Development of Efficient Inelastic Scattering in Atomistic Tight Binding

Master's Defense

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Overview

1. Motivation
2. Acoustic and Optical Phonon Scattering Verification
3. Effect of scattering on MOSFETs
4. Effect of scattering on TFETs
5. Why does scattering increase tunneling current?
6. Numerical Details
7. Conclusions/Future Work

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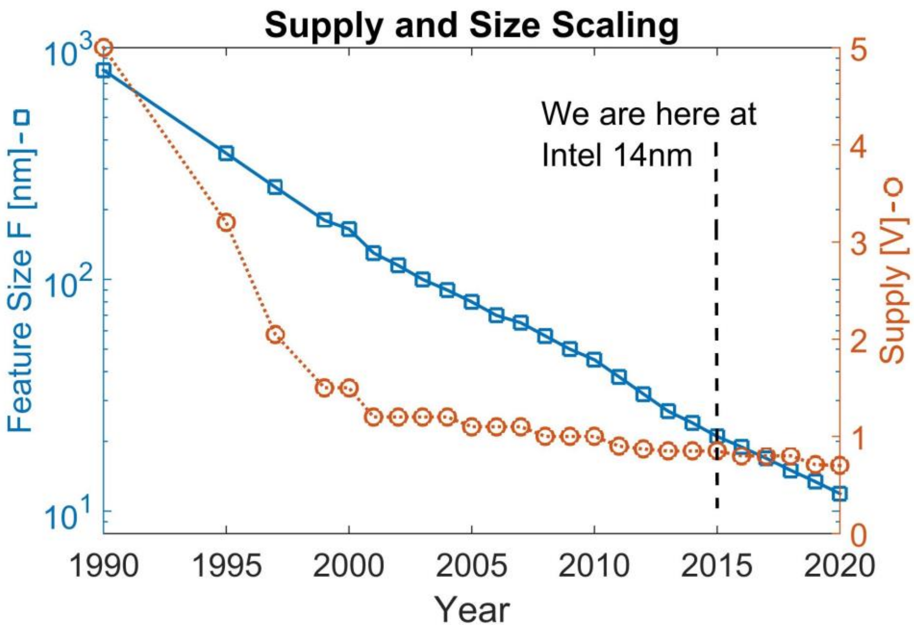
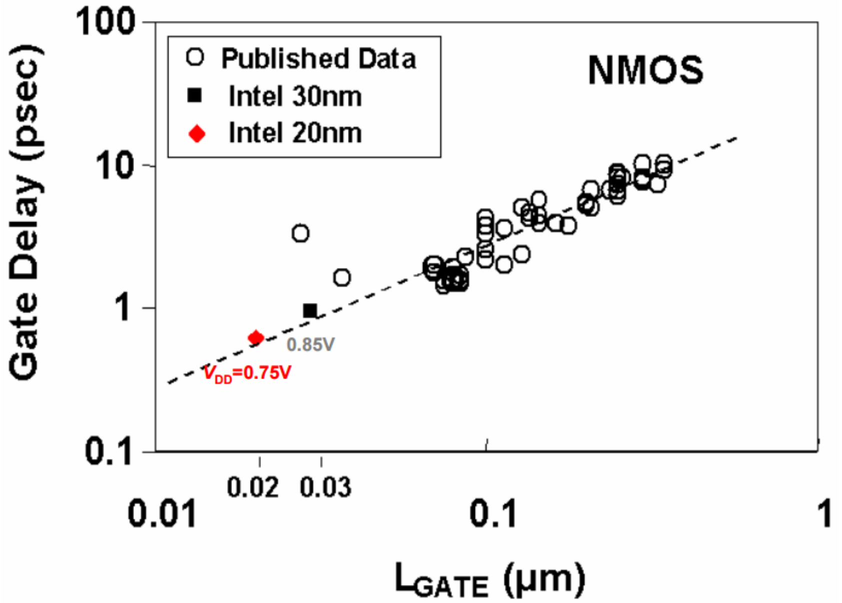
Length Downscaling

Figure courtesy of Tarek Ameen



Gate Delay ∝ $C_{gate}V_{DD}/I_{sat}$

http://www-inst.eecs.berkeley.edu/~ee40/fa03

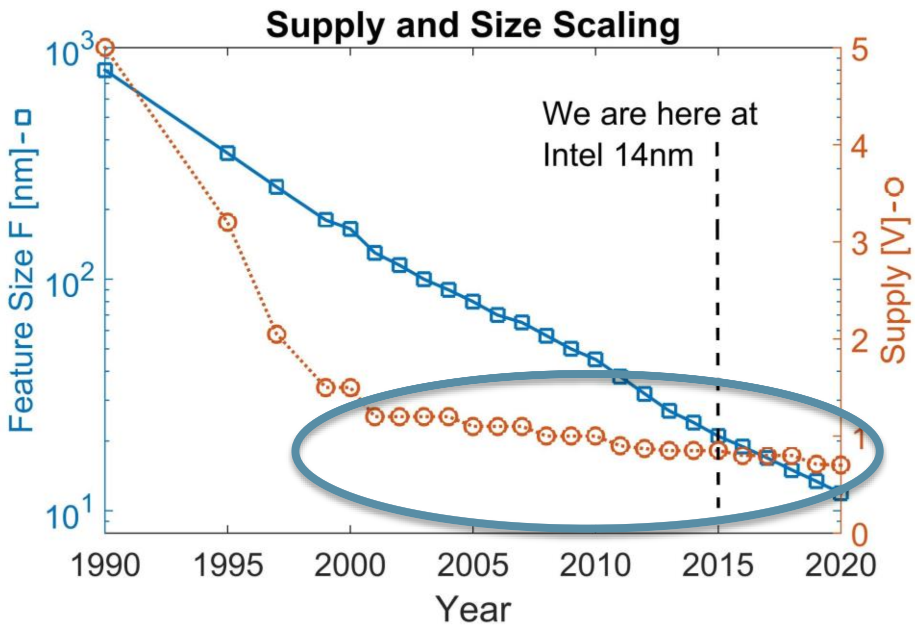


Length Scaling:
1) Area ↙ transistor density ↗
2) Capacitance ↙ Delay ↙

Length continues to scale smoothly

Voltage Downscaling

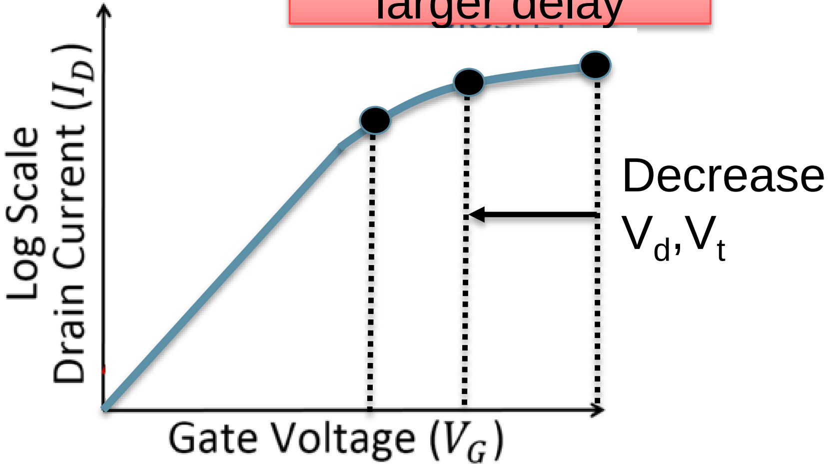
Figure courtesy of Tarek Ameen



Why has voltage scaling saturated around 2000?

Gate Delay $\propto C_{gate}V_{DD}/I_{sat}$

I_{sat} decreases, larger delay



Keep off-current constant

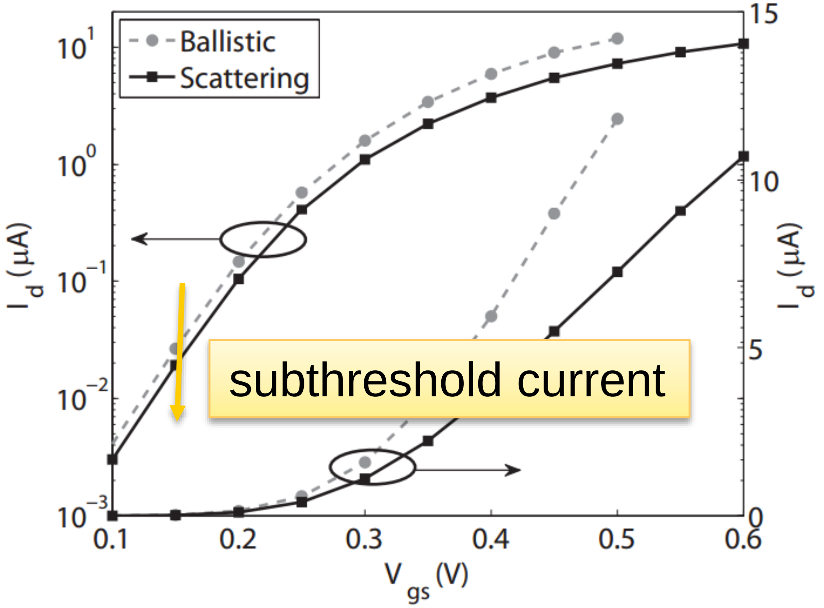
Voltage Scaling:
- ↙ Static Power $\propto V_{DD}I_{off}$
- ↙ Dynamic Power $\propto V_{DD}^2$

Voltage scaling has saturated

Importance of Scattering

“that the **reduction [compared to ballistic]** of the device drain current, …is more important in the ON-state than in the OFF-state of the transistor”

M. Luisier and G. Klimeck Phys. Rev. B **80**, 155430 (2009).

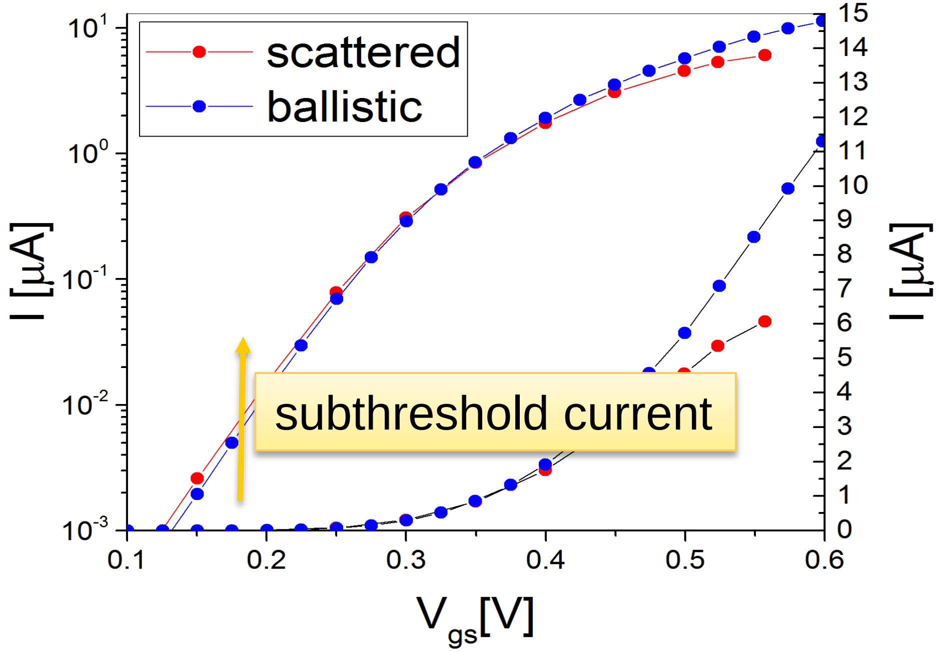


subthreshold current

3 nm circular silicon nanowire, $V_{ds}=0.6$V
$sp^3d^5s^*$ tight binding basis
confined phonon model

ballistic yields an **underestimation** of the subthreshold current up to 20%

Data from A. Esposito, M. Frey, et. al. JCEL vol. 8 (2009).



subthreshold current

3.2 nm square silicon nanowire, $V_{ds}=0.5$V
non-parabolic effective mass basis
bulk phonon model

Conflicting trends for subthreshold current in literature. How to resolve this?

Critical Questions:

1) How to increase electrostatic gate control?
2) What is the limit of gate length scaling?
3) How to continue supply voltage scaling?
4) What is the importance of scattering in gate length and supply voltage scaling?

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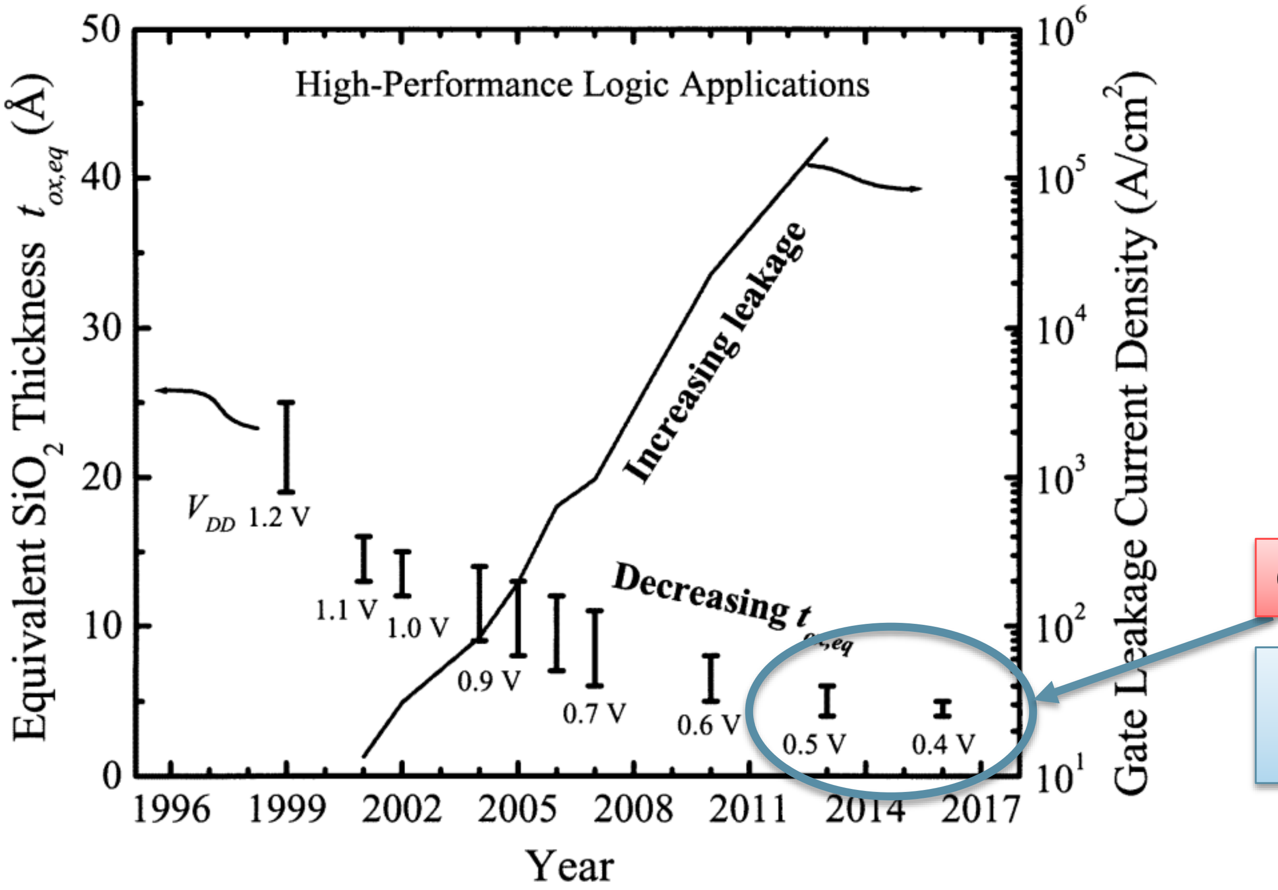
↗ Capacitance by Oxide Thickness Scaling

↗ capacitance ↙ oxide thickness

↗ gate leakage



Oxide scaling is saturated

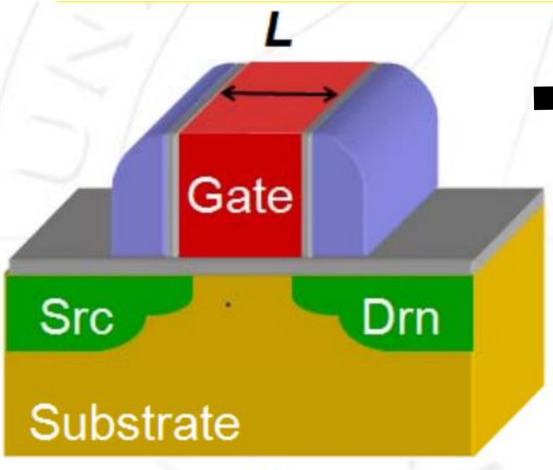
Leakage puts a limit on minimum oxide thickness

IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 50, NO. 4, APRIL 2003

↗ Capacitance for Better Gate Control

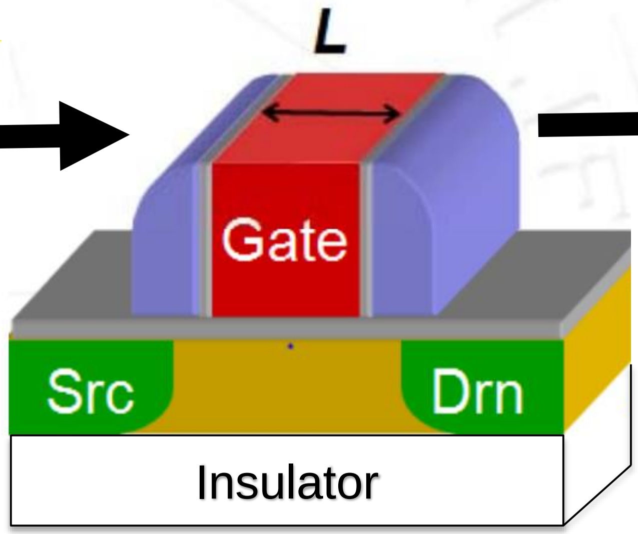
Geometry Trend

increased electrostatic control

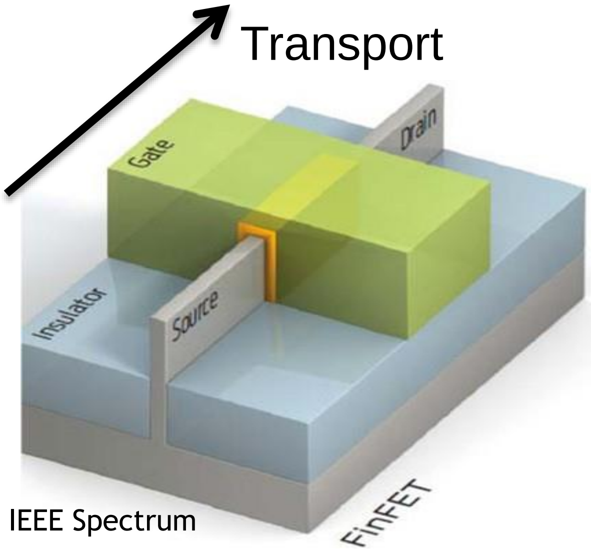


Bulk

ESSDERC 2012

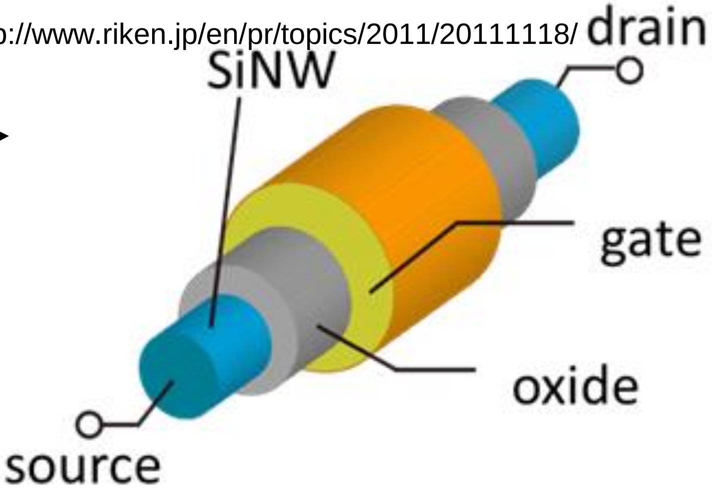


SOI



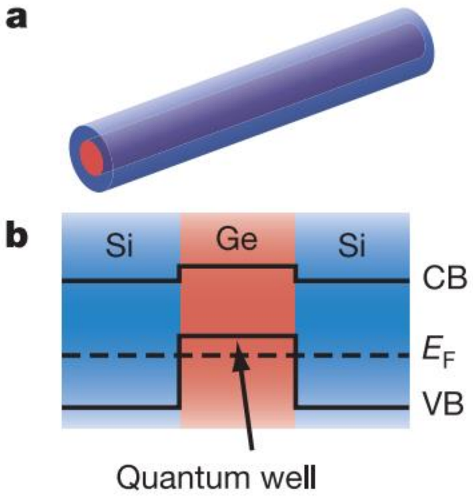
IEEE Spectrum

http://www.riken.jp/en/pr/topics/2011/20111118/

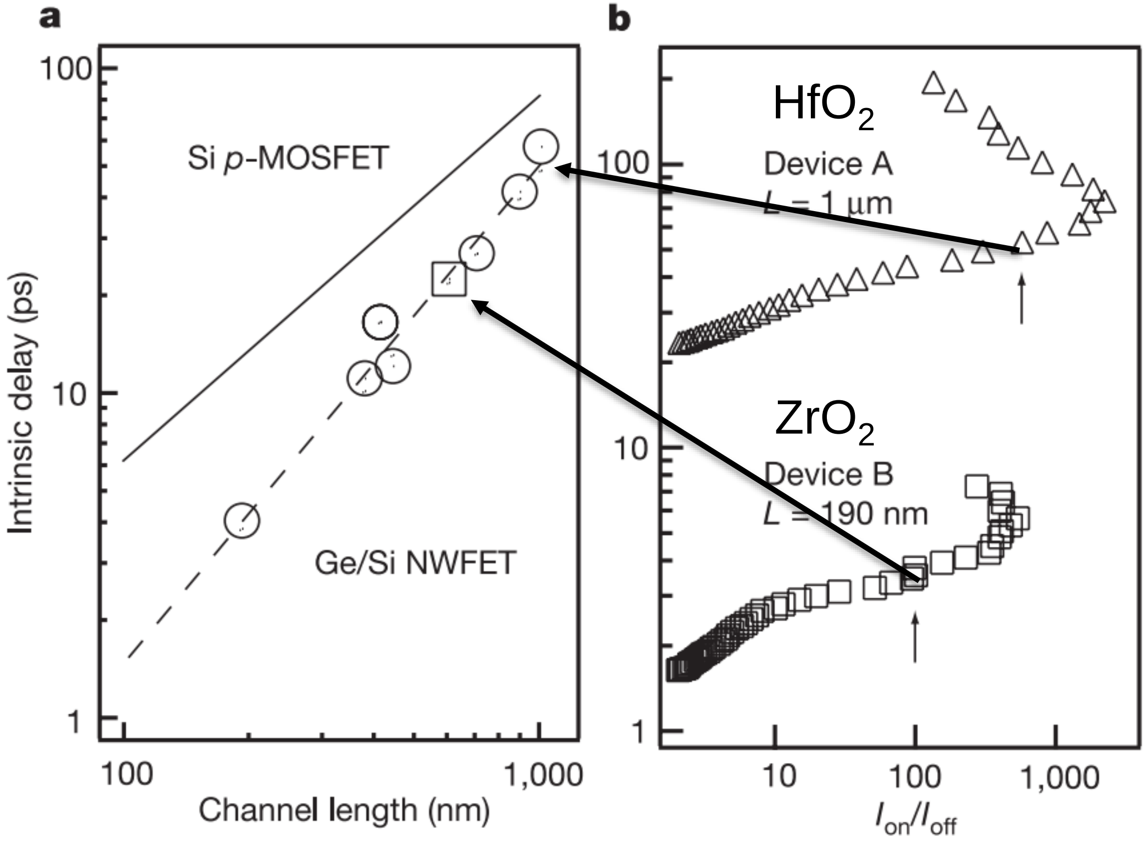


Nanowires candidate for best electrostatic control

Nanowire Performance Comparison



J. Xiang Nature Vol. 441 (2006).



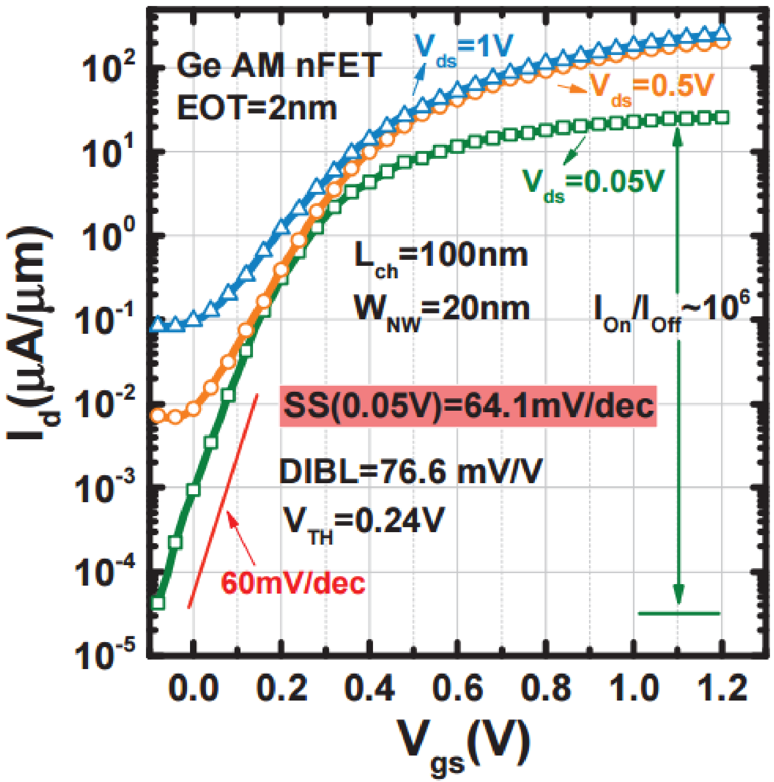
Nanowire increased drive current and reduced delay

p-MOSFET data from Chau, R. *et al.* Benchmarking nanotechnology for high-performance and low-power logic transistor applications. *IEEE Trans. Nanotechnol.* **4**, 153–158 (2005).

Experimental Evidence of Excellent Nanowire Performance

Experimental evidence of nearly perfect S.S achieved for nanowires

Peide Ye IEDM 2015



"delivering more than 30% and 40% reduced SS over FinFETs and planar MOSFETS"

nanowires have excellent gate control

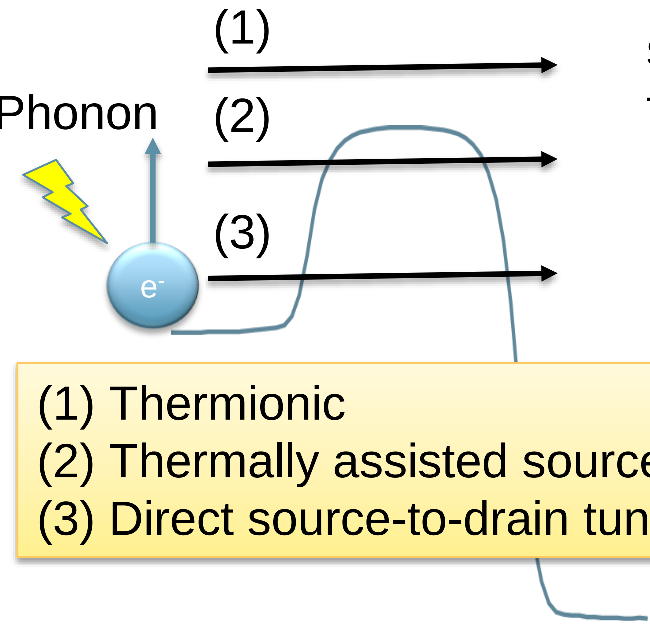
Critical Questions:

1) How to increase electrostatic gate control?
2) **What is the limit of gate length scaling?**
3) How to continue supply voltage scaling?
4) What is the importance of scattering in gate length and supply voltage scaling?

Gate Length Scaling Limits

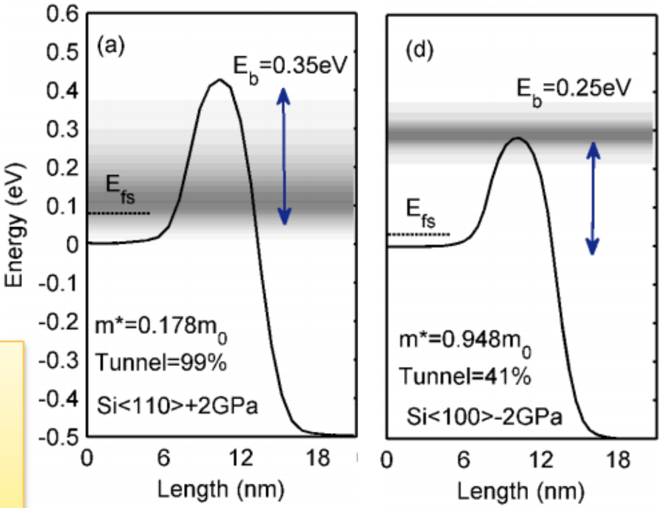
ITRS demands scaling of gate length

J. Wang, M. Lundstrom, IEDM 2002: “The results show that source-to-drain tunneling does set an ultimate scaling limit



Bandgap engineering shown to decrease tunneling leakage..

S. Mehrota et. al. IEEE Trans. Vol. 60, No.7 (2013).



(1) Thermionic
(2) Thermally assisted source-to-drain tunneling
(3) Direct source-to-drain tunneling

Need to include scattering to account for thermally assisted source-to-drain tunneling

Critical Questions:

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Supply Voltage Scaling

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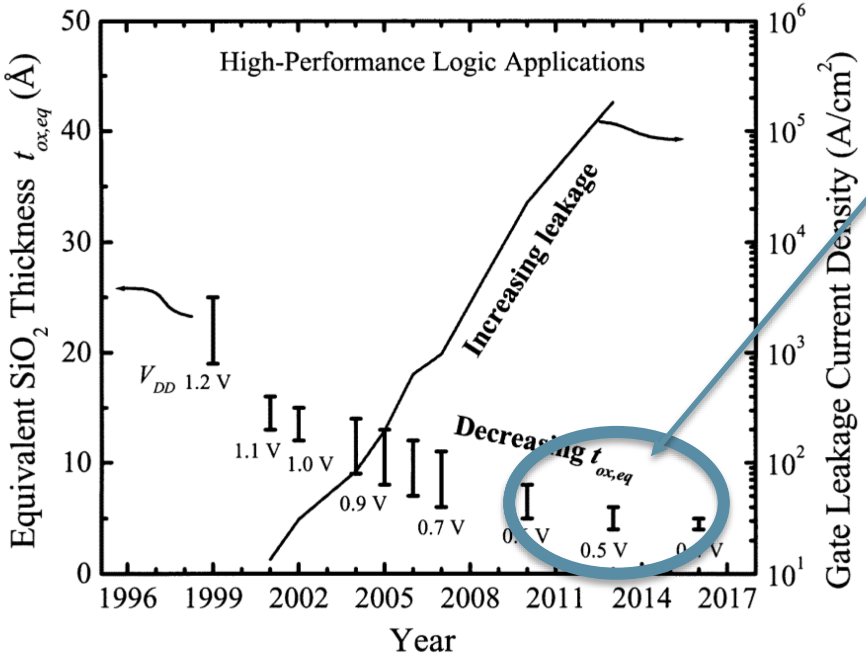
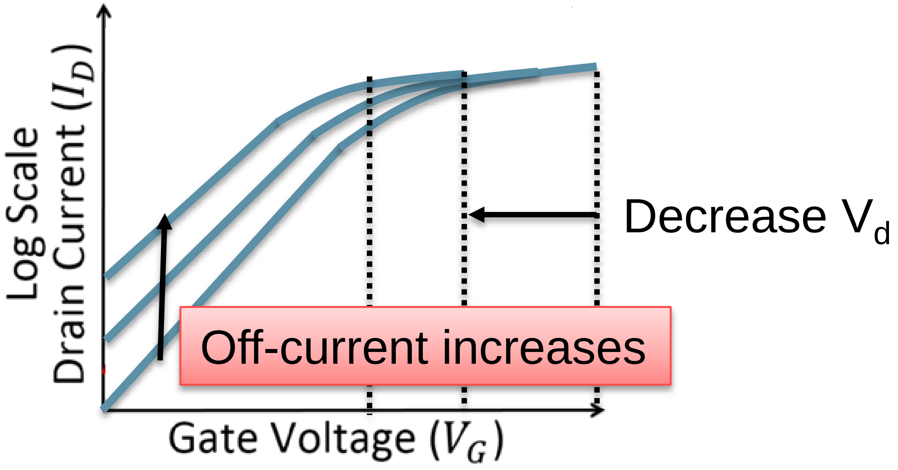


Note: prediction of EOT depends on scaled voltage (dictated by ITRS)

Reality:
Since ~2000, supply voltage is no longer scaling. Why?

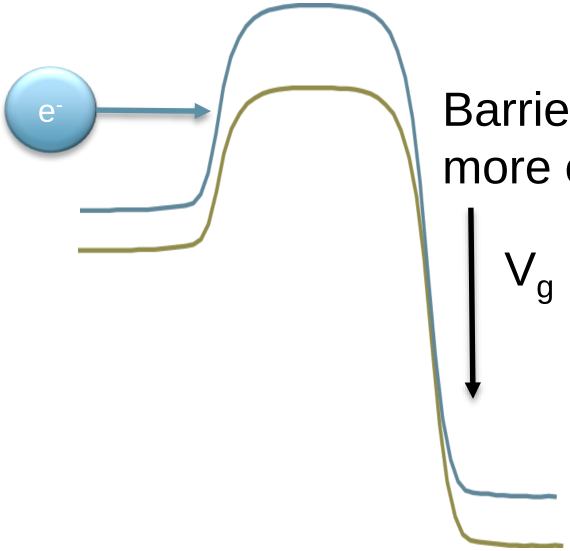


Decrease V_d

Off-current increases

Physical Limitation of Subthreshold Slope

Typical MOSFETs :
Barrier controlled device



Barrier lowered more electrons

V_g

- Sub-threshold slope: fundamental limit of how fast the device can turn on
- This is limited by the Fermi band-tail

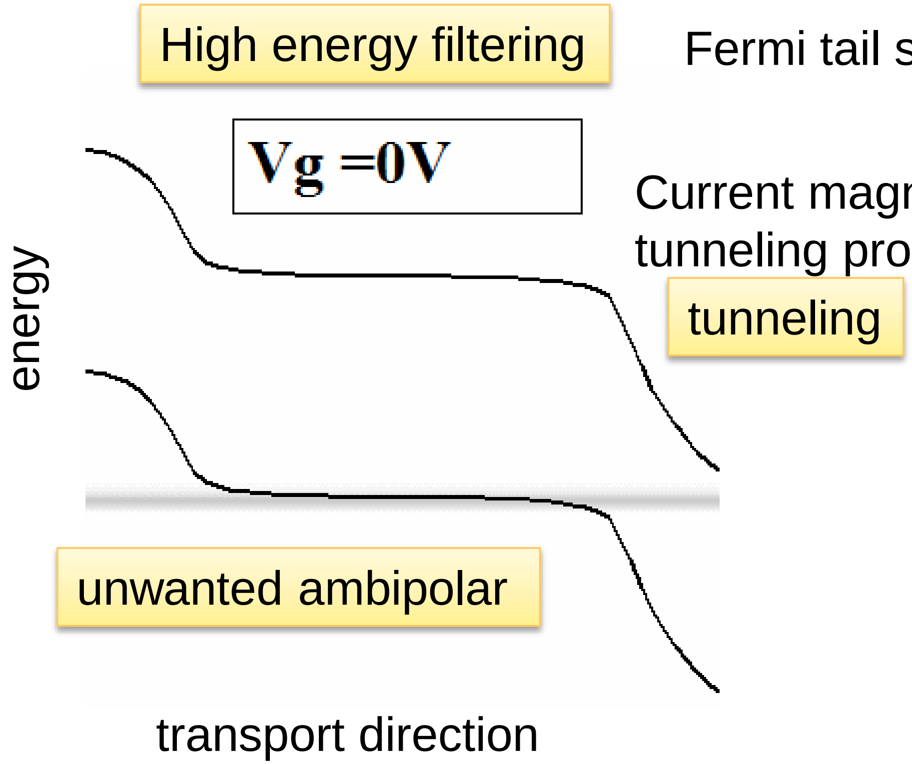
$$S.S = \ln(10)\frac{kT}{q}\left(1 + \frac{C_{ch}}{C_{ox}}\right)$$

$$min(S.S) = 60\ mV/dec$$

How can we overcome this limit?

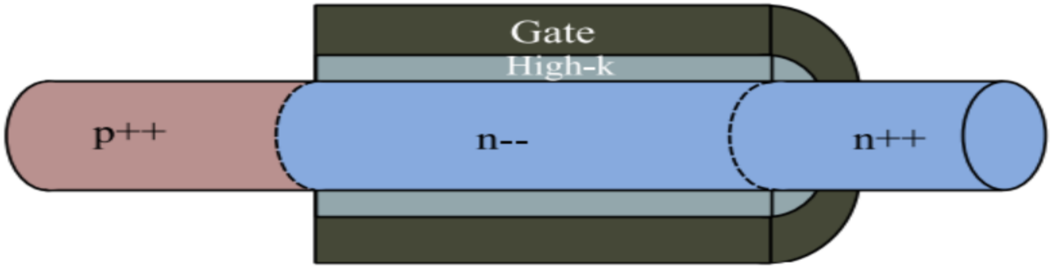
The Need for Tunneling Dominated Transistors

Spectral current for InAs TFET



Fermi tail suppressed

Current magnitude related to tunneling probability



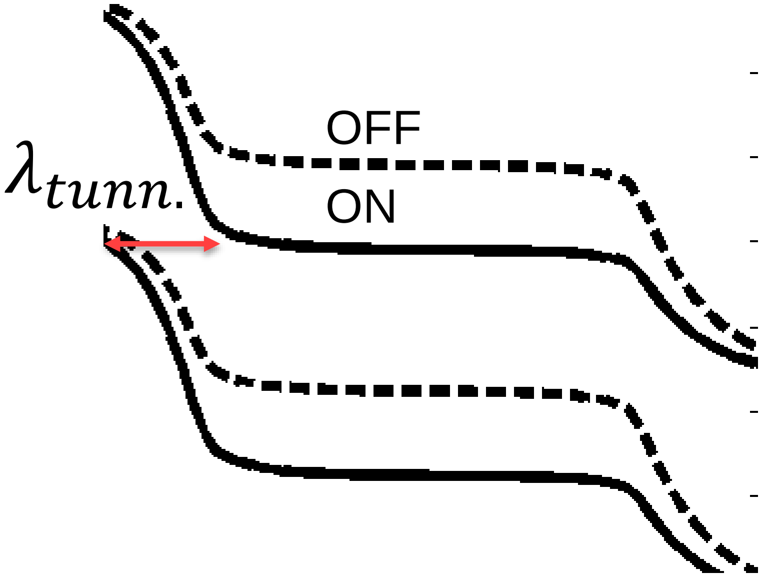
Nanowire Potential Candidate for TFETs

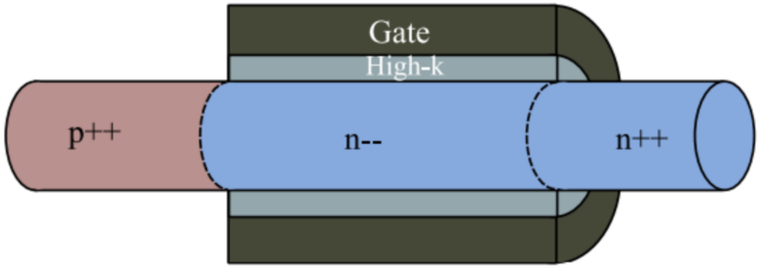
Gate-all-around (GAA) nanowire TFET

1. Excellent Gate Control

I_{on} increases with steeper source to channel transition
(decreased tunneling distance $\lambda_{tunn.}$)

2. Best subthreshold-slope compared to MOSFET





What is the importance of scattering in TFETs?

Critical Questions:

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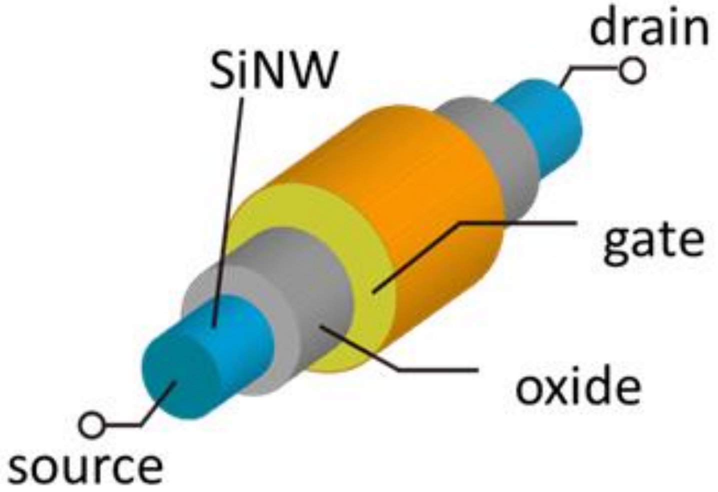
Modeling Quantum Effects + Scattering

Method:
Non-equilibrium Green's Function (NEGF) + scattering in the Self-consistent Born Approximation (SCBA)

Atomistic Resolution with Semi-empirical tight binding

http://www.riken.jp/en/pr/topics/2011/20111118/



Schematic of Gate All Around (GAA) Nanowire

Scattering in the Self-consistent Born Approximation

What is a scattering self-energy?

- Contribution to particle's energy due to interaction with the system.
- Complex matrix

  Real Part $\Sigma^{\mathrm{R}} \sim \Delta E$

  Imaginary Part Σ^{R} ~ related to lifetime of particle.

What is self-consistent Born?

- Interactions treated as (weak) perturbations
- Leads to self-consistent loop to stabilize charge/current

Why include scattering?

Effect of Scattering on Transistors

Three Major Effects from scattering :

- Resistive (decreases on-current)
- Increases tunneling current
- Broadens/fills resonant states

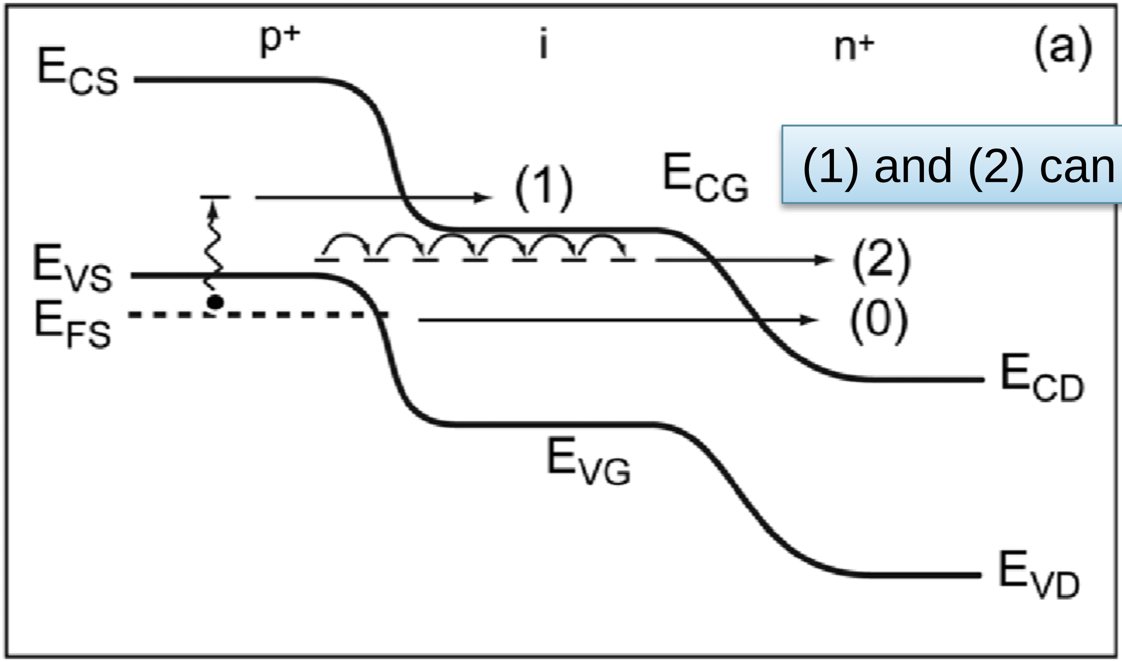
Tunneling mechanisms for TFET

(0) Direct, coherent tunneling

(1) Thermally excited carriers tunneling

(2) Tunneling via channel band-tails

Khayer, JAP 110, 074508 (2011).



(1) and (2) can only be covered with scattering

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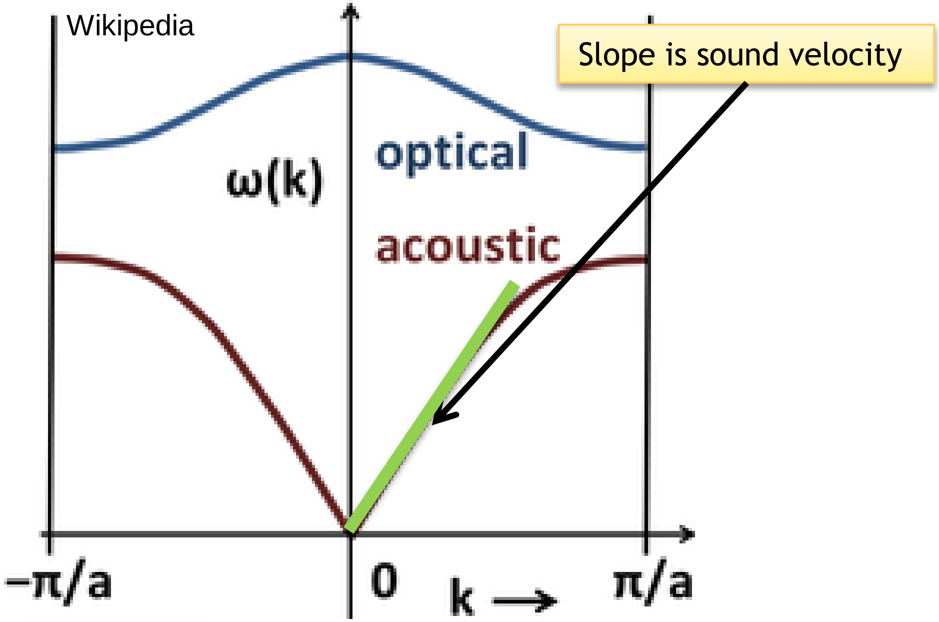
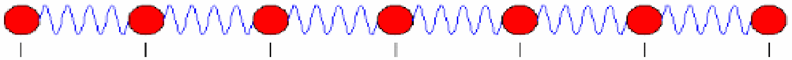
Acoustic Perturbing Potential

- With bulk phonons, the perturbing potential (electron-phonon interaction strength) is solved analytically
- Assume:
  - linear dispersion
  - bulk phonons in equilibrium
  - elastic
  - high temperature



long wavelength acoustic phonon



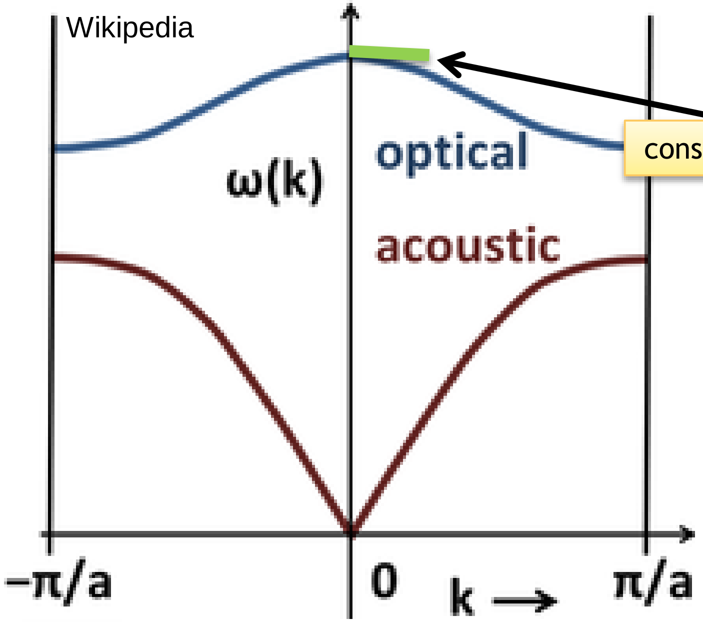
Scattering Parameters:
- ion mass density
- sound velocity
- deformation potential

http://exafs.ucsc.edu/simulations

NEMO5

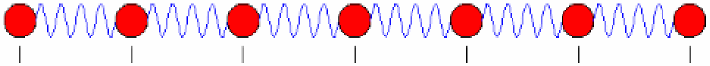
Optical Perturbing Potential

- With bulk phonons, the perturbing potential (electron-phonon interaction strength) is solved analytically
- Assume:
  - flat dispersion
  - bulk phonons in equilibrium
  - inelastic



constant phonon freq.

Optical phonon



http://exafs.ucsc.edu/simulations

Scattering Parameters:

- ion mass density
- phonon frequency
- optical deformation constant

Acoustic and Optical Scattering Rate Verification

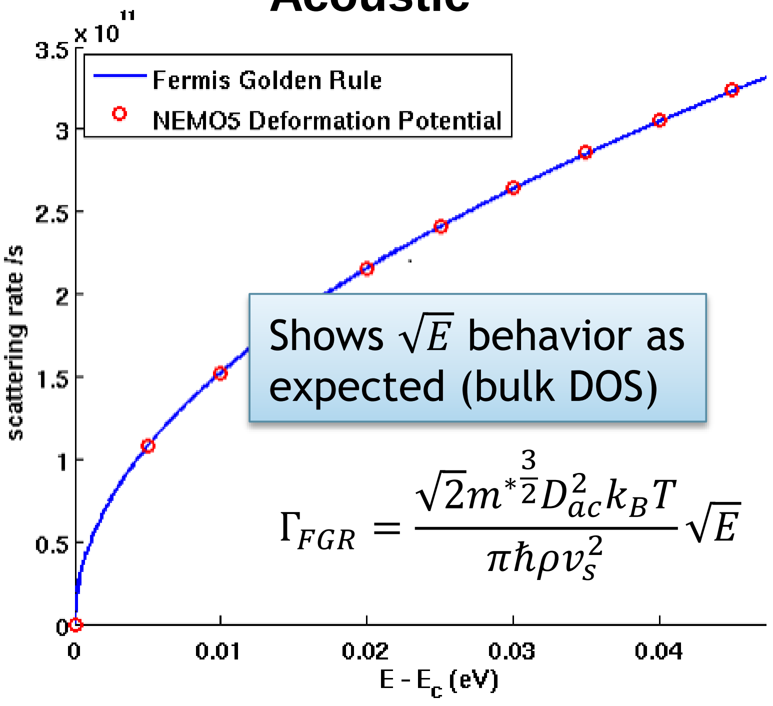
Both can be verified against analytical expressions for scattering rate

$$\Gamma_{NEGF}(E) = -\frac{2}{\hbar} Im\{\Sigma^{R}_{\text{scatt}}(E)\}$$

Bulk GaAs Material Parameters$*$:

- Deformation potential D_{ac} = 8.8
- Sound velocity v_s = 4726 m/s
- Material density ρ = 5317 kg/m^3
- Phonon energy E_0 = 35 meV
- Optical Coupling constant D_{op} = 110 eV/nm

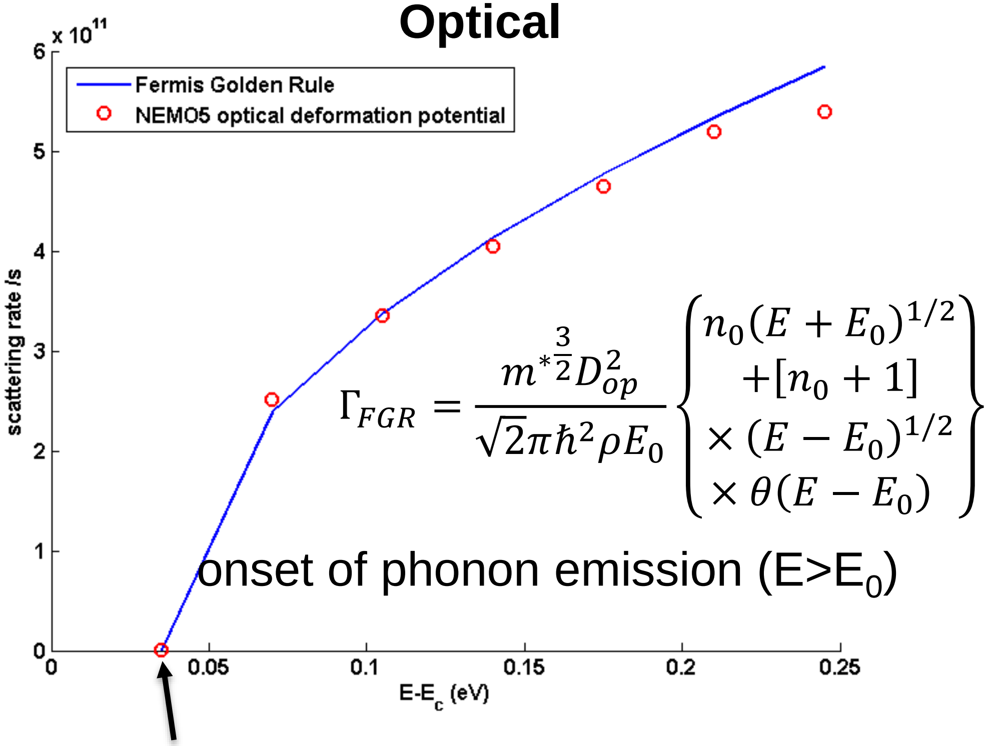
Acoustic



Shows $\sqrt{E}$ behavior as expected (bulk DOS)

$$\Gamma_{FGR} = \frac{\sqrt{2} m^{*\frac{3}{2}} D_{ac}^2 k_B T}{\pi \hbar \rho v_s^2} \sqrt{E}$$

Optical



$$\Gamma_{FGR} = \frac{m^{*\frac{3}{2}} D_{op}^2}{\sqrt{2}\pi \hbar^2 \rho E_0} \left\{ \begin{array}{c} n_0 (E + E_0)^{1/2} \\ +[n_0 + 1] \\ \times (E - E_0)^{1/2} \\ \times \theta(E - E_0) \end{array} \right\}$$

onset of phonon emission ($E>E_0$)

35 meV

*Landolt-Bornstein Database- Springer Materials

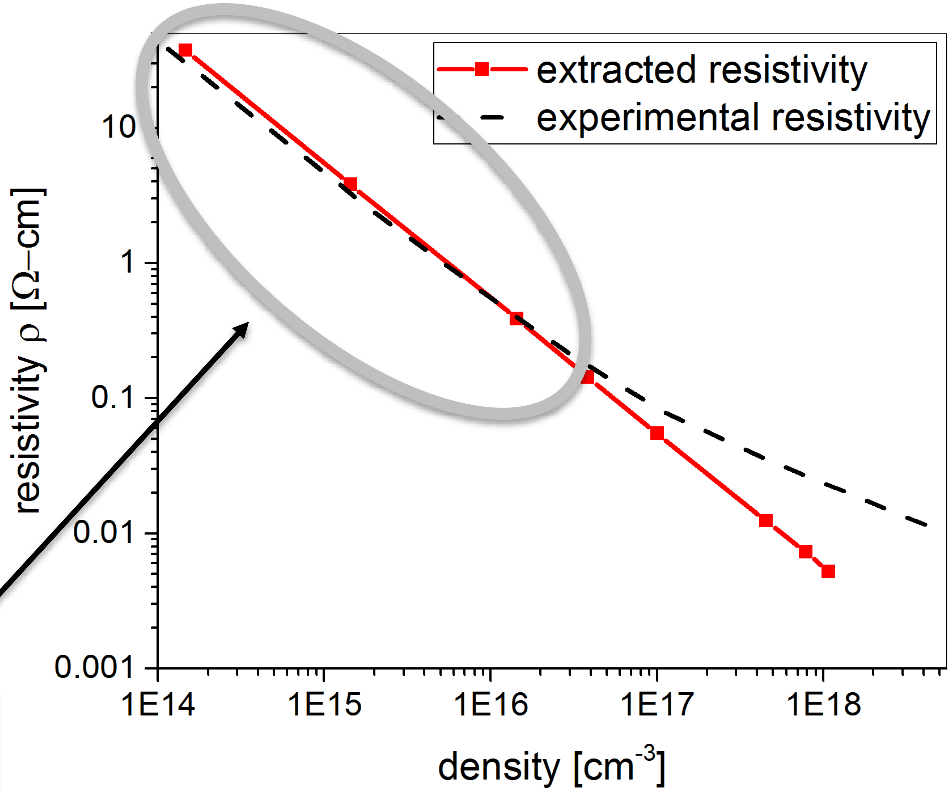
Extracted Resistivity

Device: homogeneous silicon bar in effective mass

Steps:

1. Calculate current of differer lengths with small applied potential (5 meV)
2. Calculate slope of resistance vs. length

Deviation due to neglect of electron-electron and impurity scattering



Matches well for phonon-limited range

Experimental data from NIST

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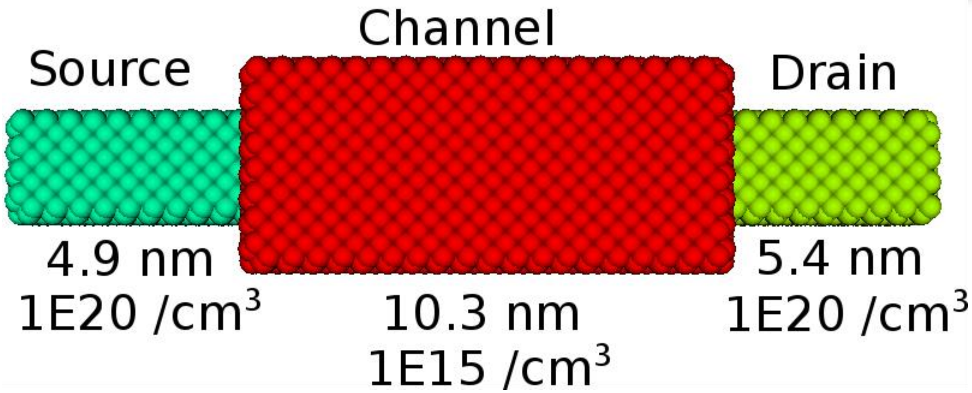
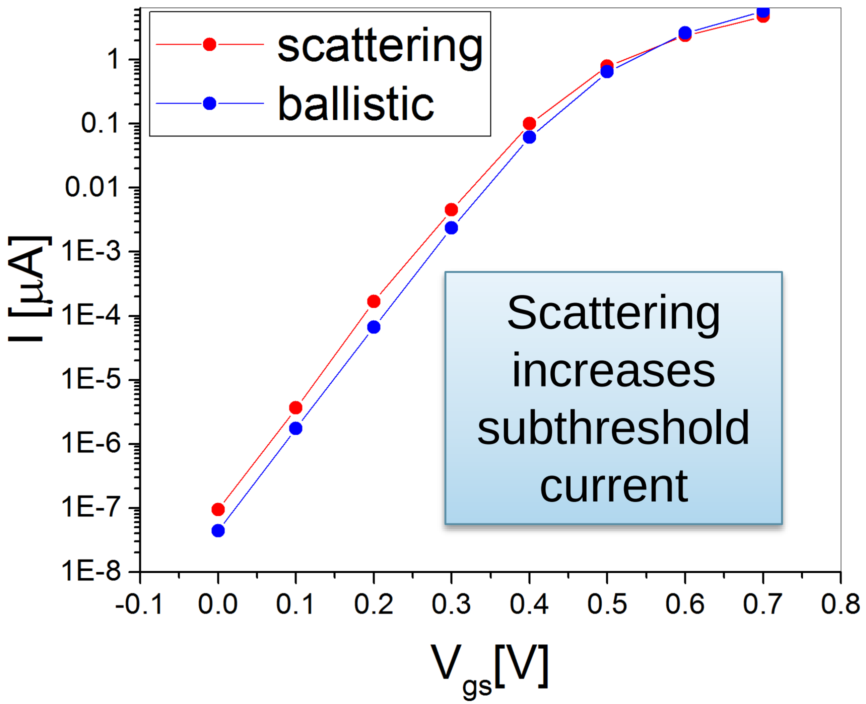
Effect of scattering on MOSFETs



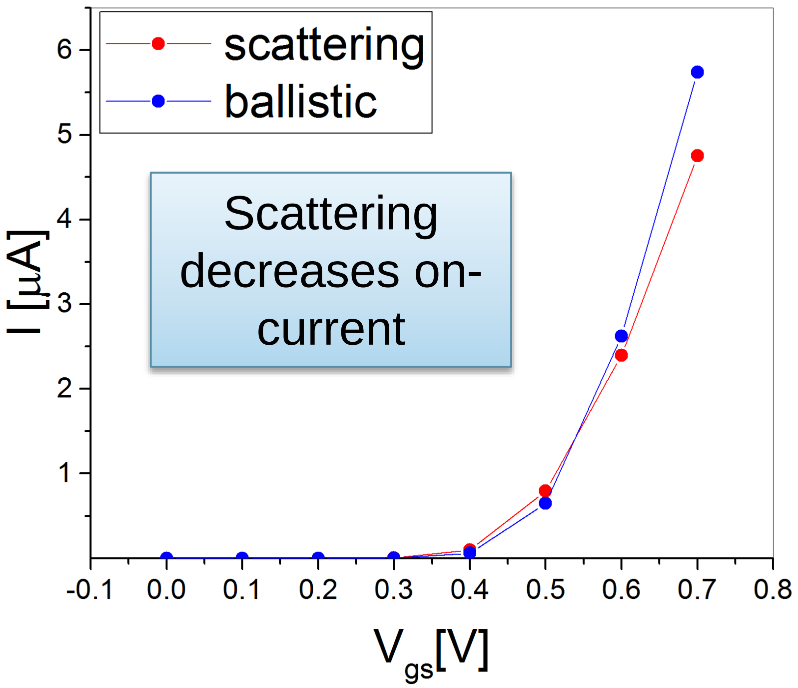
Si, 2nm cross section $sp^3d^5s^*$

NIN nanowire elastic
acoustic + optical



Scattering increases subthreshold current

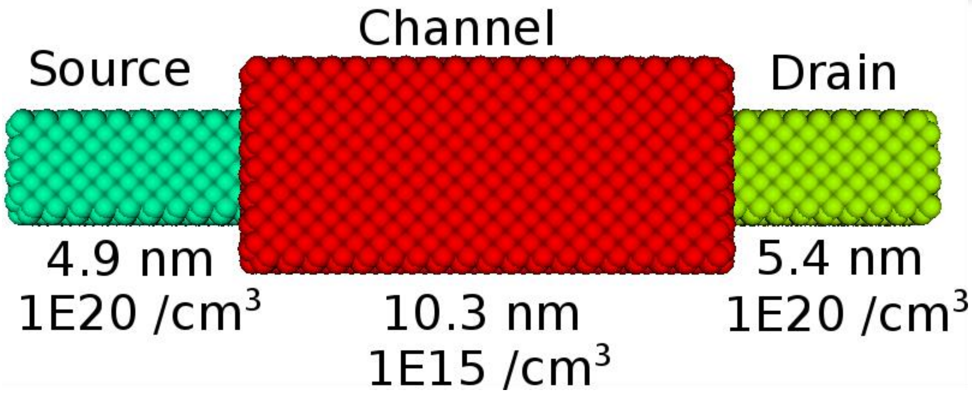
Log



Scattering decreases on-current

Linear

Varying Scattering Strength

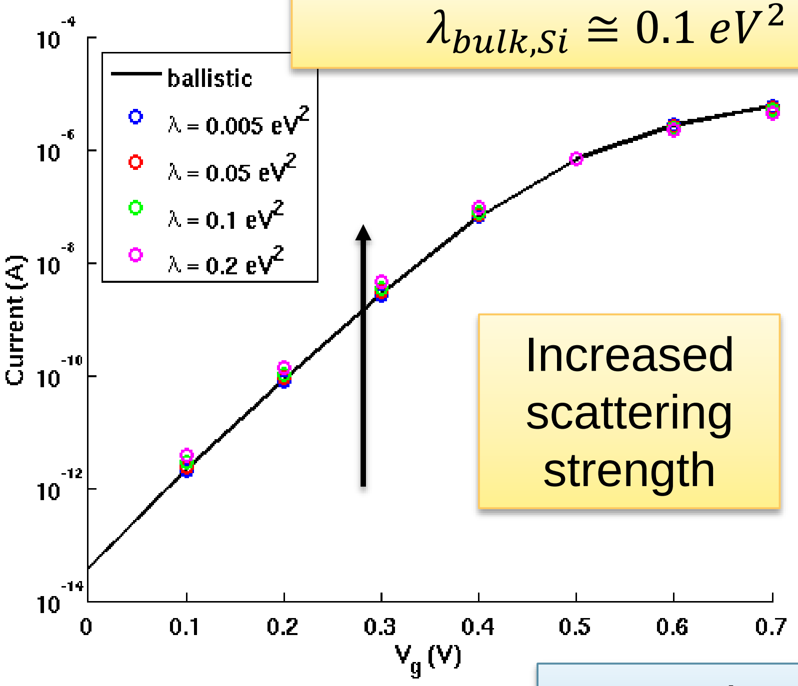


Si, 2nm cross section $sp^3d^5s^*$

NIN nanowire elastic acoustic

$$\lambda = \frac{k_B T D_{ac}^2}{\hbar v_s^2 \rho} \delta_{\alpha,}$$

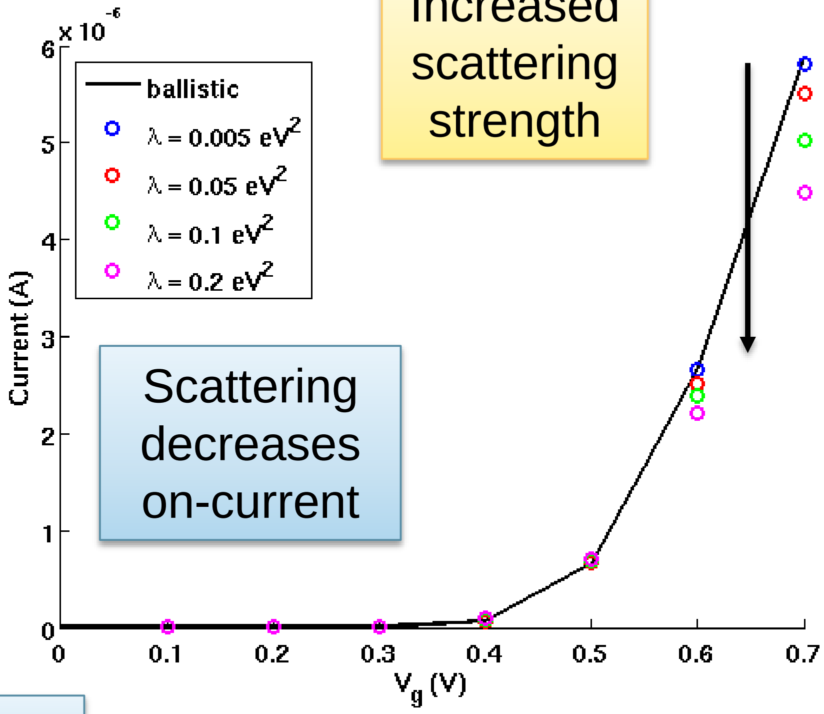
$$\lambda_{bulk,Si} \cong 0.1\ eV^2$$



Increased scattering strength

Log

Scattering increases subthreshold current



Increased scattering strength

Scattering decreases on-current

Linear

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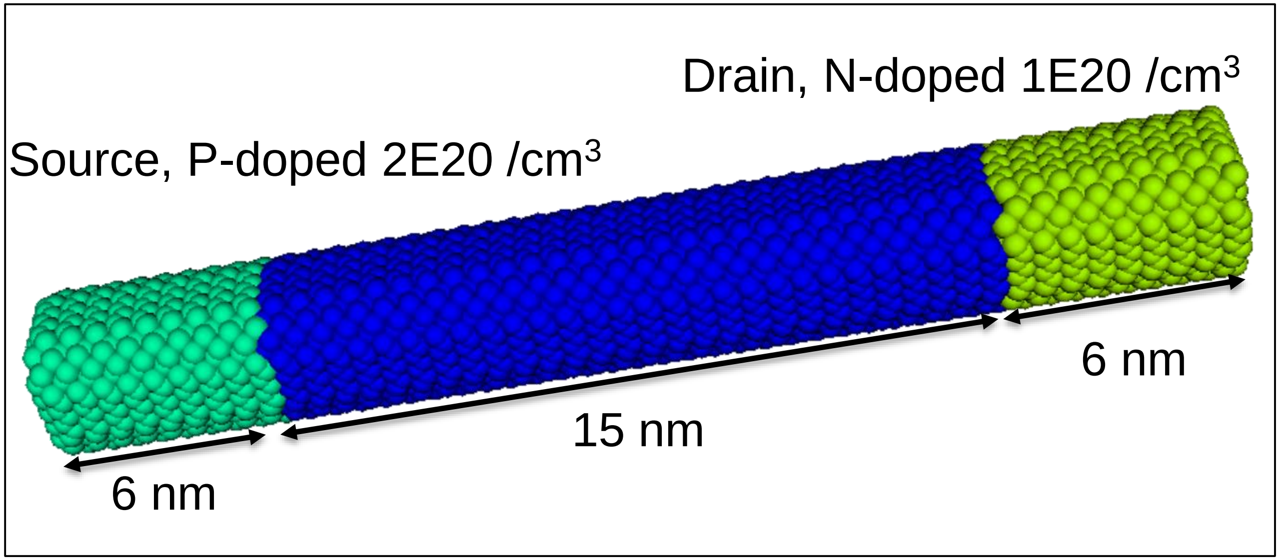
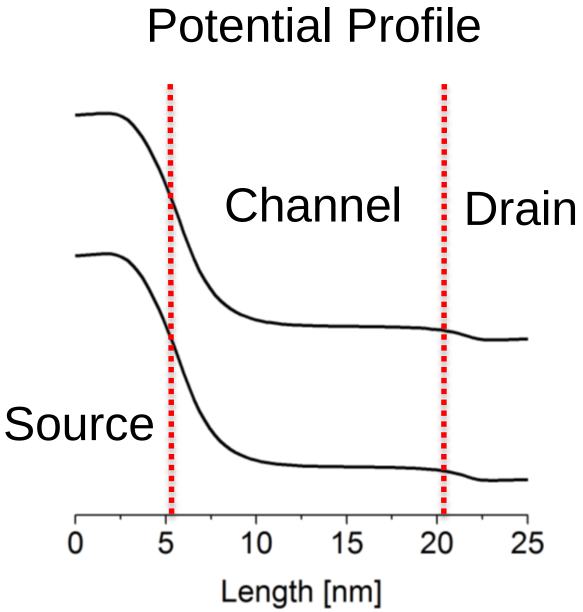
TFET Scattering in NEMO5

Question:
What is the impact of scattering on TFETs?

Circular Si TFET 3nm cross section



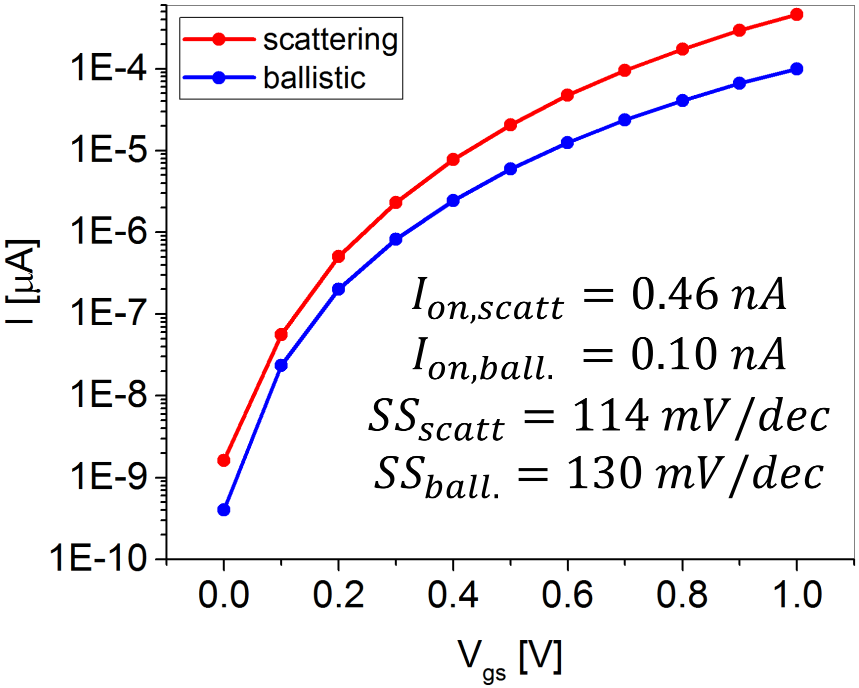


1nm thick oxide covers entire device (not shown)
V_{ds} = 1.0V

Deformation potential phonons included:
Elastic acoustic and inelastic optical phonon self-energies

Silicon TFET with scattering

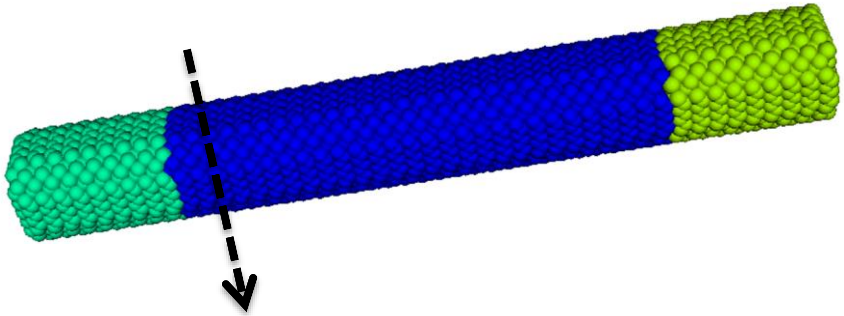
TFET IV characteristics



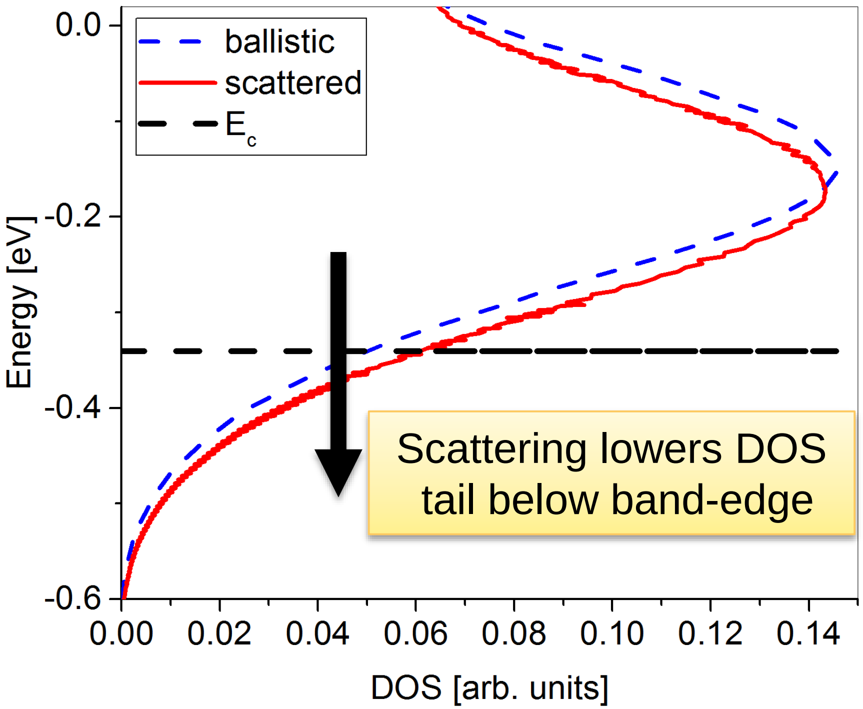
$$I_{on,scatt} = 0.46\,nA$$
$$I_{on,ball.} = 0.10\,nA$$
$$SS_{scatt} = 114\,mV/dec$$
$$SS_{ball.} = 130\,mV/dec$$

Impact of incoherent scattering:
- Increase and shift of band tails
- Increase of tunneling current by ~4x

Realistic TFET performance prediction questionable without scattering



Density of states along cut



Scattering lowers DOS tail below band-edge

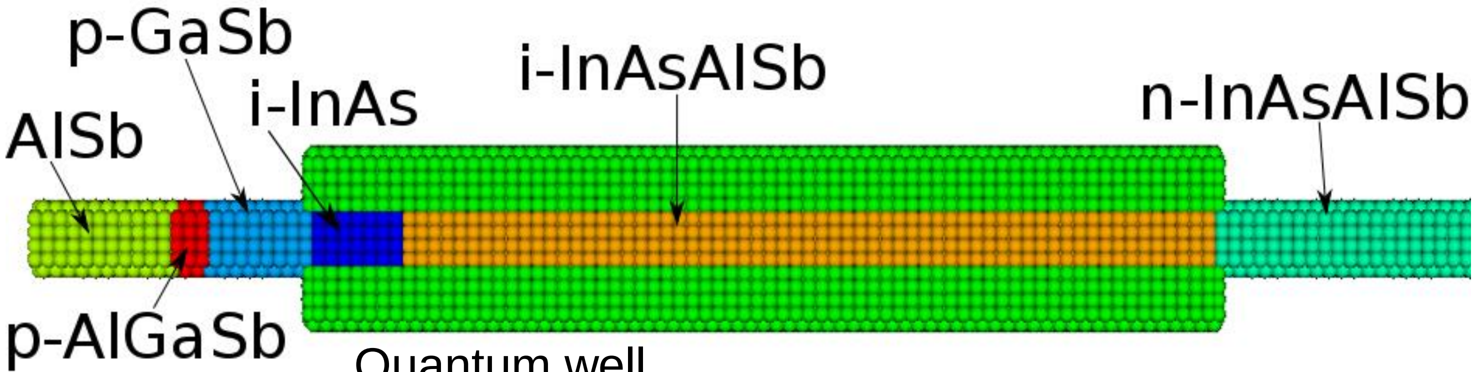
III-V ultra-thin body TFET

Question:
What is the impact of scattering on a III-V resonant TFET?

Dominant scattering in III-Vs is polar optical phonon (POP) scattering

Assumptions:
Treat POP scattering as diagonal (non-polar optical) with increased scattering strength. Why? It is numerically feasible but loses nonlocality information.

TFET design by Pengyu Long, et. al. DRC (2016).



Graded source

Quantum well
(further energy filtering
by resonance filtering)

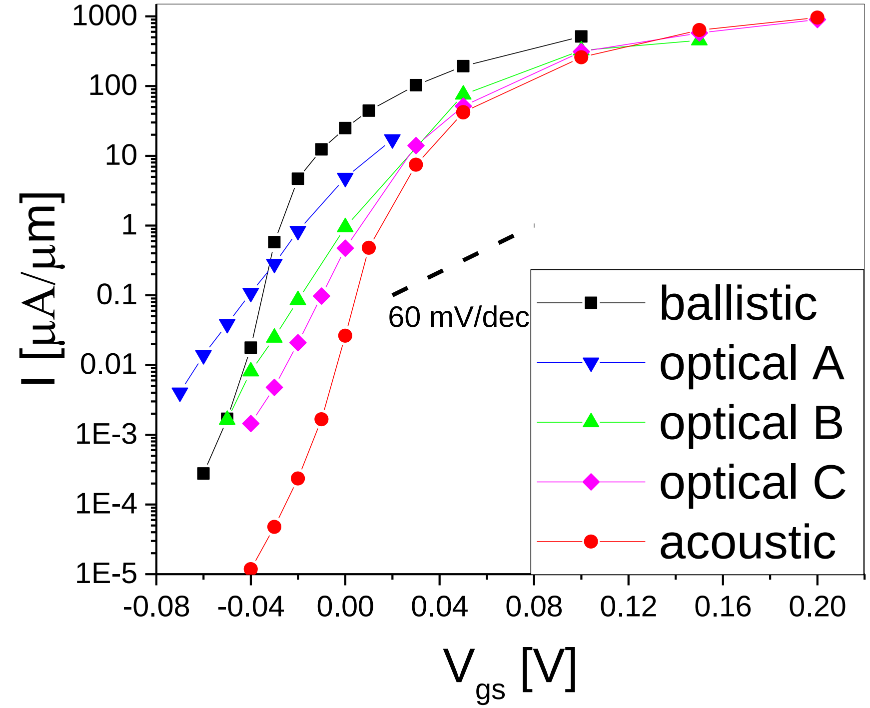
Scattering used to help assess design feasibility

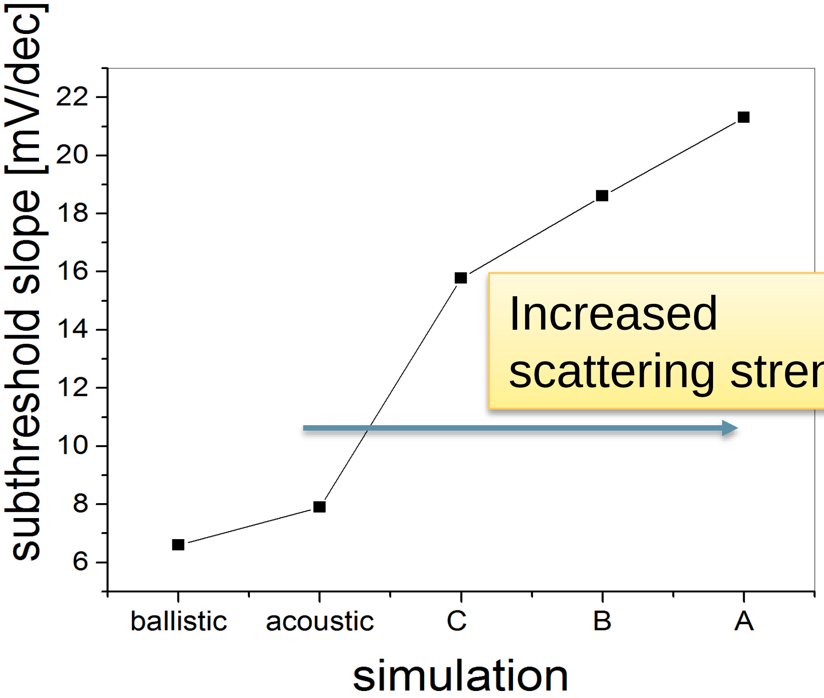
Scattering Effective on Resonance TFET

Optical phonon energy 35 meV

	Strength
ballistic	
Optical A	Optical C * 4
Optical B	Optical C * 1.85
Optical C	Si non-polar def. const.
acoustic	Bulk GaAs parameters

Simulations ran by Devin Verrick





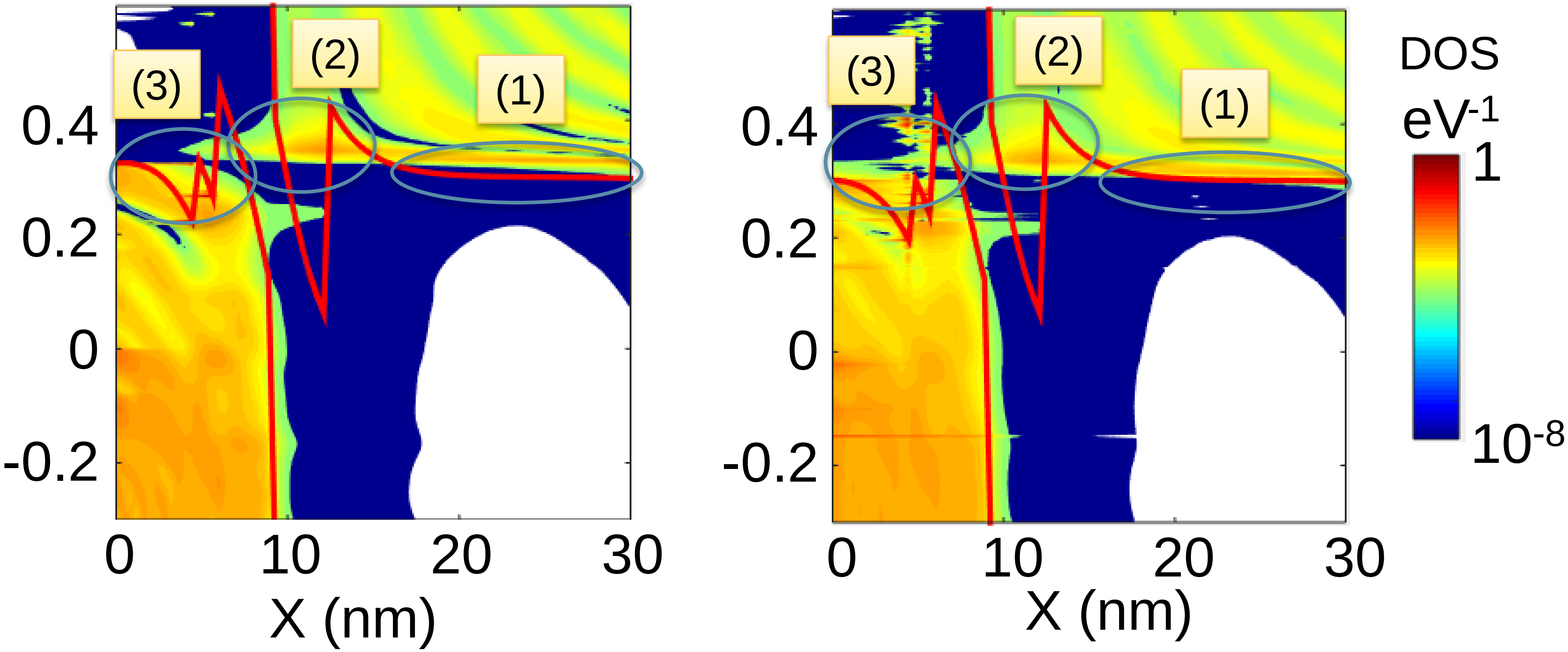
Increased scattering strength

Scattering degrades subthreshold slope

Why does the S.S degrade?

Similar to ballistic – scattering weak

Simulations and figures by Devin Verrick



Acoustic

Acoustic + NPO 220eV/nm

Inelastic scattering increases:
(1) Tunneling below band edge
(2) Penetration of resonance state into bandgap
(3) Coupling of source hole states to resonance states

Overview

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Approximations for Σ^R_{scatt}

Approximation I: Neglect $Re\{\Sigma^R\}$ completely. Solve $\Sigma^>$ and $\Sigma^<$

$\Sigma^R \cong \frac{1}{2}(\Sigma^> - \Sigma^<)$ then neglecting the principal value integral.

broadening only

Approximation II: Keep part of $Re\{\Sigma^R\}$ by solving Σ^R and $\Sigma^<$ then neglecting the (partial) principal value integral.

broadening + part of energy shift

Full: Solve principal value integral

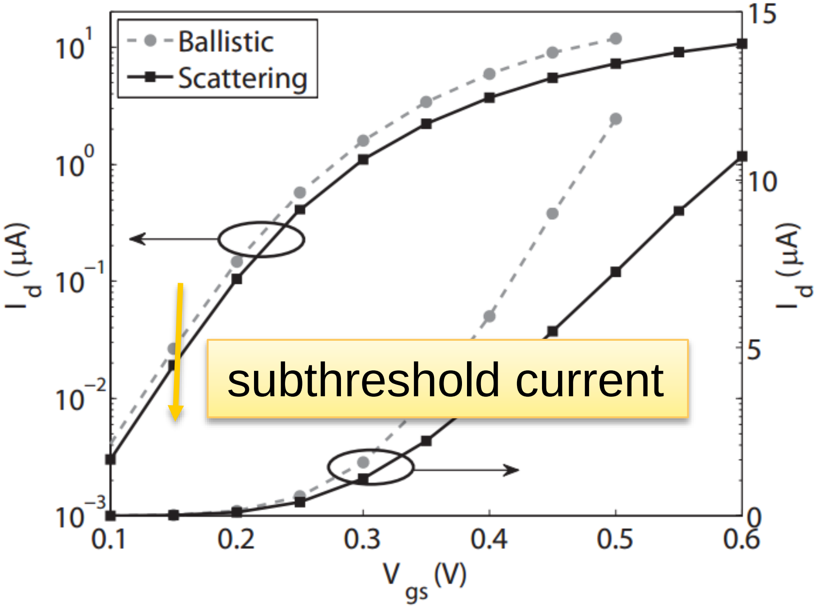
broadening + energy shift

Using Approximation II is a compromise between efficiency and accuracy

Comparing Results from Literature

Approximation I

M. Luisier and G. Klimeck Phys. Rev. B **80**, 155430 (2009).



subthreshold current

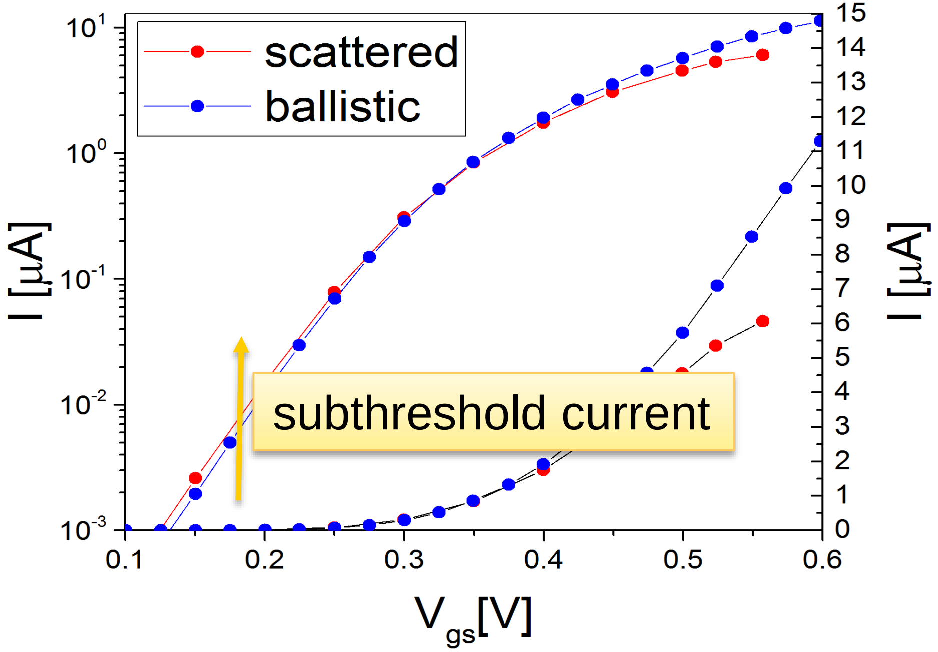
3 nm circular silicon nanowire, V_{ds} =0.6V

$sp^3d^5s^*$ tight binding basis

confined phonon model

Approximation II

Data from A. Esposito, M. Frey, et. al. JCEL vol. 8 (2009).



subthreshold current

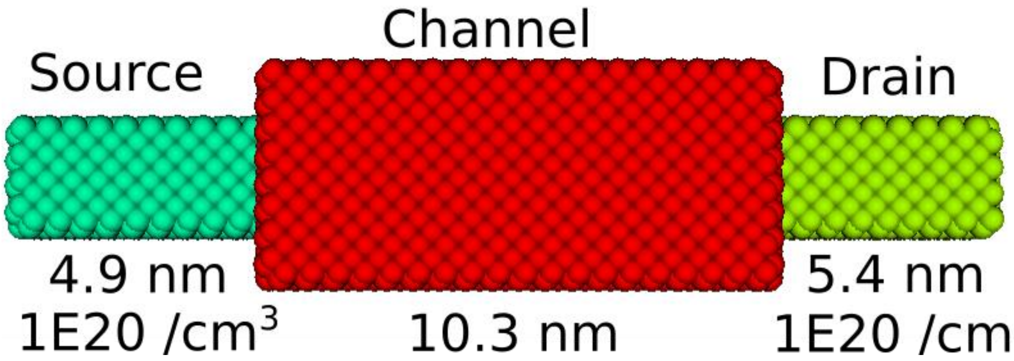
3.2 nm square silicon nanowire, V_{ds} =0.5V

non-parabolic effective mass basis

bulk phonon model

Maybe the difference in approximations made lead to conflicting trends?

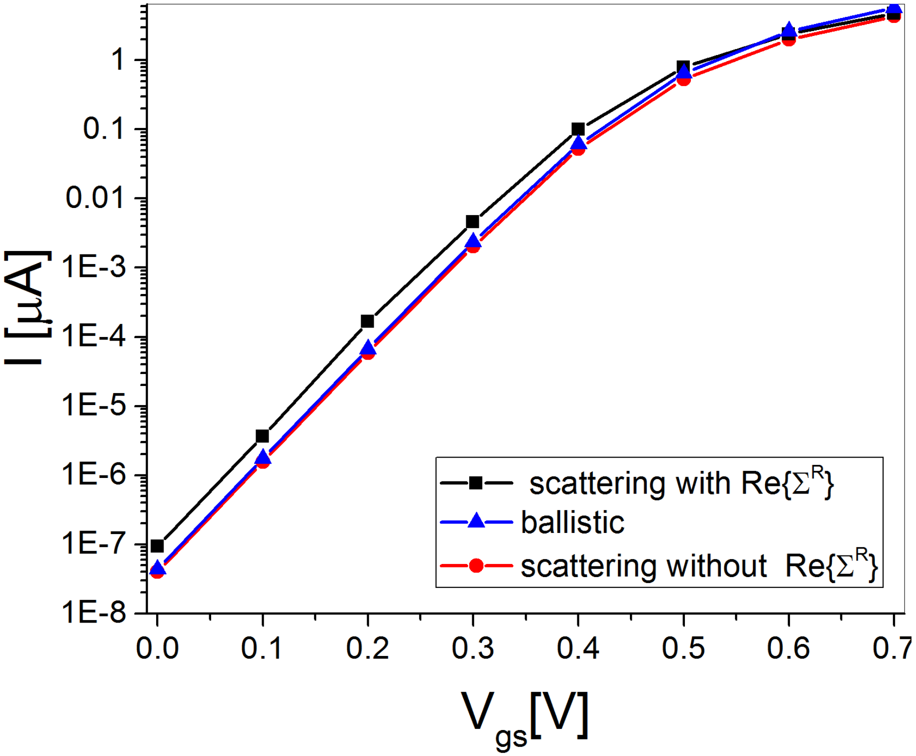
Comparing Approximation I and II for MOSFETs

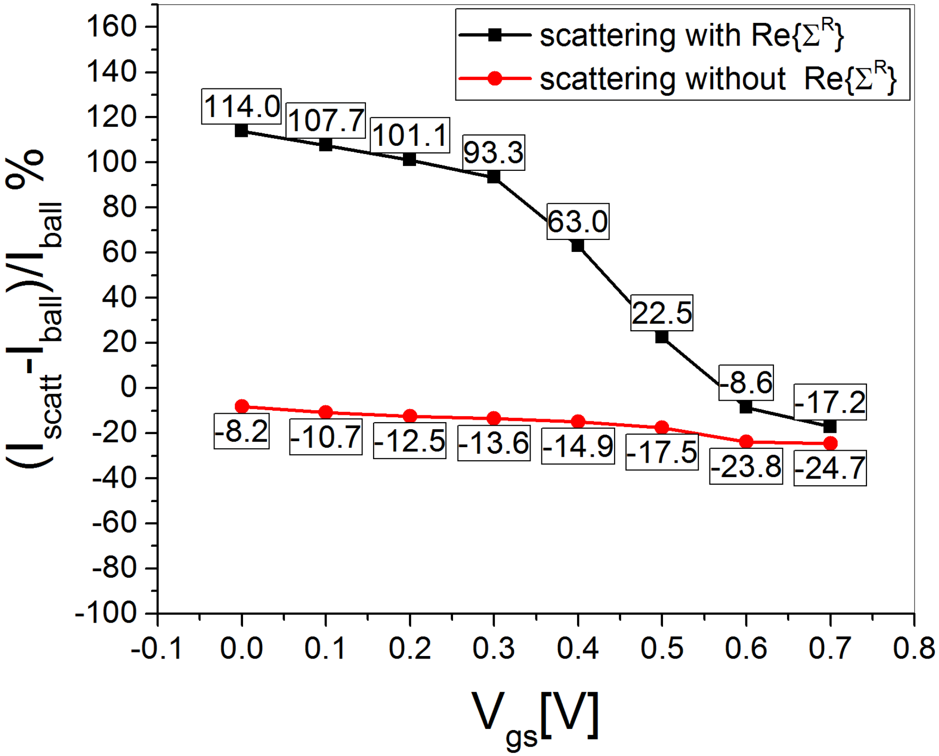


Si, 2nm cross section sp^{3}d^{5}s*

NIN nanowire

Relative Error $(I_{scatt}-I_{ball})/I_{ball}$



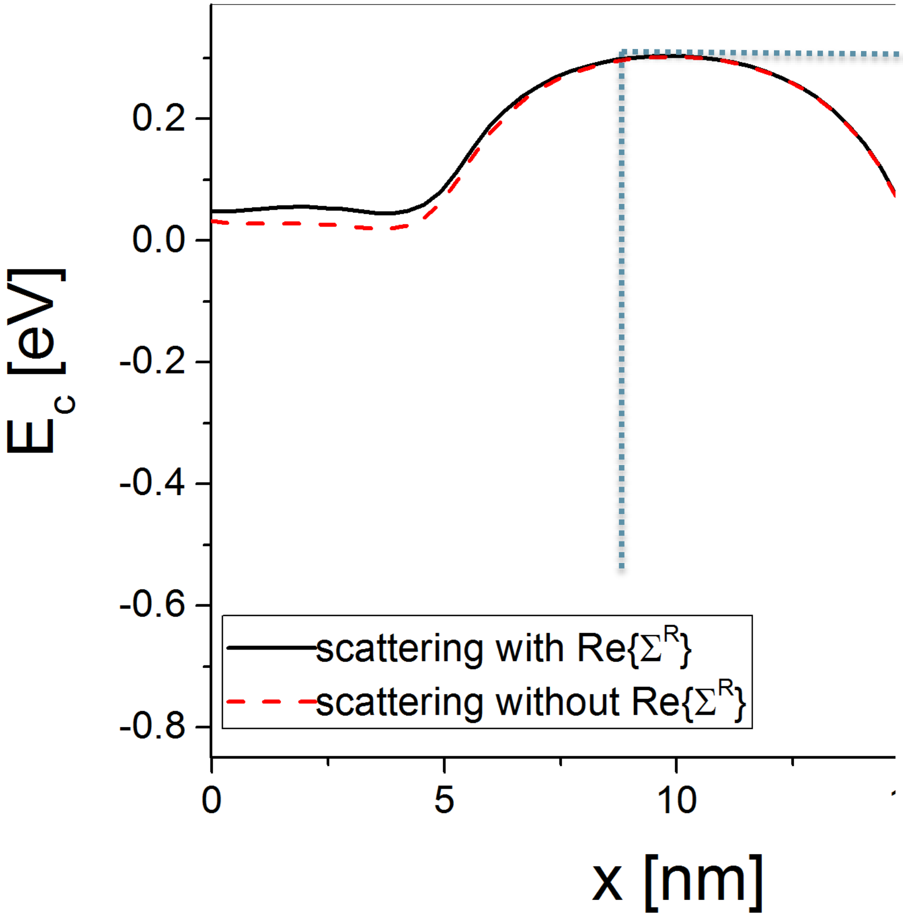


Neglecting $\mathrm{Re}\{\Sigma^R\}$ (Approximation I) underestimates subthreshold current

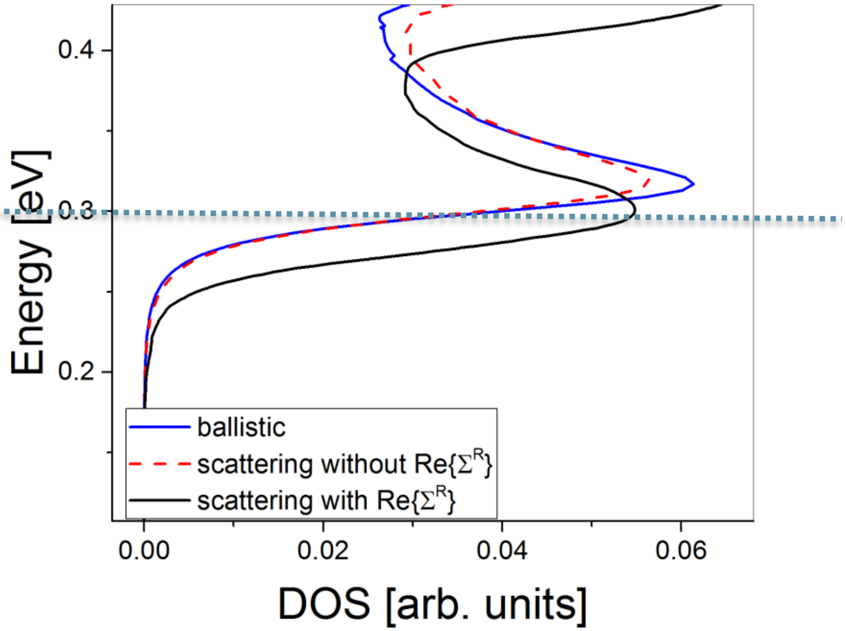
DOS comparison

V_{gs}=0.2V, V_{ds}=0.8V

1D conduction band edge through center of device



DOS lowered beneath band edge

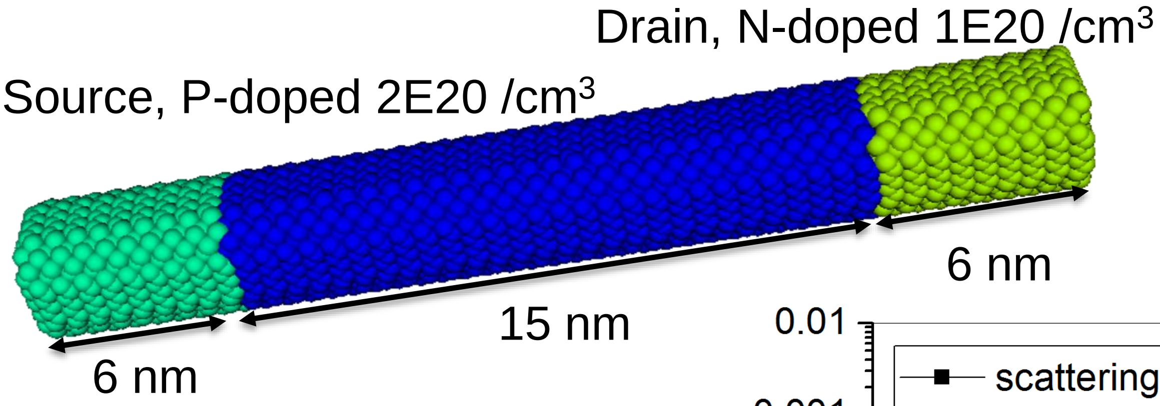


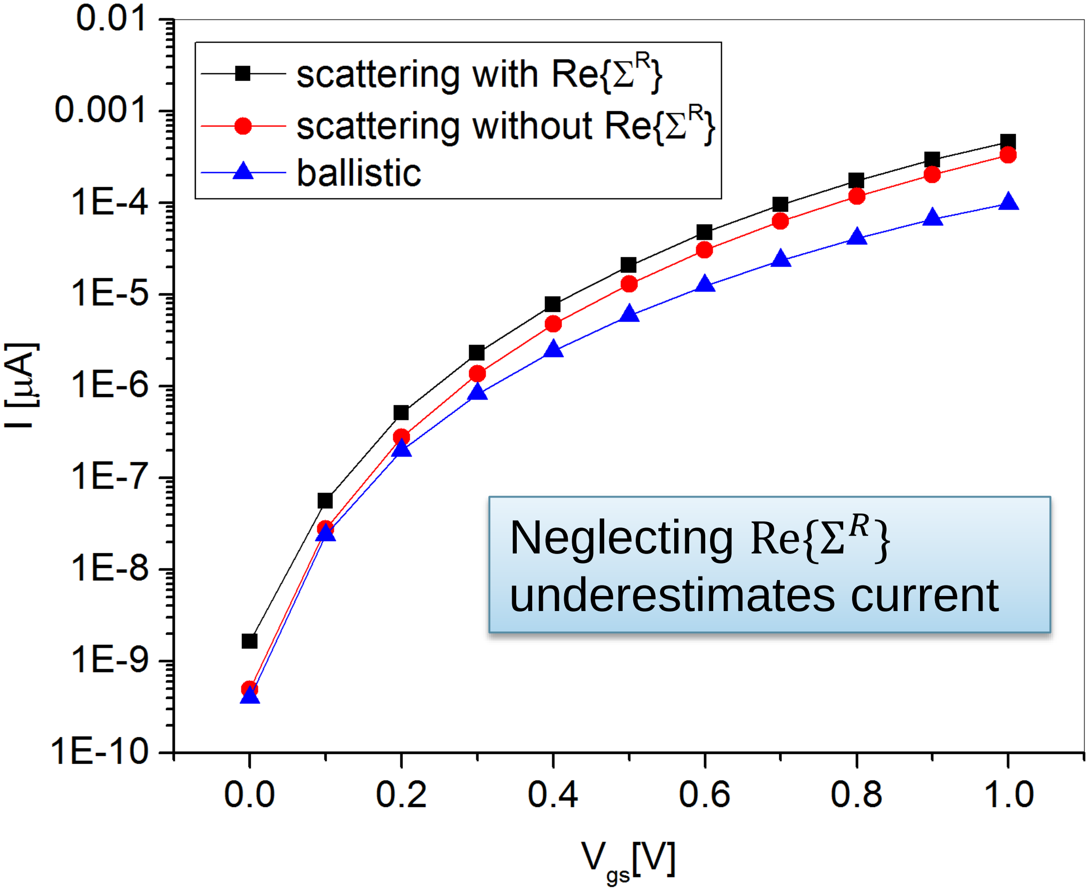
¾ Peak – Full Maximum
Ballistic 38 meV
With $Re\{\Sigma^R\}$ 50 meV
Without $Re\{\Sigma^R\}$ 45 meV

Comparing Approximation I and II for Si TFET

Si, 3nm diameter $sp^3d^5s^*$

PIN nanowire TFET





Neglecting Re{Σ^R} underestimates current

Overview

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7) Conclusions/Future Work

Strong Scaling Results

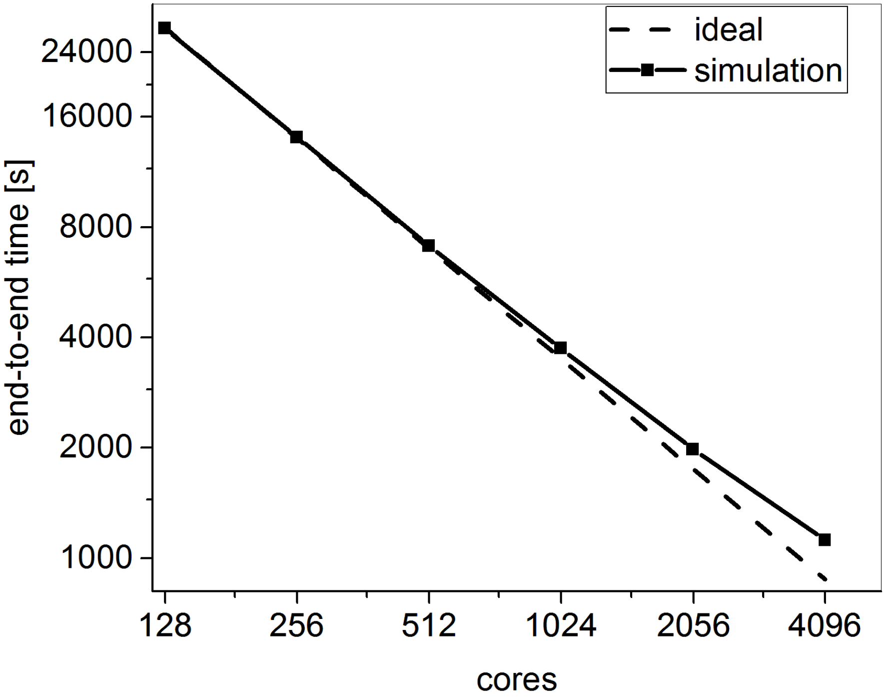
Scattering self-energies requires energies that can be on different MPI processes:

$$\Sigma^{<}(E) = \frac{\hbar}{2\rho\omega_o}\delta(\overrightarrow{x_3} - \overrightarrow{x_4})\,[N_{op}G^{<}(E - E_{op}) + (N_{op} + 1)\,G^{<}(E + E_{op})]$$

Requires communication of diagonal matrices

Note: for UTB simulations there is an additional wave-vector k integral that increases communication

Device: Si TFET used for IV



Reasonable scaling despite complex communication

Other Development Work for Electron-Phonon Scattering in NEMO5

- Stabilized Recursive Green's Function algorithm
- Improved Poisson convergence with improved Jacobian
- Interpolated scattering self-energies to decrease number of scattering iterations needed and improve current conservation
- Implemented dynamical convergence to decrease number of scattering iterations needed
- Current conservation in optical phonon scattering with inhomogeneous energy grid
- Improved resonance mesh suitable for resonant devices

NEMO5

Stabilized Recursive Green’s Function algorithm

Objective:

Efficient implementation of recursive Green’ function (RGF) algorithm suitable for scattering

Problem:

Initial implementation of RGF in NEMO5 following OMEN was unstable when scattering was included.

Approach:

- Systematic analysis of RGF equations to find source of instability.
- Remove assumptions of symmetries only valid with infinite precision
- Preserve symmetry of equations in each recursive iteration

Results/Impact:

Found instabilities and improved RGF algorithm to allow scattering simulations.

Before:

$$\boldsymbol{G^{<}_{i,i} = g^{<}_{i,i} + g^{R}_{i,i} H_{i,i+1} G^{<}_{i+1,i+1} H_{i+1,i} g^{A}_{i,i} + g^{R}_{i,i} H_{i,i+1} G^{R}_{i+1,i+1} H_{i+1,i} g^{<}_{i,i}}$$

$$\boldsymbol{-\left(g^{R}_{i,i} H_{i,i+1} G^{R}_{i+1,i+1} H_{i+1,i} g^{<}_{i,i}\right)^{\dagger}}$$

After:

$$\boldsymbol{G^{<}_{i,i} = g^{<}_{i,i} + g^{R}_{i,i} H_{i,i+1} G^{<}_{i+1,i+1} H_{i+1,i} g^{A}_{i,i} + g^{R}_{i,i} H_{i,i+1} G^{R}_{i+1,i+1} H_{i+1,i} g^{<}_{i,i}}$$

$$\boldsymbol{+\left(g^{R}_{i,i} H_{i,i+1} G^{R}_{i+1,i+1} H_{i+1,i} (g^{<}_{i,i})^{\dagger}\right)^{\dagger}}$$

Additionally: $\boldsymbol{G^{<}_{i,i}}$ is anti-symmetrized each iteration.

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Improved Convergence with improved Jacobian

Objective:

Convergence of NEGF-Poisson equations with minimum number of iterations

Problem:

Ballistic Jacobian typically used in NEMO5 is not suitable for scattering

Approach:

- Balance between number of iterations needed and calculation time of Jacobian
- Found best balance is to use a mixture of ballistic Jacobian (extra NEGF solution) and approximate scattering Jacobian

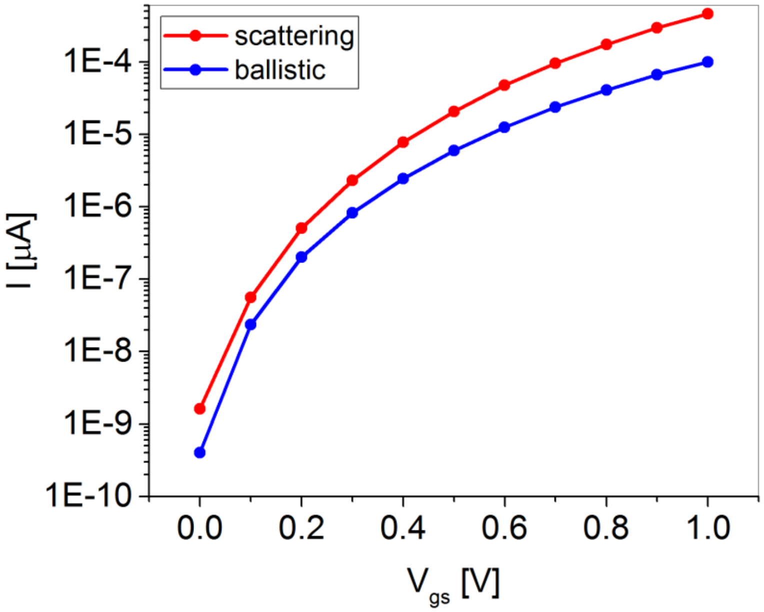
Results/Impact:

Convergence achieved for previously not converging simulations.

Jacobian where λ is a mixing parameter

$$J(\vec{x}) = \Im\left\{\lambda \int G^{<}_{ballistic}\left(\vec{x}, E, \frac{\partial f_{S,D}}{\partial E}\right) dE + \int (1-\lambda) G^{<}_{scattered}\left(\vec{x}, E, f_{S,D}\right) dE\right\}$$

Si circular nanowire TFET



Interpolate Scattering Self-energy

Objective:

Minimum number of self-consistent Born approximation (SCBA) iterations to reach converged result

Problem:

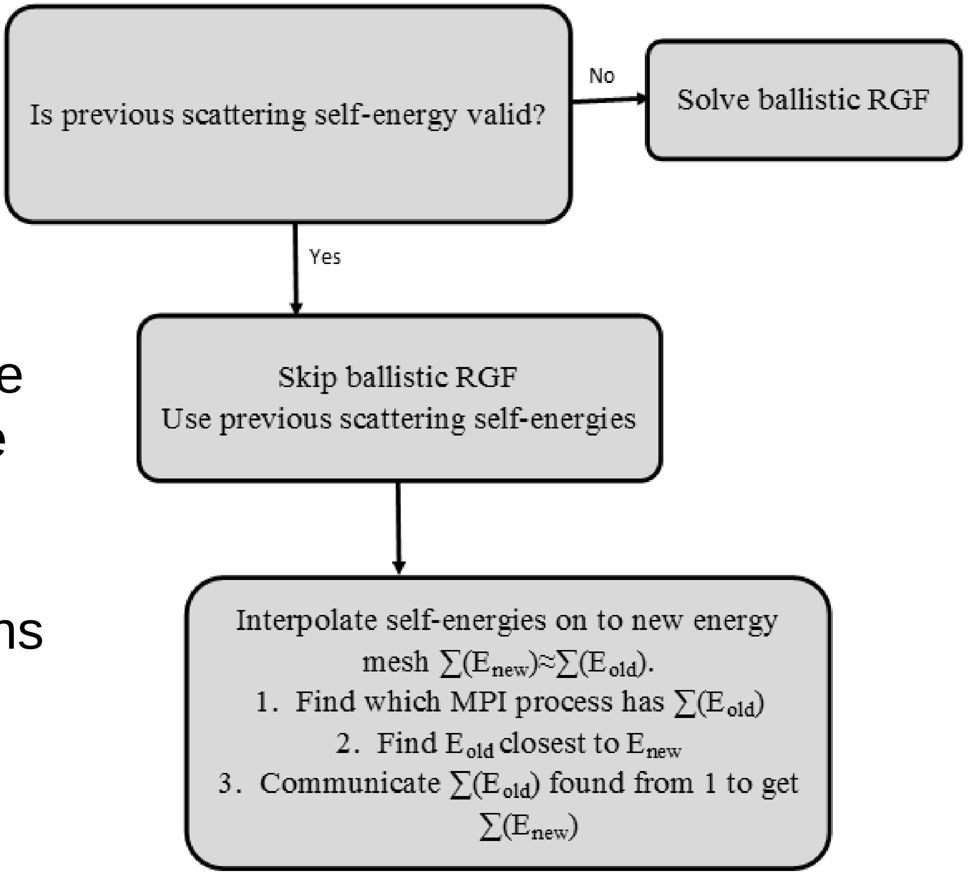
Typically for current conservation, self-consistent Born needs 20-40 computationally expensive iterations

Approach:

- Reuse previously converged SCBA results to accelerate convergence of updated Poisson potentials

Results/Impact:

- Discovered that when Poisson-NEGF loop is close to convergence previously scattered results can be interpolated on to the updated energy mesh
- Reduced number of SCBA iterations by about 3



Interpolate self-energies on to new energy mesh $\sum(E_{new}) \approx \sum(E_{old})$.

1. Find which MPI process has $\sum(E_{old})$
2. Find E_{old} closest to E_{new}
3. Communicate $\sum(E_{old})$ found from 1 to get $\sum(E_{new})$

Dynamical Convergence Criterion for Self-consistent Born Loop

Objective:

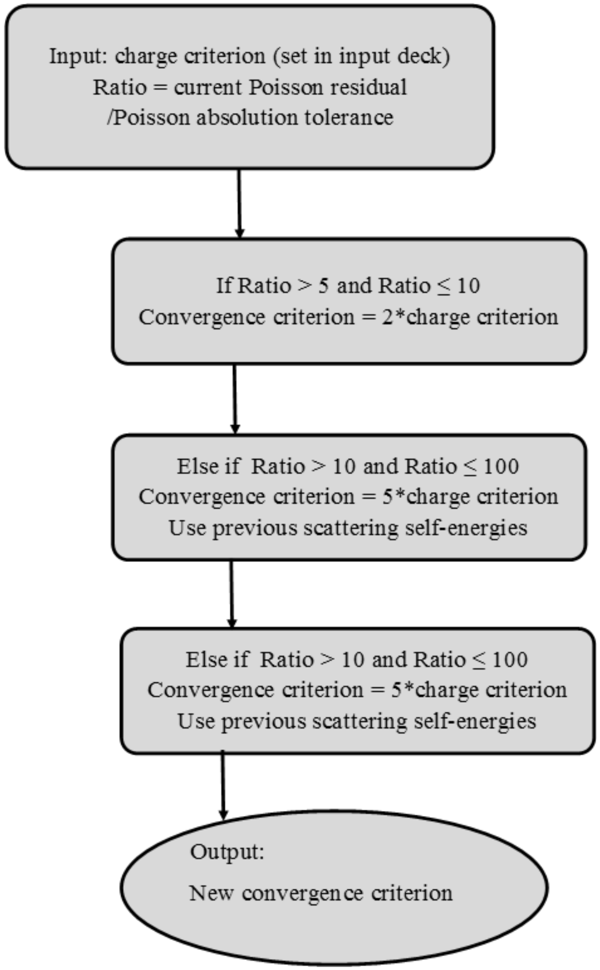
Minimum number of self-consistent Born approximation (SCBA) iterations to reach converged result.

Problem:

Typically for current conservation, self-consistent Born needs 20-40 computationally expensive iterations

Approach:

- Reduce the number of SCBA iterations without making additional approximations



Input: charge criterion (set in input deck)
Ratio = current Poisson residual
/Poisson absolution tolerance

If Ratio > 5 and Ratio ≤ 10
Convergence criterion = 2*charge criterion

Else if Ratio > 10 and Ratio ≤ 100
Convergence criterion = 5*charge criterion
Use previous scattering self-energies

Else if Ratio > 10 and Ratio ≤ 100
Convergence criterion = 5*charge criterion
Use previous scattering self-energies

Output:
New convergence criterion

Results/Impact:

Reduced number of SCBA iterations by approximately half

Current conservation in optical phonon scattering with inhomogeneous energy grid

Objective:

Current conservation criterion for converged self-consistent Born results

Problem:

For efficient simulations, an inhomogeneous energy mesh must be used but the energy mesh will not be commensurate with phonon energies, thus current conservation is not trivial.

Approach:

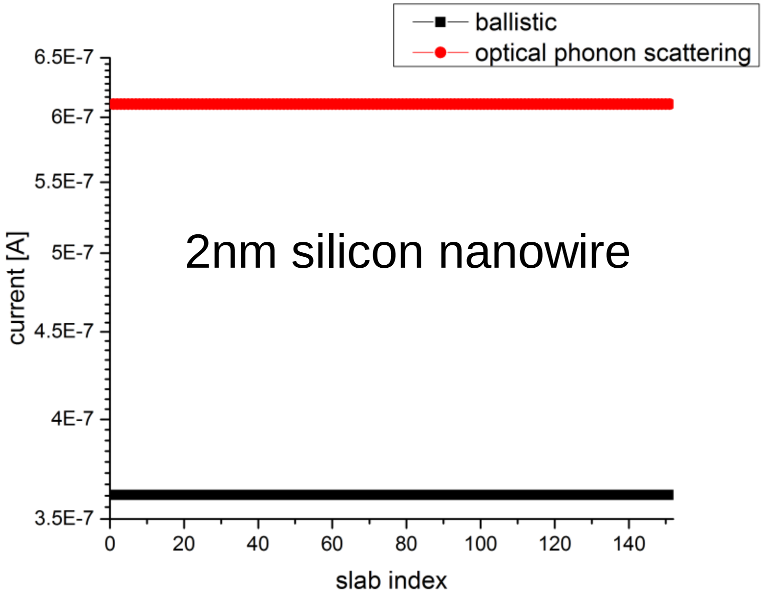
- Ensure detailed balance is always met and use this constraint to form constraints on interpretation of scattering self-energies

$$\int \left(\Sigma^{<}(E)G^{>}(E) - G^{<}(E)\Sigma^{>}(E)\right)dE = 0$$

“in-scattering must balance out-scattering”

Results/Impact:

Current Conservation with general energy mesh



Improved Convergence with improved Jacobian

Objective:

Convergence of NEGF-Poisson equations with minimum number of iterations

Problem:

Scattering introduces resonance shifts that must be properly resolved

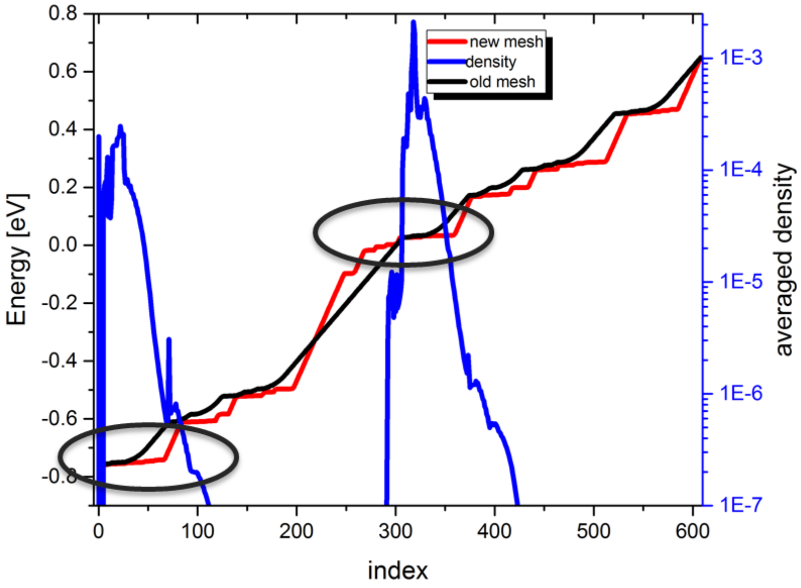
Approach:

- Use device information in order to resolve resonances due to scattering
- Adapt energies to shifts in resonances

Results/Impact:

- Improved convergence of NEGF-Poisson loop.
- Resonances due to scattering are properly resolved

Si circular nanowire TFET



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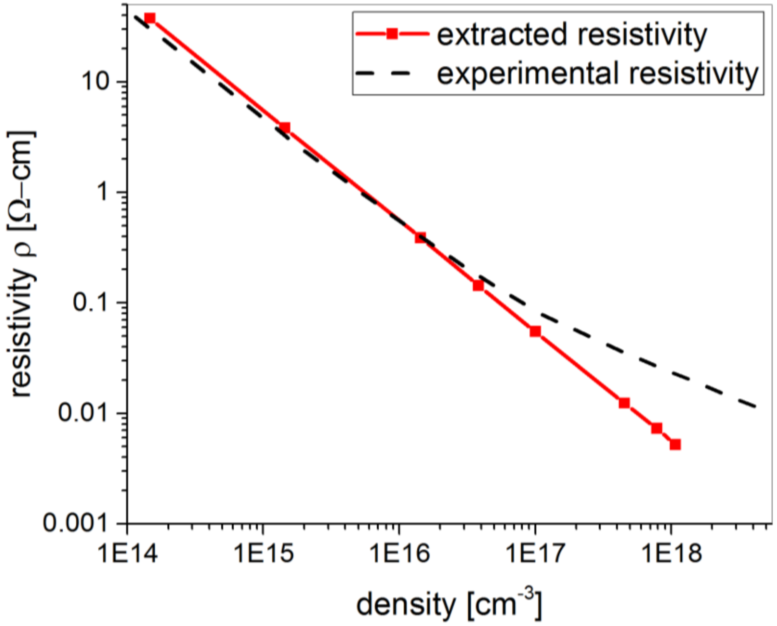
6) **Conclusions/Future Work**

Conclusions

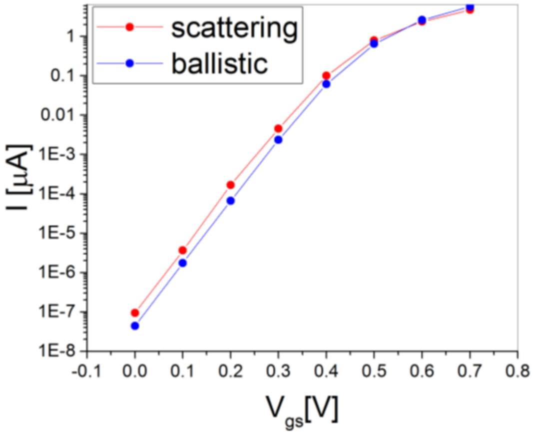
- Efficient implementation of scattering introduced
- Verification of implementation with comparison to Fermi’s golden rule and to experimental resistivity



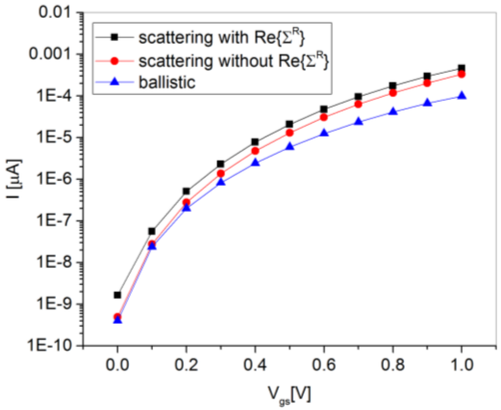
- Effect of certain approximations made in literature assessed

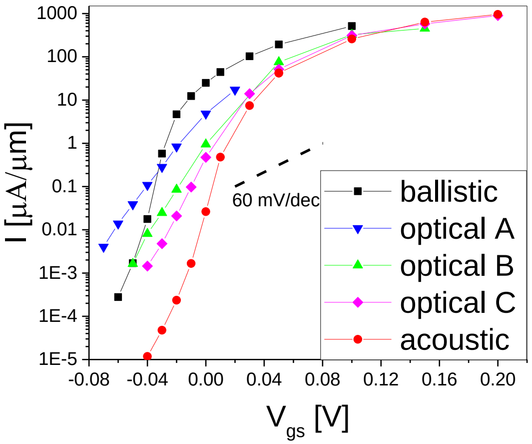
Conclusions cont.

- MOSFET IV results with and without scattering



- TFET IV results with and without scattering compared





Future Work

- Further assessment of approximations made e.g. local POP, bulk phonons.
- Comparison to heuristic models e.g. Klimeck's 1994 model "equilibrium-nonequilibrium" model
- Include other scattering mechanisms as scattering self-energies (e.g. roughness)
- Scattering model (phonons, roughness etc.) suitable for 2D materials e.g. TMDs

Acknowledgements/Questions

Thanks to:

my committee members: Professors Gerhard Klimeck, Supriyo Datta, Tillmann Kubis

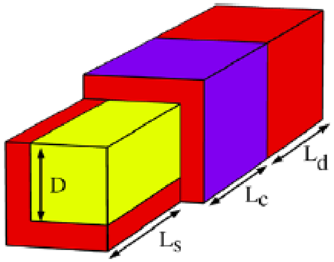
Administrative Staff

My groupmates and friends

Questions?

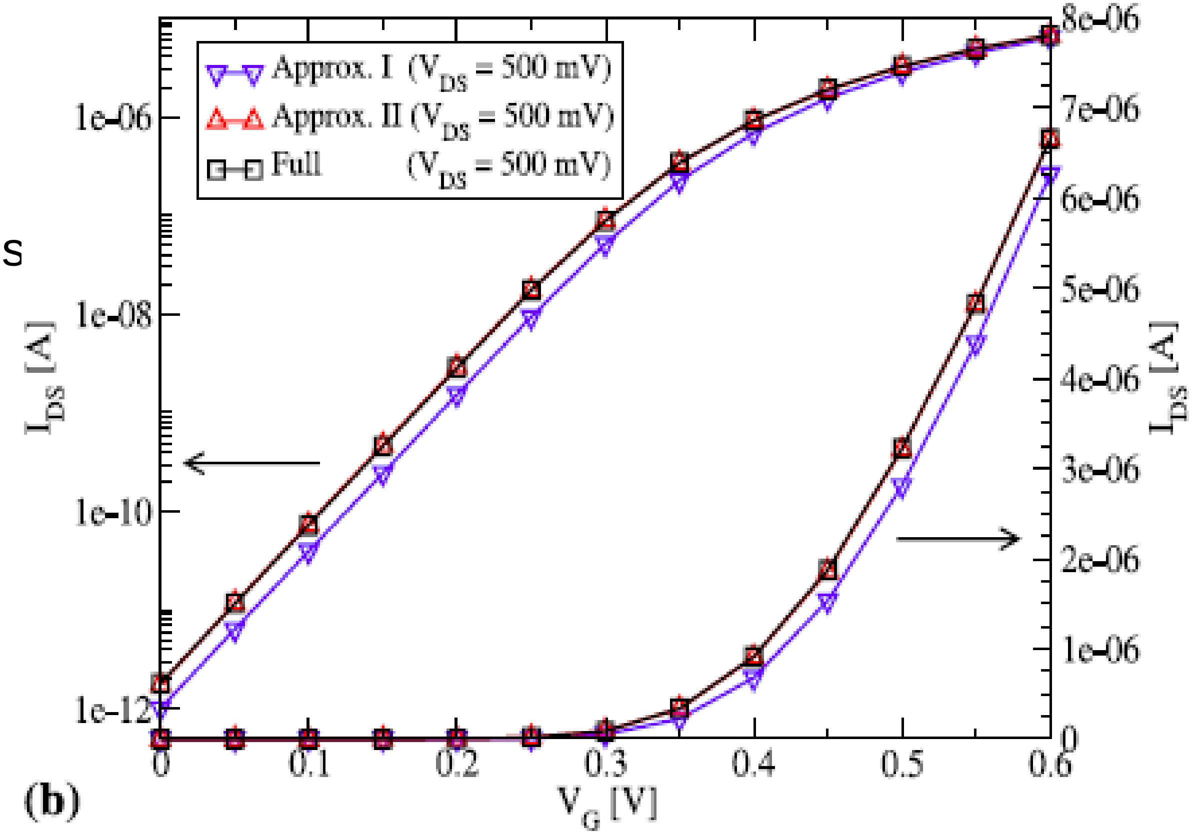
Backup Slides

Qualitative Comparison to Literature



Si Nanowire in effective mass
Lc = 15 nm, D = 3.26 nm

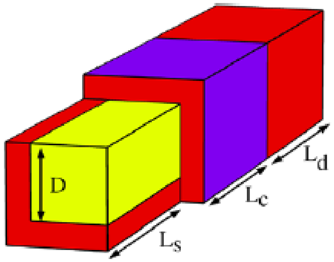
Approx. I – $Re\{\Sigma^R\} = 0$
Approx. II - $Re\{\Sigma^R\} \cong 0$
But neglect PVI
Full – includes PVI



Neglecting $Re\{\Sigma^R\}$ leads to underestimation of off-current.
NEMO5 uses Approx. II

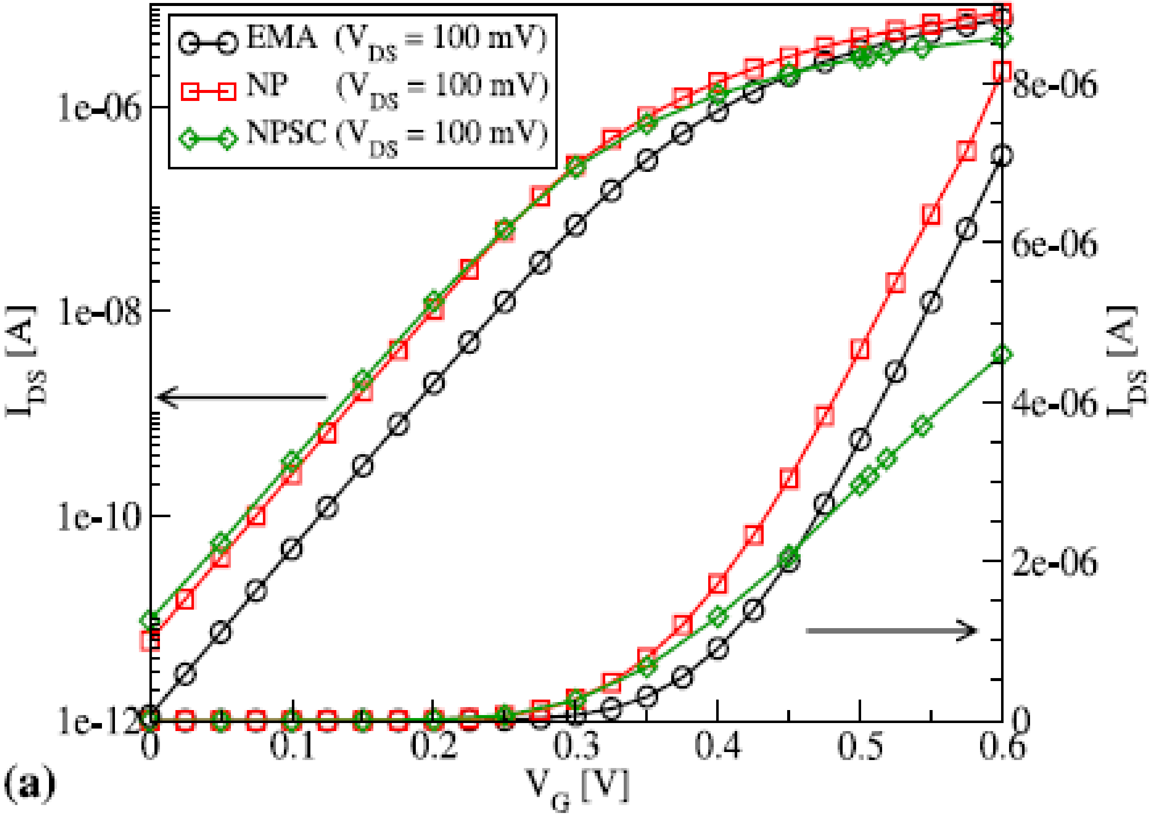
Esposito, Frey J.Comput Electron (2009).

Qualitative comparison to literature



Si Nanowire in effective mass
Lc = 15 nm, D = 3.26 nm

NP – ballistic nonparabolic EM
NPSC –
Nonparabolic EM + Approx II
$Re\{\Sigma^R\} \cong 0$ But neglect PVI



Same trend as seen in NEMO5!

Esposito, Frey J.Comput Electron (2009).

Acoustic Phonon Scattering Self-energies

$$\Sigma^{<}_{ac}(\alpha, \beta, E) = \frac{D^2 k_b T}{\rho v_s^2} \delta_{\alpha,\beta} G^{<}(\alpha, \beta, E)$$

*"Quantum Transport in Semiconductor Nanostructures" , T. Kubis PhD thesis (2009).

Optical Phonon Scattering Self-energies

N_{op} is independent of q (flat optical phonon band)
Long wavelength limit $q \to 0$
Discrete energies for emission and absorption $(E \pm E_{op})$ where $E_{op} = \hbar\omega_o$

emission absorption

$$\Sigma^{<}(E) = \frac{\hbar}{2\rho\omega_o}\delta(\overrightarrow{x_3} - \overrightarrow{x_4})\,[N_{op}G^{<}\left(E - E_{op}\right) + (N_{op} + 1)\,G^{<}\left(E + E_{op}\right)]$$

$$\Sigma^{R}(E) = \frac{\hbar}{2\rho\omega_o}\delta(\overrightarrow{x_3} - \overrightarrow{x_4})\,[N_{op}G^{R}\left(E + E_{op}\right) + (N_{op} + 1)\,G^{<}(E - E_{ac}) + \frac{1}{2}G^{<}(E - E_{ac}) - \frac{1}{2}G^{<}(E + E_{ac})]$$

Neglecting principal value integral

*"Quantum Transport in Semiconductor Nanostructures" , T. Kubis PhD thesis (2009).

Approximations for Scattering Real/Imag. Part

Approximation I:

$$\Sigma^R(E) = \frac{1}{2}\left(\Sigma^>(E) - \Sigma^<(E)\right) + \cancel{iP\int \frac{dE'}{2\pi}\frac{\left(\Sigma^>(E') - \Sigma^<(E')\right)}{E - E'}}$$

neglect

emission

absorption

Approximation II:

$$\Sigma^<(E) = \frac{\hbar}{2\rho\omega_o}\delta(\overrightarrow{x_3} - \overrightarrow{x_4})\,[N_{op}G^<\left(E - E_{op}\right) + (N_{op} + 1)\, G^<\left(E + E_{op}\right)]$$

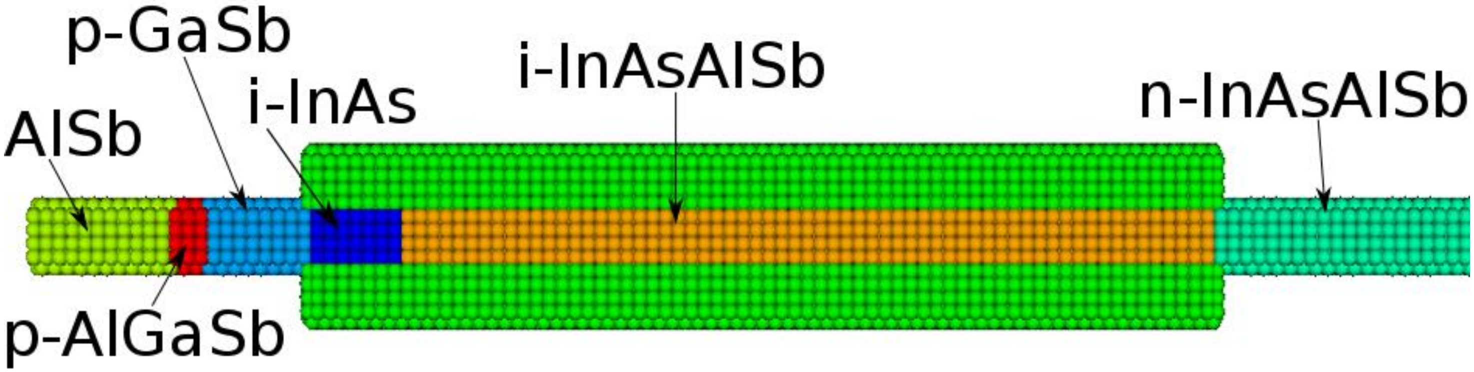
$$\Sigma^R(E) = \frac{\hbar}{2\rho\omega_o}\delta(\overrightarrow{x_3} - \overrightarrow{x_4})\,[N_{op}G^R\left(E + E_{op}\right) + (N_{op} + 1)\, G^<(E - E_{ac})$$

$$+ \frac{1}{2}G^<(E - E_{ac}) - \frac{1}{2}G^<(E + E_{ac})]$$

$$\cancel{+iP\int \frac{dE'}{2\pi}\left(\frac{G^<(E - E')}{E' - \hbar\omega_o} - \frac{G^<(E - E')}{E' - \hbar\omega_o}\right)}$$

neglect

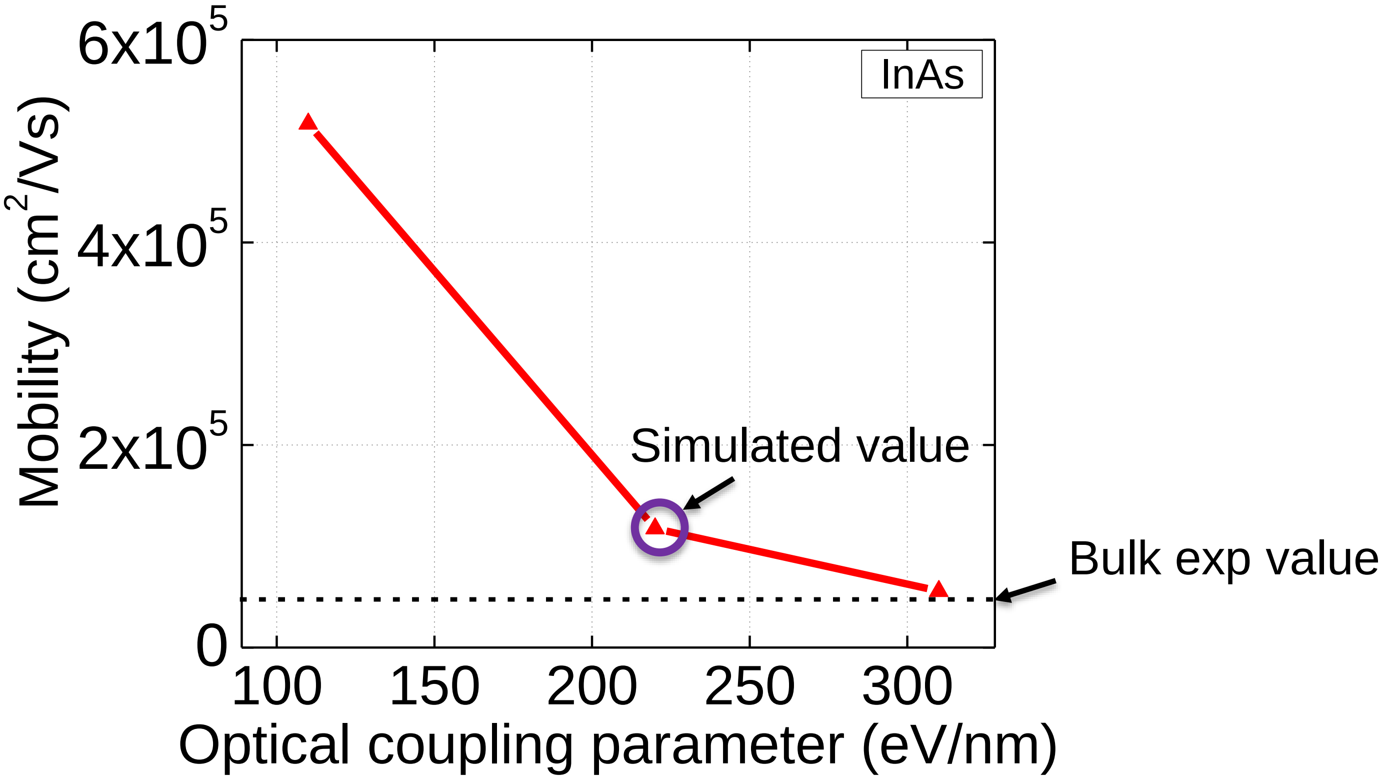
*"Quantum Transport in Semiconductor Nanostructures" , T. Kubis PhD thesis (2009).



Region-Material	Length [nm]	Doping [cm-3]
1 - AlSb	4.57	$3x10^{19}$
2 - $Al_{0.5}Ga_{0.5}Sb$	1.2	$6x10^{19}$
3 - GaSb	3.2	$5x10^{19}$
4 - InAs	3.4	$1x10^{15}$
5 - AlInAsSb	27.1	$1x10^{15}$
6 - AlInAsSb	17.3	$5x10^{19}$

Mobility calculations

Simulations and figures by Devin Verrick



Scattering strength still too weak by ~2.