Operations on Sets

- Union of Sets
  - $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection of Sets
  - $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Difference of Sets
  - $A - B = A \cap B'$
- Use Venn Diagram to depict suitable region for above set operations.

Example

1. Let $A = \{2, 4, 8, 10\}$

   $B = \{2, 4, 7, 8, 9\}$

   $C = \{5, 8, 10\}$ where

$S = \{1,2,3,4,5,6,7,8,9,10\}$

Find: $A \cup B$

$A \cap B'$

$A - C$

$C' \cap (A \cup B)$

Set Identities

A U B= B U A	A ∩ B=B ∩ A	Commutative
(A U B) U C = A U (B U C)	(A ∩ B) ∩ C = A ∩ (B ∩ C)	Associative
A U (B ∩ C) = ((A U B) ∩ (A U C)	A ∩ (B U C) = (A ∩ B) U (A ∩ C)	Distributive
A U Φ =A	A ∩ S = A	Identity
A U A′ = S	A ∩ A′ = Φ	Complement

Cartesian Product

- Let A and B be two subsets of S. The Cartesian product (cross product) denoted by A **X** B is defined by

  $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

- *Let A = {2,4} and B={1,3}*

  Find: A^2

  *A **X** B, B **X** A*

Counting

- Also termed as **Combinatorics**
  - Example : A person is to chose one appetizer out of 5 choices, one main course out of 10 choices, and one dessert out of 4 choices (if you must know they are: crème brulee, cannoli, tiramisu, and rosogolla). How many different ways can he set up his meal?

Multiplication Principle

- If there are $\mathbf{n_1}$ possible outcomes for first event and $\mathbf{n_2}$ possible outcomes for second event, there are $\boldsymbol{n_1 . n_2}$ possible outcomes for the sequence of two outcomes.
- Principle generalizes to any number of events.

Example

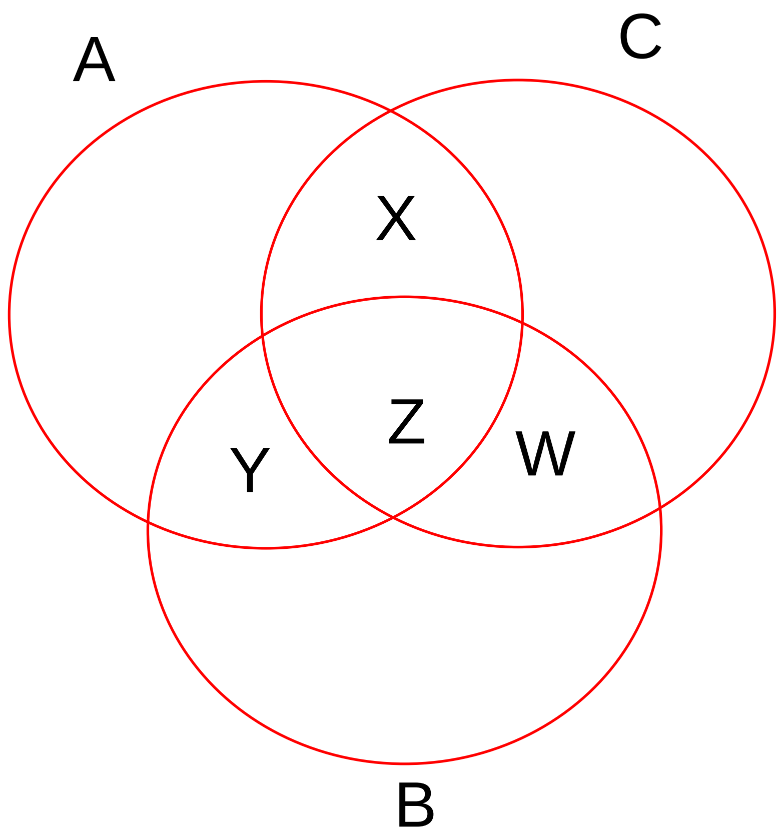
- How many 5 letter palindromes can be constructed using English alphabets?

Principle of Inclusion and Exclusion

- $|A_1 \cup A_2 \cup \dots\dots\dots\dots\dots\dots\dots \cup A_n|$

$$= \sum_{1 \le j \le n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| \ldots\ldots + (-1)^{n+1} |A_i \cap A_j \cap \ldots\ldots A_n|$$

Illustration Using Venn Diagram



Principle of Inclusion-Exclusion

- Example problem:
- A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 in both Spanish and Russian, and 14 in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages?

Pigeon Hole Principle

- What will happen if we try to put ***r*** balls in ***n*** boxes when ***r > n*** ?
- Principle
  - If more than *k* pigeons fly into *k* pigeonholes then at least one pigeonhole will have more than one pigeon.
  - If more than *k items are placed into k bins then at least one bin will contain more than one item,*

Example

- How many times a single dice must be rolled to guarantee getting same value twice ?

Permutations and Combinations

- Ordered arrangement of objects is called a Permutation
- P(n,r) is permutations of r distinct objects chosen from n distinct objects.
  - $P(n,r) = n!/(n-r)!$ For $n \geq r \geq 1$

Examples

- Permutations of 3 distinct objects
  - Solution: P(3,3)=3!/(3-3)! = 3! = 6
- Seven swimmers compete for G, S and B medals. How many different ways the awards can be made ?

Combinations

- How to chose *r* distinct objects from a set of *n* objects.
  - Order does not matter while choosing
  - Denoted by C(n,r)
  - *r* objects can be ordered in *r*! ways.
  - $C(n,r) = P(n,r)/r! = n!/(n-r)!.r!$

Example

- Seven swimmers compete and only 3 can be winners. How many different ways can the winners be chosen ?
  - Solution : C(7,3)
- How many ways a committee of 7 people consisting of 4 females and 3 males be formed out of a group of 10 females and 5 males ?

Duplicates ?

- What will happen if objects are not distinct i.e. they are same .
- Number of Permutations of r alike objects is only one.
- No. of distinct permutations of n objects out of which $r_1\ r_2\ r_{3....}\ r_k$ are alike is given by

$$\frac{n!}{r_1!\,r_2!\,r_3!\ldots r_k!}$$

Permutations and Combinations with Repetition

- Permutations with repetitions is simply n^r where r objects are chosen from a set of n objects.

- Combinations can be a little hard to find.

Not well stated

  - How would you put n objects into r boxes ?
  - Better: How would you put objects of n distinct kinds into r boxes ?
  - $C_{rep}(n, r) = C(r+n-1,r) = (r+n-1)!/r!(n-1)!$

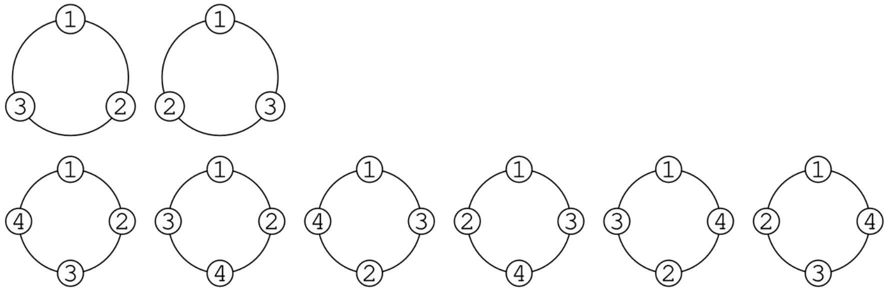
Examples

- A person X would like to invite 7 dinner guests out of a total of 14 friends. Six of her friends are males and rest are females. She would like to have at least one of each.

Examples (Cont.d)

- How many different ways can you seat 10 men and 8 women if no two women are to sit together ?

Circular Permutations



The number of permutations of n elements in a circle is $(n\text{-}1)!$ This is instead of the usual $n!$ since all cyclic permutations of objects are equivalent because the circle can be rotated.

Binomial Theorem

$(a + b)^n = C(n, 0)a^n + C(n, 1)a^{n-1}b + C(n, 2)a^{n-2}b^2 + \ldots + C(n, j)a^{n-j}b^j + \ldots + C(n, n)b^n$

In short: $(x + a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$

Examples: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Sample problem: Find the coefficient of x^2 in $(3x - 5)^4$