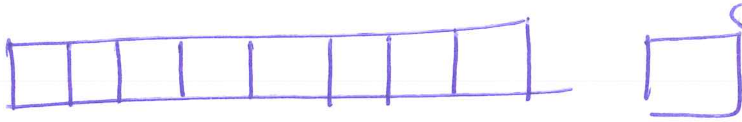
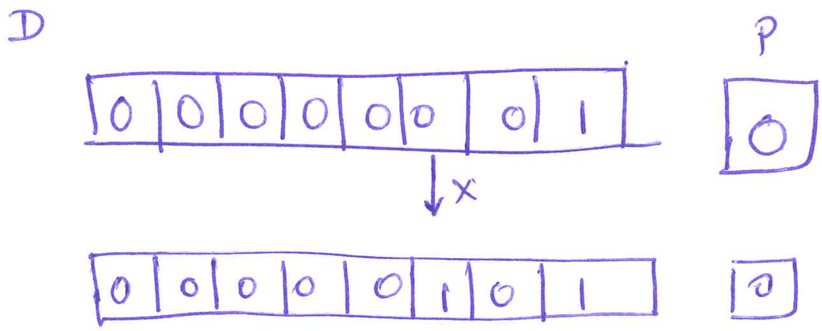
Error detection, error correction coding

Parity



odd parity ⇒ Data + Parity will have odd number of 1's



⇒ Even number of 1's

⇒ Detect error

Any odd number of bits in error

⇒ Can detect with parity

Same detection capability : odd or even parity

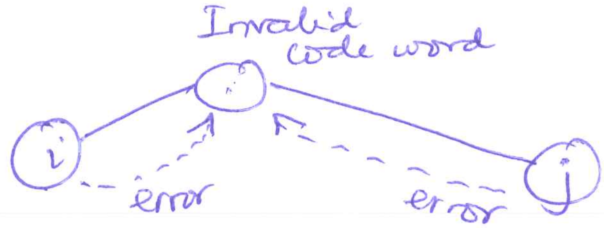
$$\text{Hamming distance (code)} = \min_{i,j} d(\text{Code}(i), \text{Code}(j))$$

i, j are <u>valid</u> code words

Hamming distance (Parity code) = 2

To detect <u>all</u> d bit errors, HD(code) $\geq d+1$

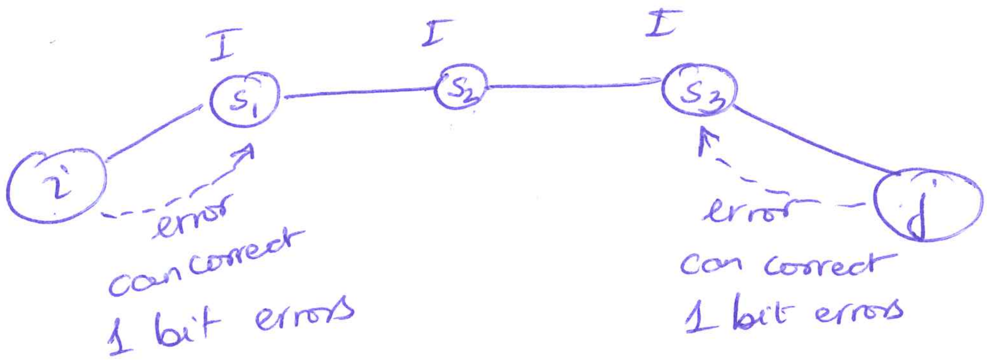
<u>Parity</u>



To correct all c bit errors, HD(code) $\geq 2c+1$

<u>Example</u>

$c = 1$



2 bit errors → can detect but cannot correct

8	7	6	5	4	3	2	1
X	d_3	d_2	d_1	p_3	d_0	p_2	p_1

4 data bits

3 parity bits

Efficiency (code) $= \frac{4}{7}$

$$p_1 = XOR(d_0, d_1, d_3)$$

$$p_2 = XOR(d_0, d_2, d_3)$$

$$p_3 = XOR(d_1, d_2, d_3)$$

d_3	d_2	d_1	p_3	d_0	p_2	p_1
1	0	0	1	1	0	0

X

1	0	1	1	1	0	0

Calculated	Stored
$\hat{p}_1 = 1$	$p_1 = 0$
$\hat{p}_2 = 0$	$p_2 = 0$
$\hat{p}_3 = 0$	$p_3 = 1$

d_1 is the culprit