ECE 661 Homework 1 : Solution

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1. Derive in just 3 steps the intersection of two lines l_1 and l_2, with l_1 passing through the points (0,4) and (3,0), and l_2 passing through the points (8,-2) and (10,8).

   (sol.)

   - Using homogeneous coordinate representation,

   $$l_1 = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -12 \end{bmatrix}$$

   $$l_2 = \begin{bmatrix} 8 \\ -2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 2 \\ 84 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 42 \end{bmatrix}$$

   $$x = l_1 \times l_2 = \begin{bmatrix} 4 \\ 3 \\ -12 \end{bmatrix} \times \begin{bmatrix} -5 \\ 1 \\ 42 \end{bmatrix} = \begin{bmatrix} 138 \\ -108 \\ 19 \end{bmatrix} = \begin{bmatrix} \frac{138}{19} \\ -\frac{108}{19} \\ 1 \end{bmatrix}.$$

   $\therefore$ The physical coordinate of this intersection point is $(\frac{138}{19}, -\frac{108}{19})$ in $\mathbb{R}^2$.

2. Given a conic $x^2 + y^2 + 4x + 2y - 29 = 0$,

   (a) where does the tangent to this conic at the perimeter point $\mathbf{x} = (1, 4)$ intersect with the y-axis?

   (sol.)

   This conic in homogeneous coordinate representation is

   $$C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & -29 \end{bmatrix}.$$

- The tangent line l to the conic at $\mathbf{x} = (1,\, 4)$ is

$$l = C\mathbf{x} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & -29 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -23 \end{bmatrix}.$$

- The intersection point $\mathbf{z}$ of the tangent to the conic with the y-axis is

$$\mathbf{z} = \begin{bmatrix} 3 \\ 5 \\ -23 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -23 \\ -5 \end{bmatrix}.$$

$\therefore$ The physical coordinate of this intersection point is $(0,\, \frac{23}{5})$ in $\mathbb{R}^2$.

(b) find the coordinates of the intersection of the tangents to this conic at points $\mathbf{x}_1 = (1,\, 4)$ and $\mathbf{x}_2 = (3, -4)$.

(sol.)

Using homogeneous coordinate representation,

$$C = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & -29 \end{bmatrix}$$

- The tangent line l_1 to the conic at $\mathbf{x}_1 = (1,\, 4)$ is

$$l_1 = C\mathbf{x} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & -29 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ -23 \end{bmatrix}.$$

- The tangent line l_2 to the conic at $\mathbf{x}_2 = (3,\, -4)$ is

$$l_2 = C\mathbf{x} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & -29 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ -27 \end{bmatrix}.$$

- The intersection point $\mathbf{z}$ of the tangent to the conic with the y-axis is

$$\mathbf{z} = \begin{bmatrix} 3 \\ 5 \\ -23 \end{bmatrix} \times \begin{bmatrix} 5 \\ -3 \\ -27 \end{bmatrix} = \begin{bmatrix} -204 \\ -34 \\ -34 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}.$$

$\therefore$ The physical coordinate of this intersection point is (6,1) in $\mathbb{R}^2$.

3. The intersection of a plane through a double cone results in a degenerate conic. This conic is composed of two lines l and m, with l passing through the points (-1,-2) and (2,4), and m passing through the points (1,-3) and $\underline{(3,9)}$. What is the description of this degenerate conic as a 3x3 matrix C in homogeneous representation?

   (sol.)

   - $l = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, $m = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -12 \\ 2 \\ 18 \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \\ 9 \end{bmatrix}$
   - $C = lm^T + ml^T = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -6 & 1 & 9 \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \\ 9 \end{bmatrix} \begin{bmatrix} -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 24 & -8 & -18 \\ -8 & 2 & 9 \\ -18 & 9 & 0 \end{bmatrix}$

   $\therefore$This is a degenerate conic becasue C has a rank 2. However, two lines are not on the conic so that this composition is not appropriate.

4. The intersection of a plane through a double cone results in a degenerate conic. This conic is composed of two lines l and m, with l passing through the points (-1,-2) and (2,4), and m passing through the points (1,-3) and $\underline{(-3,9)}$. What is the description of this degenerate conic as a 3x3 matrix C in homogeneous representation?

   (sol.)

   - $l = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, $m = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \times \begin{bmatrix} -3 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -12 \\ -4 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$
   - $C = lm^T + ml^T = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -12 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$

$\therefore$This is an appropriate degenerate conic composed by two lines on the conic.