Outline

- Quantifiers and predicates
- Translation of English sentences
- Predicate formulas with single variable
- Predicate formulas involving multiple variables
- Negation of predicate formulas
- **Text book chapter 1.3**

Weakness in propositional logic

Recall the weakness outlined for propositional logic:

- The logic fails to model the internal structure of propositions.
  - e.g. Every positive number is greater than zero.
- Let's define a logic that models some of this internal structure.
- Requires a richer abstraction of the world.

Quantifiers, Predicates, Domain

- $(\forall x)(x>0)$
- Quantifer: How many objects have a certain property - "for every" or "for some"
- Predicate: Property that a variable may have
- Domain of interpretation: Collection of objects from which the variable is taken
- Universal quantifier: $\forall$ Existential quantifer: $\exists$
- Truth value of a predicate logic formula depends on all three

Predicate Well Formed Formulas (WFF)

- Predicate WFFs built by combining predicates with quantifiers, grouping symbols, and the logical connectives seen before
- Example: $(\forall x)[(\exists y)(P(x) \wedge Q(y)) \rightarrow R(x)]$
- Scope of a quantifier
- Interpretation for an expression with predicates has:
    1. Domain of interpretation
    2. Assignment of property of objects to each predicate
    3. Assignment of particular object to each constant symbol

Interpreting Truth Values: Example

1. $(\forall x)\ P(x)$

- Where, P(x): x is positive, domain is all integers greater than 10
- This is true because all integers greater than 10 are positive

2. $(\exists y)\ G(y,0)$

- Where, G(a,b): a > b, domain is all integers, (and the constant '0' means the integer zero)

Interpreting Truth Values: Example

- Consider binary predicates

1. $(\forall x)(\exists y)\ Q(x, y)$

Where, Q(x, y): x+y = 0, domain is all integers

2. $(\forall x)(\exists y)\ Q(x, y)$

Same Q(x, y), domain is all positive integers

3. $(\exists x)(\forall y)\ Q(x, y)$

Where, Q(x, y): x+y = 0, domain is all integers

Translation: Use of universal quantifier

- "Every movie by George Lucas is great" $\Rightarrow$ "For any movie, if it is made by George Lucas, it is great"
- *Domain*: All movies
- Represented as: $(\forall x)\ (GL(x) \rightarrow G(x))$ [GL(x): movie made by George Lucas, G(x): x is great]
- Where, GL(x): movie made by George Lucas,

  G(x): x is great,
- Note that $(\forall x)\ (GL(x) \wedge G(x))$ is wrong
- That means "All movies are made by George Lucas and are great"

Translation: Use of the existential quantifier

- "There is a movie made by George Lucas that is great" $\Rightarrow$ "There exists at least one movie which is made by George Lucas and which is great"
- Represented as: $(\exists x)(GL(x) \wedge G(x))$
- The representation $(\exists x)(GL(x) \rightarrow G(x))$ is incorrect
- That would be true if there was any movie that was not made by George Lucas. It would also be true if there was no movie made by George Lucas.
- Remember: $(\forall, \rightarrow)$ and $(\exists, \wedge)$ almost always go together

Example predicate formulas

- All students are intelligent.
- Some intelligent students like music.
- Everyone who likes music is a stupid student.
- Only intelligent students like music.

Predicates

S(*x*): *x* is a student

I(*x*): *x* is intelligent

M(*x*): *x* likes music

Predicate formulas involving multiple variables

- Some vegetables are sweeter than (all) fruits.
  - F(x): x is a fruit
  - V(x): x is a vegetable
  - S(x,y): x is sweeter than y
- The representation for the above statement is $(\exists x)(V(x) \wedge (\forall y)(F(y) \rightarrow S(x,y)))$

Negation of compound predicate statements

- $[(\forall x)A(x)]' \leftrightarrow (\exists x)A'(x)$
- Example: ["Everything is beautiful"]′ ↔"There is something which is not beautiful"
- $[(\exists x)A(x)]' \leftrightarrow (\forall x)A'(x)$
- Example: ["Something is beautiful"]′ ↔"Everything is not beautiful"

Outline

- Inference rules in predicate logic
- Example uses
- Gotchas
- More examples
- Example proof using deduction method
- Translation of English sentences
- **Text book chapter 1.4**

Derivation Rules

Rules from propositional logic can still be used in predicate logic proofs.

e.g. modus ponens:

$$\frac{P \quad P \rightarrow Q}{Q}$$

P and *Q* can be predicate logic formulas:

…

8. $[(\forall x)(\forall y)\text{loves}(x,y)] \rightarrow [(\exists x)\text{happy}(x)]$ …
9. $[(\forall x)(\forall y)\text{loves}(x,y)]$ …
10. $[(\exists x)\text{happy}(x)]$ 8,9,mp

New Inference Rules

Name & Abbrev		If you have:	You may conclude:	When:
Existential Generalization	eg	$P(a)$	$(\exists x)P(x)$	*x* must not appear in *P(a)*
Existential Instantiation	ei	$(\exists x)P(x)$	$P(a)$	Must be the first rule that introduces *a*
Universal Generalization	ug	$P(x)$	$(\forall x)P(x)$	1. P(x) not derived from a hyp with x as free variable 2. *P(x)* not derived by ei from wff with x as free variable
Universal Instantiation	ui	$(\forall x)P(x)$	$P(a)$	*a* is a constant

P is any formula, *P*(*a*) is the same formula with constant *a* replacing each free *x*.

Example Uses

- Premises:
  Every man is mortal. There is a man.
  - Prove: There is a mortal.

1.	$(\forall x)\text{man}(x) \rightarrow \text{mortal}(x)$	premise
2.	$(\exists x)\text{man}(x)$	premise
3.	$\text{man}(a)$	2,ei
4.	$\text{man}(a) \rightarrow \text{mortal}(a)$	1,ui
5.	$\text{mortal}(a)$	3,4,mp
6.	$(\exists x)\text{mortal}(x)$	5,eg

Example Uses

- Premises: Every man is mortal.
  Every mortal is alive.
  - Prove: Every man is alive.

1.	$(\forall x)\text{man}(x) \rightarrow \text{mortal}(x)$	premise
2.	$(\forall x)\text{mortal}(x) \rightarrow \text{alive}(x)$	premise
3.	$\text{man}(a)$	ded. thm premise
4.	$\text{man}(a) \rightarrow \text{mortal}(a)$	1, ui
5.	$\text{mortal}(a) \rightarrow \text{alive}(a)$	2, ui,
6.	$\text{mortal}(a)$	3,4, mp
7.	$\text{alive}(a)$	5,6, mp
8.	$\text{man}(a) \rightarrow \text{alive}(a)$	3,7,ded. thm.
9.	$(\forall x)\text{man}(x) \rightarrow \text{alive}(x)$	8, ug

Deduction Method Subproof

Idea: Try to prove $\text{man}(a) \rightarrow \text{alive}(a)$ for some *a* not in the premises, then use the ug rule

Gotchas in applying proof rules

1.	$(\forall x)[P(x) \rightarrow Q(x)]$	hyp
2.	$(\exists y)P(y)$	hyp
3.	$P(a) \rightarrow Q(a)$	1, ui
4.	$P(a)$	2, ei
5.	$Q(a)$	3,4, mp

1.	$P(x)$	hyp
2.	$(\forall x)P(x)$	1, ug

1.	$(\forall x)(\exists y)Q(x,y)$	hyp
2.	$(\exists y)Q(x,y)$	hyp
3.	$Q(x,a)$	2, ei
4.	$(\forall x)Q(x,a)$	3, ug

Example proofs

$$(\forall x)[P(x) \wedge Q(x)] \rightarrow (\forall x)[Q(x) \wedge P(x)]$$

Example proof using deduction method

$(\forall y)[P(x) \rightarrow Q(x,y)] \rightarrow [P(x) \rightarrow (\forall y)Q(x,y)]$

Translation of English Sentences

Every ECE student works harder than somebody, and everyone who works harder than any other person gets less sleep than that person. Maria is an ECE student. Therefore, Maria gets less sleep than someone.

Use E(x), W(x,y), S(x,y), *m*