ECE 661: Homework 5
Due October 28th

Problem:

In the last homework, weighted RANSAC was used to obtain a good (inlier) set of correspondences from a larger, noisy set. The inlier correspondences were then used to solve a linear lest squares problem to obtain the final homography. In this homework, the goal is to use the inlier correspondences in a nonlinear least squares problem which minimizes the reprojection error in both images.

In the last homework, the DLT algorithm was used to find the homography which minimized an algebraic cost function. In this homework, we wish to find the homography H which minimizes the geometric cost function given by Equation 4.6 in the textbook (pg. 94)

$$C(\mathrm{H}) = \sum_i d\left(x_i', \mathrm{H}x_i\right)^2 \tag{1}$$

where $d()$ is Euclidean distance.

Equation (1) can be rewritten as a nonlinear least squares problem with the form

$$C(\mathbf{p}) = \|\mathrm{X} - F(\mathbf{p})\|^2 \tag{2}$$

where X is a set of data points and $F(\mathbf{p})$ is a nonlinear function with parameters $\mathbf{p}$. Note that the minimization is carried out only over the parameters $\mathbf{p}$; the data points and nonlinear function are fixed. The first part of the problem is to determine what X, $F(\mathbf{p})$, and $\mathbf{p}$ should be in Equation (2) in order to give the same cost as Equation (1).

Once the cost function is in the form of Equation (2), there are several iterative methods available to minimize it. In this homework you will try 2 standard methods, gradient descent and Gauss-Newton, as well as two hybrid methods, Levenberg-Marquardt and dog leg. A brief description of each method is given in the supplemental material at http://engineering.purdue.edu/ece661/homework/hw5_supp.pdf

This homework requires a lot of matrix manipulation and you are encouraged (but not required) to use MATLAB to simplify the coding. If you choose to use C++, the OpenCV C++ interface includes a nice matrix class which will simplify the coding. Note: you will be resuing these nonlinear least squares solvers in a future homework with a different cost function so be sure to write the code in a general way.

Solution:

You should turn in a report in pdf format of your homework solution using electronic turn-in. The report should include:

1. A description of the nonlinear least squares problem you have solved.
2. A description of your implementation of the 4 iterative solving approaches. Be sure to talk about how you set the free parameters and how these affect the convergence.
3. A comparison of the 4 methods. What are there relative convergence rates? How sensitive are they to initial conditions (e.g. try starting with a poor estimate for the homography H, do they still converge?)
4. The results when you apply all 4 algorithms to the pair of images at
   http://engineering.purdue.edu/ece661/homework/ECE661_hw3_images.zip
   (a) For the "prime" image, plot x_i' and $\mathrm{H}x_i$ for each correspondence in different colors.
   (b) As in the last homework, you should transform one of the inputs to look like the other input.
   (c) Be sure to compare the results from the 4 nonlinear optimization methods as well as the original result based on the DLT algorithm.
5. Your source code.