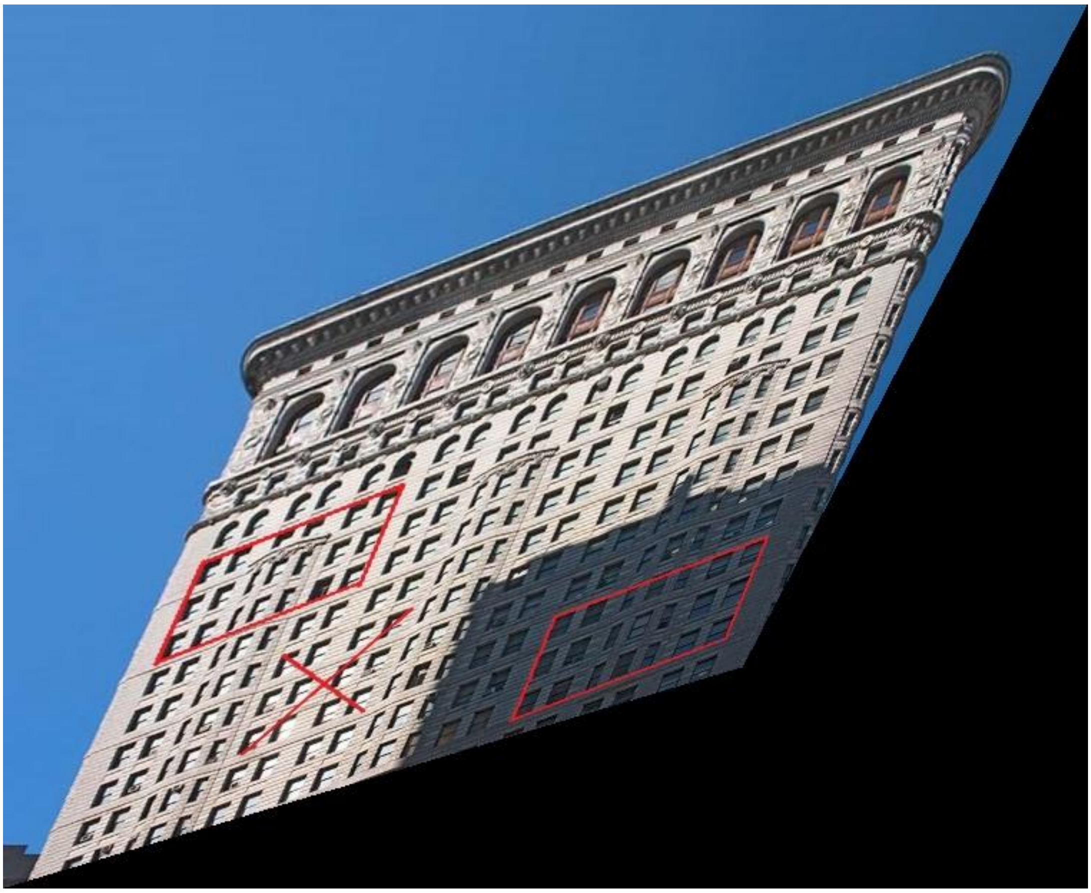
one-step method



Set4-Img2

original

projective removed



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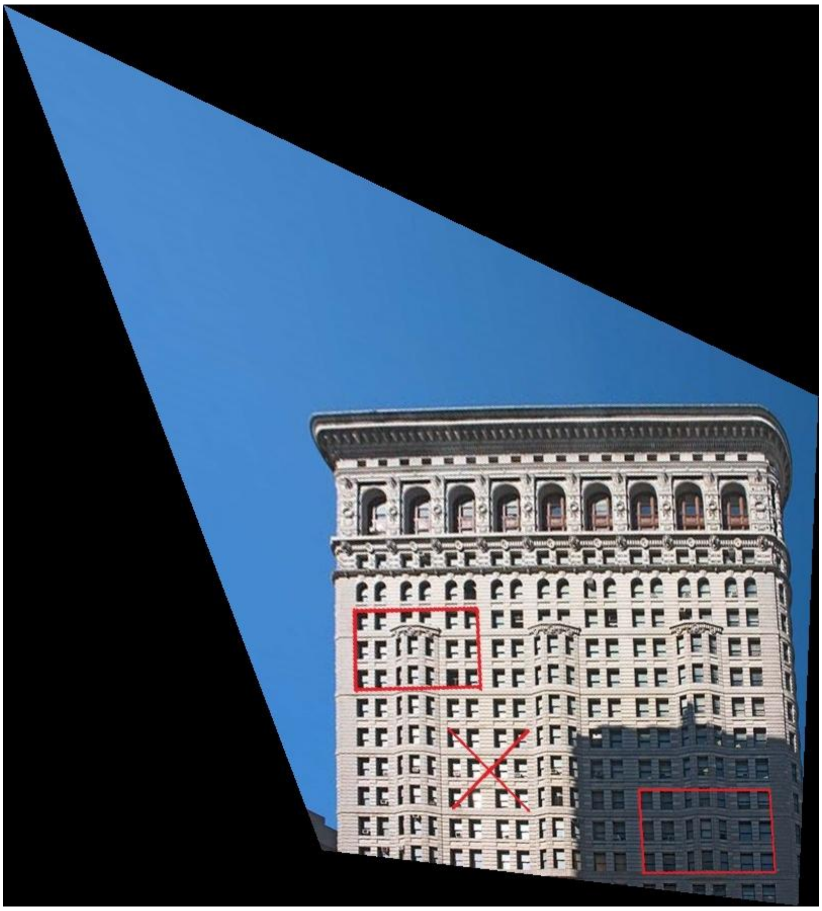
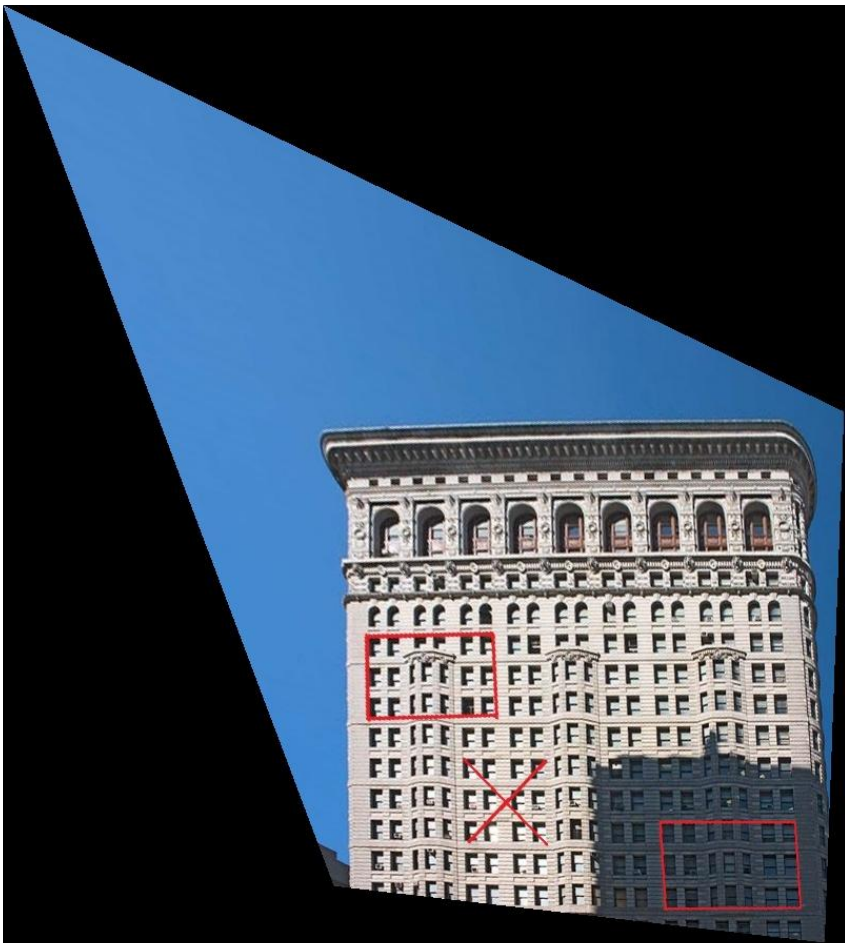
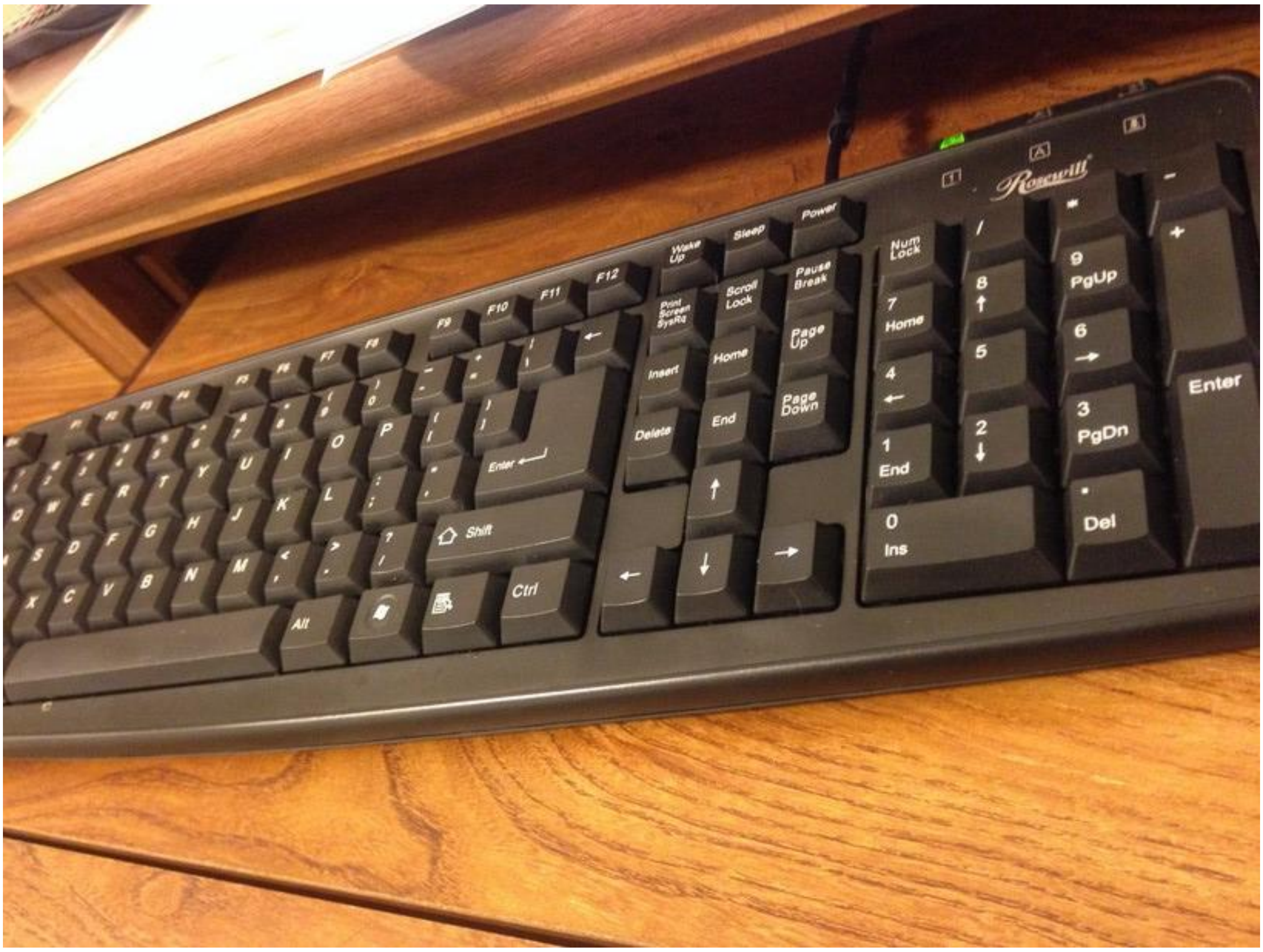


one-step method



My Image 000

original



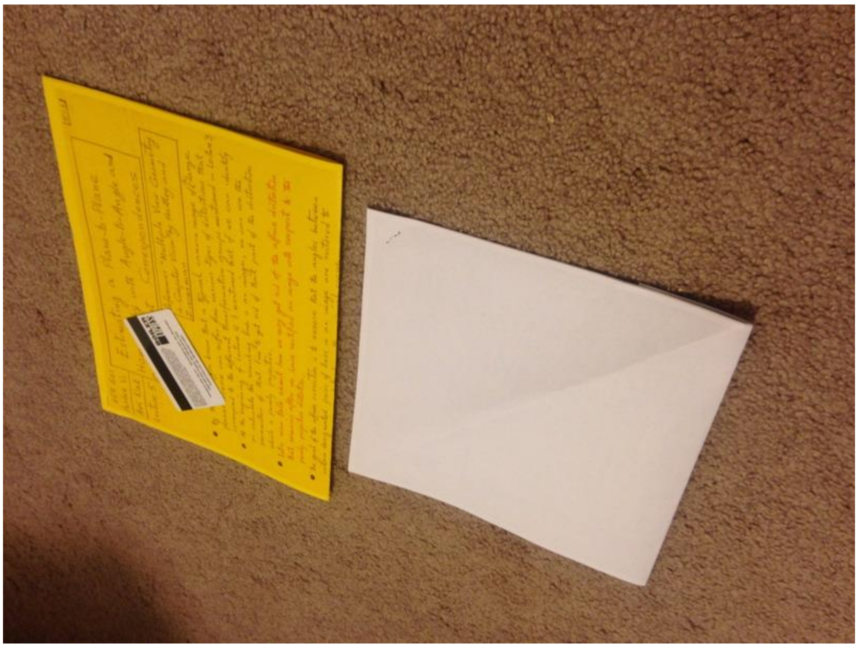
projective removed

affine removed

one-step method

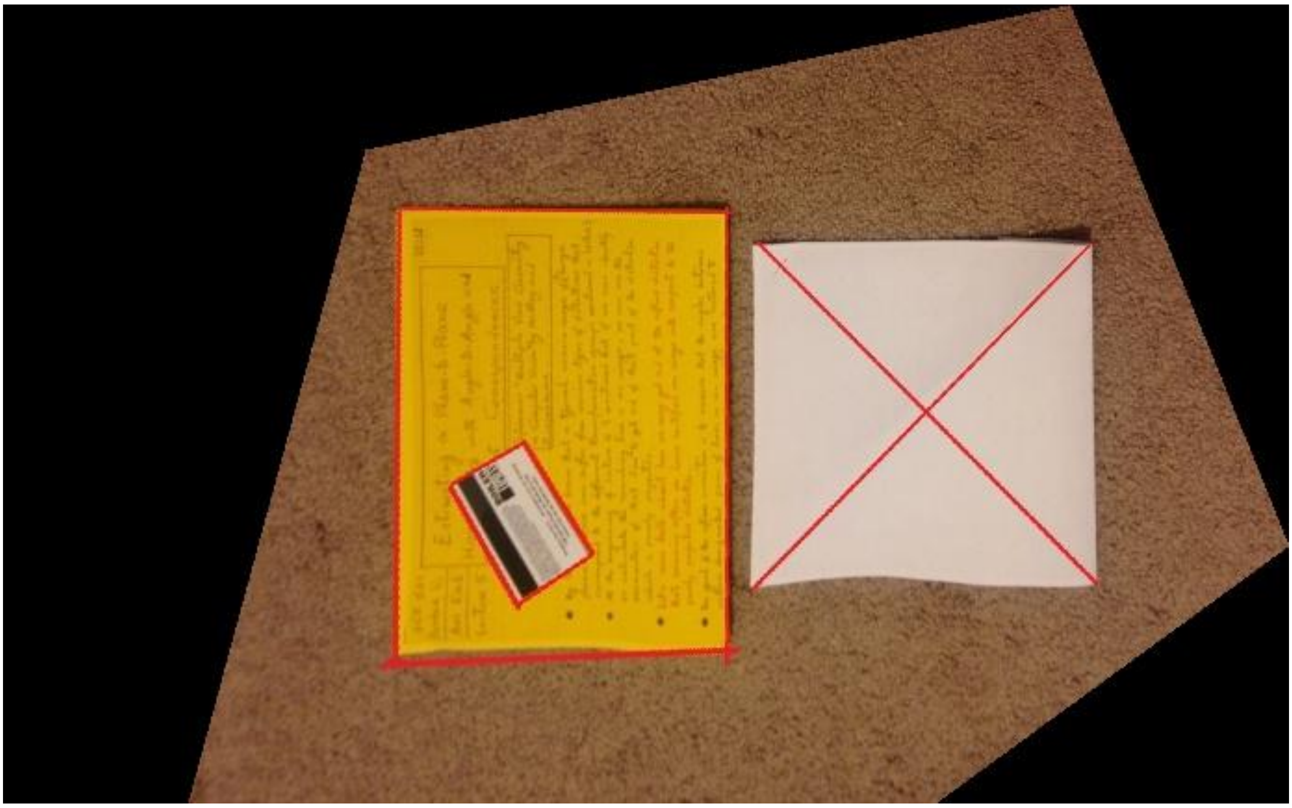
My Image 001

original

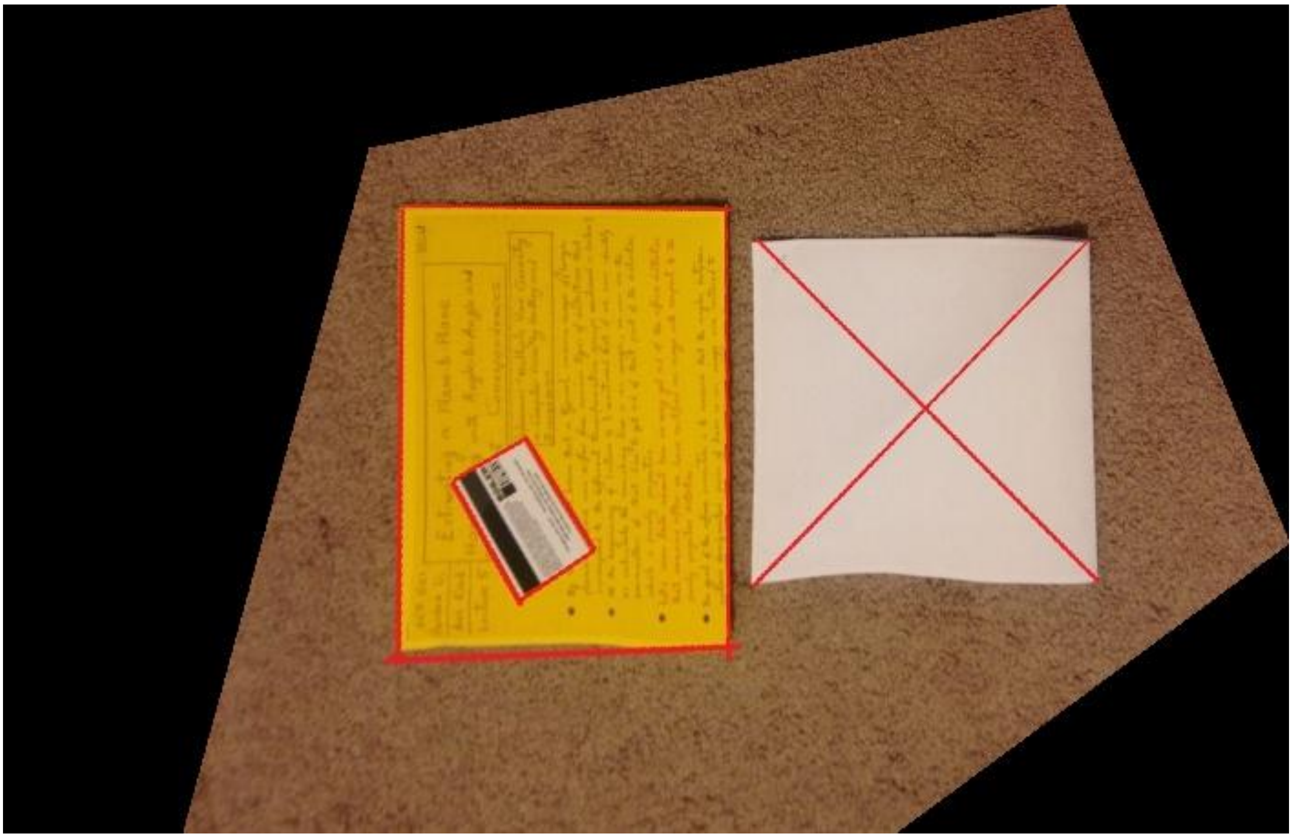


projective removed

affine removed

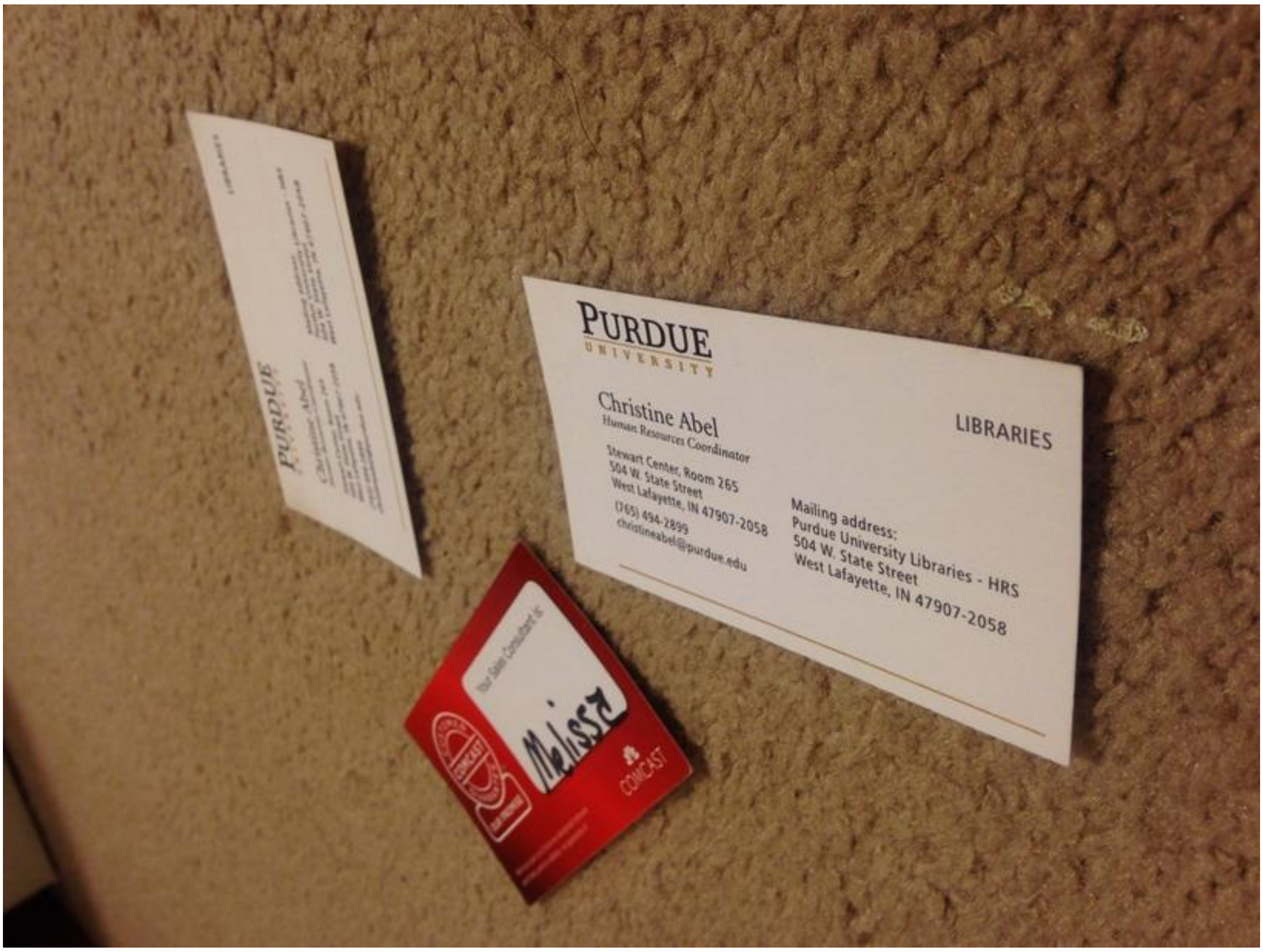


one-step method



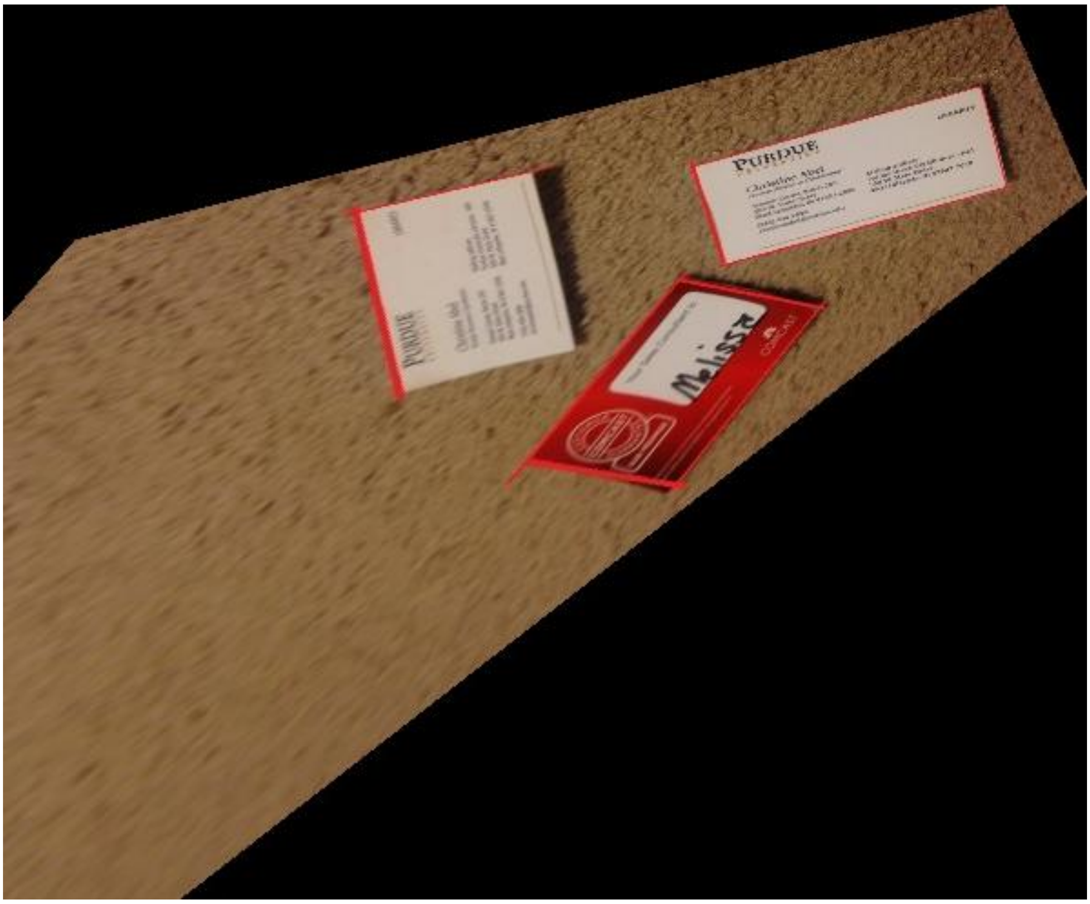
My Image 002

original



projective removed

affine removed



one-step method