Exercises 1.1

Ex 3)

3. a. T b. F c. F d. F e. T f. F g. T h. T

Ex 8)

a. Either the processor is slow or the printer is fast.

c. The processor is fast but so is the printer.

f. The printer is slow and the file is not damaged.

Ex 13)

a. $H \rightarrow K$

c. $K \rightarrow H$

d. $K \leftrightarrow A$

Exercises 1.2

42. The argument is $(C \to F') \wedge (F \vee S) \to (C \to S)$

A proof sequence is:

1.	$C \to F'$	hyp
2.	$F \vee S$	hyp
3.	C	hyp
4.	F'	1, 3, mp
5.	S	2, 4, ds

47. The argument is $[((J \vee L) \to C) \wedge T' \wedge (C \to T)] \to J'$

A proof sequence is:

1.	$(J \vee L) \to C$	hyp
2.	T'	hyp
3.	$C \to T$	hyp
4.	$T' \to C'$	3, cont
5.	C'	2, 4, mp
6.	$C' \to (J \vee L)'$	1, cont
7.	$(J \vee L)'$	5, 6, mp
8.	$J' \wedge L'$	7, De Morgan
9.	J'	8, sim

Exercises 1.3

Ex 2)

e. false (may have $x = y$)

f. true (pick $y = -x$)

g. true (pick $x = 2, y = 4$)

h. false (may have $x = 0$)

Ex 11)

b. $(\forall x)(P(x) \to (\exists y)(T(y) \wedge F(x, y))$

c. $[(\forall x)(\forall y)(P(x) \wedge T(y) \to F(x, y))]'$ or $(\exists x)(\exists y)(P(x) \wedge T(y) \wedge (F(x, y)')$ or $[(\forall x)(P(x) \to (\forall y)(T(y) \to F(x, y)))]'$

Ex 16)

a. $(\forall x)[B(x) \to (\forall y)(F(y) \to L(x, y))]$ or $(\forall x)(\forall y)[(B(x) \wedge F(y)) \to L(x, y)]$

i. $((\exists x)[B(x) \wedge (\forall y)(L(x, y) \to F(y))]$

l. $[(\exists x)[B(x) \wedge (\forall y)(F(y) \to (L(x, y))')]]'$ or $(\forall x)[B(x) \to (\exists y)(F(y) \wedge L(x, y))]$

Exercises 1.4

5. One conclusion is that some flowers are weeds. The hypotheses have the form $(\exists x)(F(x) \wedge P(x) \wedge T(x))$, $(\forall x)(F(x) \wedge T(x) \rightarrow B(x))$, and $(\forall x)(F(x) \wedge B(x) \rightarrow W(x))$. By existential and universal instantiation (in that order), $F(a) \wedge P(a) \wedge T(a)$, $F(a) \wedge T(a) \rightarrow B(a)$, and $F(a) \wedge B(a) \rightarrow W(a)$. Simplification gives $F(a) \wedge T(a)$ which, using modus ponens, gives $B(a)$. Combining $F(a)$ with $B(a)$ and using modus ponens results in $W(a)$. Combining $F(a)$ and $W(a)$ and using existential generalization results in $(\exists x)(F(x) \wedge W(x))$. Other possible conclusions are: Some pink flowers smell bad, Some pink, thorny, smelly flowers are weeds.

9. a. domain is the integers, $Q(x, y)$ is "$x < y$"; for every y there is an x with $x < y$ but there is no single integer x that is less than every integer y.
   b. The use of universal generalization at step 4 is illegal because step 3 was deduced by ei from $(\exists x)Q(x, y)$ in which y is a free variable.

33. The argument is:
$(\exists x)(M(x) \wedge (\forall y)R(x, y)) \wedge (\forall x)(\forall y)(R(x, y) \rightarrow T(x, y)) \rightarrow (\exists x)(M(x) \wedge (\forall y)T(x, y))$
A proof sequence is:

1.	$(\exists x)(M(x) \wedge (\forall y)R(x, y))$	hyp
2.	$M(a) \wedge (\forall y)R(a, y)$	1, ei
3.	$M(a)$	2, sim
4.	$(\forall x)(\forall y)(R(x, y) \rightarrow T(x, y))$	hyp
5.	$(\forall y)(R(a, y) \rightarrow T(a, y))$	4, ui
6.	$(R(a, y) \rightarrow T(a, y))$	5, ui
7.	$(\forall y)R(a, y))$	2, sim
8.	$R(a, y)$	7, ui
8.	$T(a, y))$	6, 8, mp
9.	$(\forall y)T(a, y)$	8, ug
10.	$M(a) \wedge (\forall y)T(a, y)$	3, 9, con
11.	$(\exists x)(M(x) \wedge (\forall y)T(x, y))$	10, eg

37. The argument is:

$(\forall x)(F(x) \to (\exists y)(C(y) \land O(x, y))) \land (\forall x)(\forall y)(C(y) \land O(x, y) \to [D(x)]')$
$\quad \to (\forall x)(F(x) \to [D(x)]')$

A proof sequence is:

1.	$(\forall x)(F(x) \to (\exists y)(C(y) \land O(x, y)))$	hyp
2.	$(\forall x)(\forall y)(C(y) \land O(x, y) \to [D(x)]')$	hyp
3.	$F(x) \to (\exists y)(C(y) \land O(x, y))$	1, ui
4.	$F(x)$	temporary hyp
5.	$\quad (\exists y)(C(y) \land O(x, y))$	3, 4, mp
6.	$\quad C(a) \land O(x, a)$	5, ei
7.	$\quad (\forall y)(C(y) \land O(x, y) \to [D(x)]')$	2, ui
8.	$\quad C(a) \land O(x, a) \to [D(x)]'$	7, ui
9.	$\quad [D(x)]'$	6, 8, mp
10.	$F(x) \to [D(x)]'$	temporary hyp discharged
11.	$(\forall x)(F(x) \to [D(x)]')$	10, ui

Q5

Problem I

Are $A \vee (B \wedge C)$ and $(A \vee B) \wedge C$ equivalent?

Solution

They are not. Draw up the truth tables for the two and you will see that they do not match for all rows.

Side note: $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

This is the Distributive Law.

Q6

Problem II

Aristotle's Ferio

No M are P.

Some S are M.

∴ Some S are not P.

1. $(\forall x)\,(M(x) \to P'(x))$ hyp.

2. $(\exists x)\,(S(x) \wedge M(x))$ hyp.

To prove: $(\exists x)(S(x) \wedge P'(x))$

3. $S(a) \wedge M(a)$ a is a constant 2, ei

4. $M(a) \to P'(a)$ 1, ui

5. $S(a)$ 3, sim

6. $M(a)$ 3, sim

7. $P'(a)$ 6, 4, mp

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8. $S(a) \wedge P'(a)$ 5, 7, con

9. $(\exists x)\,(S(x) \wedge P'(x))$ 8, eg