1. Ex 2.1

1.1 Problem 31

$n + (n+1) + (n+2) = 3n+3 = 3(n+1)$

1.2 Problem 36

Proof by cases:

Case 1: $x=0$ or $y=0$

$x=0 \rightarrow |x|=0, xy=0, |xy|=0$

$\therefore |xy| = 0 = 0 \cdot |y| = |x||y|$

$y=0 \rightarrow$ similar to above

Case 2: $x>0, y>0$

$|x|=x$ & $|y|=y$

$xy>0$ & $|xy| = |x||y|$

$\therefore |xy| = xy = |x||y|$

Case 3: $x<0, y>0$

$|x| = -x, |y|=y$

$xy<0 \rightarrow |xy| = -xy$

$\therefore |xy| = -xy = (-x)y = |x||y|$

Case 4: $x>0, y<0$

$|x|=x, |y|=-y$

$xy<0 \rightarrow |xy| = -xy$

$\therefore |xy| = -xy = x(-y) = |x||y|$

Case 5: $x<0, y<0$

$|x| = -x, |y| = -y$

$xy>0 \rightarrow |xy| = xy$

$\therefore |xy| = xy = |x||y|$

1.3 Problem 59

Counterexample: $9 = 2^3 + 1$

$9 = (3)(3)$

$\therefore$ 9 is not a prime

2. Exercises 2.2

2.1 Problem 7

$P(1): 1^2 = \frac{1(1+1)(2+1)}{6}$ - true

Assume $P(k): 1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Show $P(k+1): 1^2 + 2^2 + \cdots + (k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$

$$1^2+2^2+\cdots+(k+1)^2 = 1^2+2^2+\cdots+k^2+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + k+1\right]$$

$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

QED

2.2 Problem 24

$P(1): a = \frac{1}{2}(2a)$ - true

Assume $P(k): a + (a+d) + \cdots + [a+(k-1)d] = \left[\frac{k}{2}\right][2a+(k-1)d]$

Show $P(k+1): a + (a+d) + \cdots + [a+kd] = \left[\frac{k+1}{2}\right][2a+kd]$

$$a+(a+d)+\cdots+[a+kd] = a+(a+d)+\cdots+[a+(k-1)d]+[a+kd]$$

$$= \left[\frac{k}{2}\right][2a+(k-1)d] + a + kd$$

$$= \left[\frac{k}{2}\right][2a+(k-1)d] + \frac{2a+2kd}{2}$$

$$= \frac{2ka + k^2d - kd + 2a + 2kd}{2}$$

$$= \frac{k(2a + kd) + (2a + kd)}{2}$$

$$= \frac{(k+1)}{2}(2a + kd)$$

QED

2.3 Problem 65

65. a. Let P(n) be the property that any word composed of a juxtaposition of n subwords has an even number of o's. Then P(1) is true because the only words with 1 subword are the words moon, noon, and soon, all of which have 2 o's. Assume that P(k) is true and consider P(k+1). For any word composed of k + 1 subwords, break the word into two parts compoased of k subwords and 1 subword. By the inductive hypothesis, the part with k subwords has an even number m of o's. The part with 1 subword has 2 o's. The total number of o's is therefore m + 2, an even number. This verifies P(k + 1) and completes the proof.

b. Let P(n) be the property that any word composed of a juxtaposition of n subwords has an even number of o's. Then P(1) is true because the only words with 1 subword are the words *moon*, *noon*, and *soon*, all of which have 2 o's. Assume that P(r) is true for all r, $1 \le r \le k$ and consider P(k+1). For any word composed of k + 1 subwords, break the word into two parts compoased of r_1 and r_2 subwords, with $1 \le r_1 \le k$, $1 \le r_2 \le k$, and $r_1 + r_2 = k + 1$. By the inductive hypothesis, r_1 contains m_1 o's, an even number, and r_2 contains m_2 o's, an even number. Then the original word contains $m_1 + m_2$ o's, an even number. This verifies P(k + 1) and completes the proof.

... the base case, n = 3, the polygon is a triangle, and the sum of the interior angles ...