Erlang/Gamma Distribution

Process goes through $r$ sequential phases each of which has identical exponential distribution.

Then, overall time spent by process in all phases follows Erlang distribution.

$r$-stage Erlang

$$f(t) = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{(r-1)!}, \quad t > 0, \lambda > 0, \quad r = 1, 2, \ldots$$

$$F(t) = 1 - e^{-\lambda t} \sum_{k=0}^{r-1} \frac{(\lambda t)^k}{k!}$$

Consider a system that can withstand less than $r$ peak stresses. Stresses occur as a random Poisson process.

Exponential ($\lambda$) $\equiv$ Erlang ($\lambda, 1$)
Erlang / Gamma Distribution

If \( r \) in Erlang can take non-integral values, then distribution is called Gamma.

\[
f(t) = \frac{\lambda^r}{\Gamma(r)} t^{r-1} e^{-\lambda t}
\]

where \( \Gamma(r) = \int_0^\infty x^{r-1} e^{-x} \, dx \) (gamma function)
Hyperexponential Distribution

If process goes through only one of several alternate phases and each phase is exponential, then overall time follows hyperexponential distribution.

\[ f(t) = \sum_{i=1}^{k} \alpha_i \lambda_i e^{-\lambda_i t} \]

(k phases, \( \sum \alpha_i = 1 \))

\[ F(t) = \sum_{i=1}^{k} \alpha_i (1 - e^{-\lambda_i t}) \]

\[ h(t) = \frac{f(t)}{R(t)} = \frac{\sum \alpha_i \lambda_i e^{-\lambda_i t}}{\sum \alpha_i e^{-\lambda_i t}} \]

DFR with :
Weibull Distribution

Most widely used parametric family of failure distributions.
Proper choice of shape parameter can make it a IFR, DFR or CFR distribution.

\[ f(t) = \lambda \alpha t^{\alpha - 1} e^{-\lambda t^\alpha} \]
\[ F(t) = 1 - e^{-\lambda t^\alpha} \]
\[ h(t) = \frac{\lambda \alpha t^{\alpha - 1} e^{-\lambda t^\alpha}}{e^{-\lambda t^\alpha} - 1} = \lambda \alpha t^{\alpha - 1} \]

\[ \text{EXP}(\lambda) = \text{WEI}(\lambda, 1) \]

\[ h(\alpha = 2) = 2t \]

\[ \lambda = 1 \]
\[ \alpha = 0.5 \]
\[ \alpha = 1 \]
\[ \alpha = 1.5 \]
\[ \alpha = 2 \]
Normal or Gaussian Distribution

Central limit theorem gives that if you have \( n \) mutually independent random variables, then a sample of them will have a mean which is normally distributed as \( n \to \infty \).

Examples:

- Component lifetime in wear-out phase.
- Errors in measurements

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2}, \quad -\infty < x < \infty
\]

\[X \sim N(\mu, \sigma^2)\]

Standard normal distribution \( Z \sim N(0,1) \)

\[Z = \frac{X - \mu}{\sigma}\]

\[F_X(x) = F_Z \left( \frac{x - \mu}{\sigma} \right)\]

**Problem #3**