# ECE 661 Computer Vision: Midterm Exam, Fall 2006

1. **(5 points)** Show that the intersection of the 2D line $\mathbf{l}$ and $\mathbf{l}'$ is the 2D point $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$

2. **(5 points)** Show that the line through two 2D points $\mathbf{x}$ and $\mathbf{x}'$ is $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$

3. **(5 points)** Show that under the 2D point transformation $\mathbf{x} = \mathbf{H}\mathbf{x}'$, a line transform is

$$\mathbf{l}' = \mathbf{H}^{-T}\,\mathbf{l}$$

4. **(10 points)** Show that a 3x3 homography is affine if and only if a line at infinity is mapped to a line at infinity. Your proof must go in both directions.

Hint: If $\mathbf{Z} = \begin{bmatrix} \mathbf{X} & \mathbf{y} \\ \mathbf{0}^T & 1 \end{bmatrix}$ then $\mathbf{Z}^{-1} = \begin{bmatrix} \mathbf{X}^{-1} & -\mathbf{X}^{-1}\mathbf{y} \\ \mathbf{0}^T & 1 \end{bmatrix}$,

where $\mathbf{X}$ is a 2x2 non-singular matrix, and $\mathbf{y}$ and $\mathbf{0}$ are vectors with 2 elements each.

5. **(5 points)** Show that under the 3D point transformation $\mathbf{X} = \mathbf{H}\mathbf{X}'$, a 3D plane transforms as

$$\boldsymbol{\pi}' = \mathbf{H}^{-T}\,\boldsymbol{\pi}$$

6. Let $\mathbf{L} = \mathbf{A}\mathbf{B}^T - \mathbf{B}\mathbf{A}^T$ be a 4x4 Plucker matrix that represents the 3D line joining the point $\mathbf{A}$ and $\mathbf{B}$, and $\mathbf{L}^* = \mathbf{P}\mathbf{Q}^T - \mathbf{Q}\mathbf{P}^T$ be the dual Plucker representation of the line $\mathbf{L}$.

(a) **(10 points)** Show that the plane defined by the join of the point $\mathbf{X}$ and line $\mathbf{L}$ is

$$\boldsymbol{\pi} = \mathbf{L}^*\mathbf{X}$$

(b) **(10 points)** Show that the point defined by the intersection of the line $\mathbf{L}$ with the plane $\boldsymbol{\pi}$ is

$$\mathbf{X} = \mathbf{L}\boldsymbol{\pi}$$

7. **(10 points)** Explain the notion of the algebraic distance and the geometric distance in the context of estimating a homography.

8. **(15 points)** Explain that the eigen-decomposition of the matrix

$$\mathbf{C} = \begin{bmatrix} \sum_{W} I_x^{\ 2} & \sum_{W} I_x I_y \\ \sum_{W} I_x I_y & \sum_{W} I_y^{\ 2} \end{bmatrix}$$

and subsequently thresholding of the eigenvalues **constitute** a sound basis for a corner finding algorithm. $I_x$ and $I_y$ stand for the following partial derivatives in the formula for **C:**

$$I_x = \frac{\partial I(x, y)}{\partial x} \quad \text{and} \quad I_y = \frac{\partial I(x, y)}{\partial y}$$

where I(x,y) is the image intensity at the pixel located at (x,y). W in the formula for **C** is the local window over which the summations are carried out.

9. **(10 points)** What do the different rows and the different columns of the camera matrix stand for? Provide simple algebraic proofs for your answer.

10. **(15 points)** Prove that the null vector of the camera matrix is the homogenous vector to the camera center in the world coordinate.