ELSEVIER

Journal of Hydrology 220 (1999) 48–61

Journal of Hydrology

# Uncertainty in the model parameters due to spatial variability of rainfall

I. Chaubey[a,*], C.T. Haan[a], S. Grunwald[b], J.M. Salisbury[c]

[a]Department of Biosystems and Agricultural Engineering, Oklahoma State University, Stillwater, OK 74078, USA
[b]Department Soil Science, University of Wisconsin–Madison, Madison, WI 53706, USA
[c]SEIC, Oklahoma State University, Stillwater, OK 74078, USA

Received 5 May 1998; received in revised form 12 March 1999; accepted 27 April 1999

**Abstract**

Most hydrologic/water quality (H/WQ) models that use rainfall as input assume spatial homogeneity of rainfall. Under this assumption this study assesses the variability induced in calibrated model parameters solely due to rainfall spatial variability. The AGNPS model and a network of 17 raingauges were used. Model parameters were estimated using rainfall observed at each gauge location, one at a time, as though that rainfall covered the entire catchment. A large uncertainty in the estimated parameters resulted from the spatial variability of rainfall. The uncertainty in the estimated parameters using the rainfall observed by a single gauge exceeded the rainfall measurement error. A large uncertainty in estimated model parameters can be expected if detailed variations in the input rainfall are not taken into account. 

*Keywords:* Spatial variability; Water quality; Hydrologic modeling; AGNPS; Parameter uncertainty

## 1. Introduction

Hydrologic/Water Quality (H/WQ) models are one of the most important ways to estimate the impact of land use on nonpoint water quality. During the last decade many such models requiring several input parameters have been developed for making environmental decisions. Rainfall is a key input for all H/WQ models because it activates flow and mass transport process in hydrologic systems.

In applying H/WQ models, rainfall is generally taken as spatially uniform and is assumed not to contribute to parameter and output uncertainty, even though the storms that cause the greatest movement of sediment and nutrients are rarely uniform (Young et al., 1992). Goodrich et al. (1995) noted that even though the spatial variability of rainfall plays an important role in the process of runoff generation, rainfall is assumed to be uniform in the application of models to predict hydrological behavior of small watersheds. Often the model developers and users have available rainfall measured by only one gauge, or a few gauges. The model parameters may be estimated by calibration using an average rainfall uniformly distributed throughout the watershed. The spatial variability in rainfall may introduce significant uncertainty in these parameters when they are based on a comparison of observed and predicted hydrologic responses.

* Corresponding author. Present address: A-122 Bevill, University of Alabama, Box 870206, Tuscaloosa, AL 35487-0206, USA. Tel.: +1-205-348-7747; fax: +1-205-348-1403.
*E-mail address:* ichaubey@biology.as.ua.edu (I. Chaubey)

0022-1694/99/$ - see front matter 
PII: S0022-1694(99)00063-3

Although it is acknowledged that, in general, watersheds have an integrating effect on rainfall both in time and in space, the complex relationships among the degree of spatial variability of rainfall, watershed characteristics (topography, channel network, soils, etc.), antecedent soil moisture conditions and watershed response is poorly understood (Shah et al., 1996). Very few studies have been conducted to investigate the significance of spatial variability of rainfall on H/WQ processes. Most of these studies have focused on hydrograph properties, such as, runoff volume, time to peak runoff, and peak runoff rate predictions (e.g. Dawdy and Bergman, 1969; Wilson et al., 1979; Beven and Hornberger, 1982; Corradini and Singh, 1985; Seliga et al., 1992; Faures et al., 1995; Shah et al., 1996) where the storm runoff hydrographs are shown to be sensitive to the spatial distribution and accuracy of rainfall inputs. Based on a study conducted to assess the effect of rainfall spatial variability on water quality outputs, Young et al. (1992) and Luzio and Lenzi (1995) have demonstrated sediment yield, total N and total P predictions to be sensitive to the spatial variability of rainfall.

The knowledge of uncertainty in the calibrated parameters due to rainfall spatial variability is very limited. A study conducted by Troutman (1983) attempted to assess the effect of rainfall spatial pattern on estimated model parameters. The model considered was a rainfall-runoff model. The author used a synthetic rainfall to simulate the spatial correlation pattern of an actual rainfall. Because of the simplicity of the stochastic rainfall model, the results reported may not be expected to define the variability in actual rainfall-runoff modeling applications. Hamlin (1983) mentioned that a modeled rainfall may not describe the patterns and amounts of real rainfall adequately.

The overall objective of this research was to study the variability in estimated H/WQ model parameters solely due to the spatial variability of rainfall. This will help isolate this source of variability in the model parameters from other sources.

## 2. Background

Haan (1989) gave a generic representation of hydrologic models as

$$\mathbf{O} = \mathbf{f}(\mathbf{I}, \mathbf{P}, t) + \mathbf{e} \quad (1)$$

where $\mathbf{O}$ is an $n \times k$ matrix of hydrologic responses to be modeled, $\mathbf{f}$ a collection of functional relationships, $\mathbf{I}$ an $n \times m$ matrix of inputs, $\mathbf{P}$ a vector of $p$ parameters, $t$ time, $\mathbf{e}$ an $n \times k$ matrix of errors, $n$ the number of data points, $k$ the number of responses, and $m$ the number of inputs.

Generally $\mathbf{I}$ represents inputs some of which are time varying such as rainfall, temperature, etc., while $\mathbf{P}$ represents coefficients or parameters particular to a watershed which remain constant. The values of the most of the model parameters are seldom known. They must be estimated by calibration before the model can be applied in a particular situation. Parameter uncertainty reflects incomplete models, incomplete information and incomplete parameter estimation techniques (Haan, 1989). The error term, $\mathbf{e}$ represents the difference between what actually occurs, $\mathbf{O}$, and what the model predicts, $\hat{O}$

$$\hat{\mathbf{O}} = \mathbf{f}(\mathbf{I}, \mathbf{P}, t) \quad (2)$$

Troutman (1983) classified the modeling errors into two components: (1) model errors with correct input $\mathbf{I}^*$ and $\mathbf{P}^*$ and (2) errors due to erroneous input. We can denote $\mathbf{I}^*$ as the error-free true input and $\mathbf{P}^*$ as the true parameter values for the model, and $\mathbf{I}$ and $\mathbf{P}$ as erroneous input and parameter values, respectively. Putting $\mathbf{I}^*$ and $\mathbf{P}^*$ in Eq. (1) will give the relation between actual and predicted output. Even when the true input and parameter values are known, predicted output is different from the observed output because models are simplified approximations of the processes occurring in nature. This type of error is known as model error and is not considered in this study. Only input error and its effect on estimated parameter uncertainty is studied in this research. Troutman (1982, 1983, 1985) and others discuss errors that can be expected due to erroneous rainfall input in rainfall-runoff modeling.

A problem in using an erroneous input in a H/WQ model is that the predicted output should no longer be equal to the observed data. Evaluating a model with erroneous input $\mathbf{I}$ introduces a bias in the output. In contrast, if the correct output is known, an erroneous input will influence the value of $\mathbf{P}$ and the estimated parameter values may not be the true parameter values ($\mathbf{P}^*$).

The input of interest in this research is rainfall

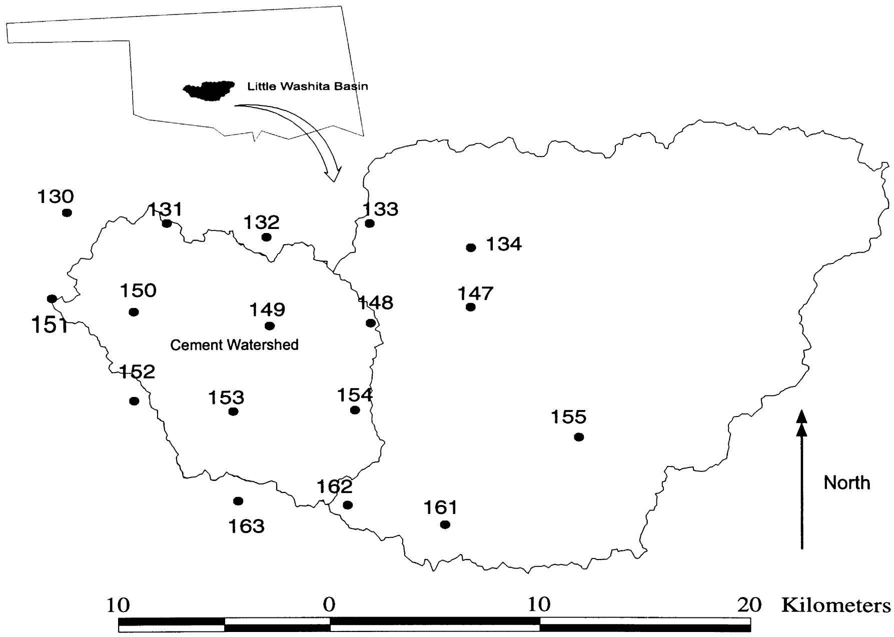


Fig. 1. Location of Little Washita basin, Cement watershed and the Micronet stations used.

depth. The outputs considered are runoff volume, total sediment yield, sediment-attached N load, and sediment-attached P load at the watershed outlet. Correct input means that the true rainfall pattern is known at every point in the watershed. Input error is present when measurements from only a small number of gauges are used when a more extensive network might be necessary to give an adequate representation of precipitation over the watershed of interest (Troutman, 1983).

## 3. Methodology

### 3.1. Description of the study area

The study was conducted using data from the Little Washita basin in Southwest Oklahoma, USA. This basin covers 610 km$^2$ and is a tributary of the Washita river in Southwest Oklahoma (Agricultural Research Service (ARS), 1991). Fig. 1 shows the location of the watershed. A network of 48 recording raingauges, known as Micronet, has been operated by the US Department of Agriculture, Agricultural Research Services (USDA–ARS) for a long time. A detailed description of the soils, topography, geology, and climate of the watershed can be found in ARS (1991). A subwatershed, known as Cement watershed, was delineated from the Little Washita basin and was used in this study. The location of the Cement watershed and the 17 Micronet stations used to capture rainfall spatial variability is shown in Fig. 1. Total area of the Cement watershed is 159 km$^2$. The watershed has a typical continental climate, characterized as moist subhumid with average annual precipitation of 747 mm. The Natural Resources Conservation Services (NRCS) have extensively surveyed the soils in the watershed and have classified 64 different soil series and 162 soil phases within these soil series. Land use and cover is primarily rangeland (63%), winter wheat (20%), and woodland (12%). Summer crops occupy about 4% of the watershed area. Impervious areas and water bodies comprise less than one percent of the total area each.

### 3.2. Description of the model

The model used to assess the effect of rainfall spatial variability was the Agricultural Non-Point Source Pollution model (AGNPS). It is an event-based model that simulates surface runoff, sediment, nutrients and pesticide transport primarily from agricultural watersheds (Young et al., 1989). The nutrients considered are soluble and sediment-attached forms of nitrogen (N) and phosphorus (P). Basic model components include hydrology, erosion, and sediment and chemical transport. The model operates on a geographic cell basis that is used to represent upland and channel conditions. Cells are uniform square areas subdividing the watershed and allowing analyses at any point within the watershed. The model requires specification of 20 different input parameters for each cell. All watershed characteristics and inputs are expressed at the cell level. Potential pollutants are routed through cells from the watershed divide to the outlet in a stepwise manner so that flow at any point between cells can be examined. More details about the model may be found in Young et al. (1989).

One of the limitations of the AGNPS model, like most of the H/WQ models, is that it allows only one value of rainfall assuming it to be homogeneous across the watershed of interest. The model was modified to input grid-based rainfall depth and energy intensity. The modifications were based on the work done by Grunwald and Frede (1997).

Preparation of the input file for AGNPS is very time intensive. For a relatively large watershed with small cell size (e.g. less than 1 ha), generating, organizing and managing the model input data and analyzing and displaying the model output data can be tedious, time-consuming and problematic. The WATERSHEDSS GRASS–AGNPS modeling tool developed by Osmond et al. (1997) was used to develop the input file. The GIS layers required were watershed boundary, topography, tillage, USLE *K* and *C* factors, hydrologic soil groups, percent sand, percent clay, nutrient application rate, land use, and a management practice map. All input layers were prepared in raster format using a 30 m cell resolution. The cell size used in AGNPS modeling was 200 m × 200 m. This cell size was used to insure the adequate representation of the watershed properties without increasing the complexity of the input file and the AGNPS run time. In a study done on the Upper Little Washita basin which encompasses the Cement watershed, Ma (1993) concluded that a cell size less than 300 m × 300 m would preserve the presence of high runoff producing areas. Once the input file for AGNPS was prepared using the GRASS–AGNPS modeling tool, the cell-based rainfall values were added to the input file.

Because of the large number of input parameters, it was not possible to study the uncertainty in all of the model parameters. Also the output of a H/WQ model is not equally sensitive to all parameters. A relative sensitivity index ($S_r$) was used to rank the model parameters in terms of their sensitivities in affecting the model outputs. The $S_r$ was defined as

$$S_r = \frac{\partial O}{\partial P}\frac{P}{O} \tag{3}$$

where $O$ is the output and $P$ is the parameter of interest. The parameters with the highest $S_r$ have the greatest impact on model output. Sensitivity analysis of AGNPS was performed using 20 parameters. The most sensitive parameters for these outputs were curve number (CN), USLE *K*, *C*, *P* factors, and land slope. For the AGNPS model, USLE *K*, *C*, and *P* factors always appear as the product KCP and thus cannot be separated for parameter estimation. Therefore, uncertainty of only one of three parameters can be analyzed and the other two parameters can be expected to show the same variability. USLE *K* factor was used in this study. Thus, the three parameters considered were CN, slope, and *K* factor.

### *3.3. Description of the rainfall events and data set*

Rainfall data for the Micronet stations were obtained from USDA–ARS. The rain gauges used in the Micronet are Belfort 5-780 series dual-traverse weighing bucket rain gauges. An automatic data logger was used to measure the rainfall amounts. Stream flow data were obtained from the US Geological Survey (USGS) (USGS http://2o.er.usgs.gov). Daily discharge in cubic feet per second was available.

A total of 9 rainfall dates (3/27/96, 3/28/96, 4/21/96, 4/23/96, 5/31/96, 6/1/96, 7/9/96, 7/10/96, and 10/27/96) were selected. The base flow was separated from the total flow to get the surface runoff. For March 27 and 28, April 21 and 23, May 31 and June 1, and July 9 and 10, it was not possible to separate the base flow from the total flow for the rainfall on each day, because often several days elapsed as the runoff volume was occurring. The total rainfall for the two

Table 1
Rainfall, runoff, sediment and nutrient values for the watershed

| Rainfall date | Rainfall (mm) | Runoff (mm) | Total sediment (Mg) | Sediment-N (kg/ha) | Sediment-P (kg/ha) |
|---|---|---|---|---|---|
| 3/27/96 | 33 | 0.5 | 242 | 0.07 | 0.03 |
| 4/21/96 | 25 | 0.8 | 443 | 0.1 | 0.06 |
| 5/31/96 | 83 | 3 | 3395 | 0.53 | 0.27 |
| 7/9/96 | 64 | 1.5 | 2367 | 0.39 | 0.2 |
| 10/27/96 | 23 | 0.3 | 68 | 0.02 | 0.01 |

days was considered as one rainfall event and was used in the analysis. Thus, the total number of rainfall events considered was five. The events are indicated by the first day of the event. Rainfall during the four days preceding the event date was also obtained to characterize the antecedent moisture conditions used in CN calculations.

### *3.4. Estimation of parameter uncertainty due to spatial variability of rainfall*

The available observed data were the rainfall and runoff volume. No measured water quality or sediment data were available. Two steps were used to estimate the parameter uncertainty due to the spatial variability of rainfall. In the first step, grid-based rainfall depths, considered as the 'true' rainfall, were captured using the Thiessen polygon method. AGNPS was calibrated for CN using observed 'true' rainfall and runoff volume by adjusting the individual cell curve numbers either all upward or downward by a constant percentage until predicted runoff volume equaled observed runoff volume. All other parameters were estimated based on the observed watershed characteristics. Runoff volume, total sediment, sediment-attached N, and sediment-attached P were obtained by running the model using calibrated CN, and 'true' rainfall values for each event. These outputs were considered as the 'observed' values for further analysis. Characteristics of rainfall, runoff, sediment, and nutrient data for all events analyzed are shown in Table 1.

In the second step, parameter uncertainty due to spatial variability of rainfall was estimated. It was assumed that each of the 17 gauges was the only gauge available for the rainfall measurement and the rainfall depth recorded by that gauge was spatially homogeneous across the watershed. Model parameters were estimated using the rainfall observed at each gauge location, one at a time, and the 'observed' runoff, total sediment, sediment-attached N, and sediment-attached P values. The objective function used in the parameter estimation was the sum of the absolute values of relative errors defined by Eq. (5) for runoff, sediment and nutrients.

A two stage "brute force" optimization procedure described by Allred and Haan (1996) was used to find the optimum parameter values. In the first optimization stage, a rough estimate of the optimum parameter set was obtained by setting a percentage by which each parameter was to be changed. The parameter values in each cell were increased or decreased by this percentage. Eight increments or decrements were performed for each parameter. Curve numbers were always increased or decreased by a whole number. For three parameters a total of 512 model runs were performed and objective function values calculated for every possible permutation of the parameters. If the optimum values of any of the three parameters were obtained at the upper or lower boundary of the parameter values, the step sizes of the parameter values were increased and the same procedure was repeated to insure that the optimum parameter estimates did not fall at the boundary values. Mathematically, the optimum parameter value can be represented as $(P_i)_j$, where $P_i$ is the average optimum value of parameter $i$ obtained at step $j$ ($j = 1,2,\ldots,8$). If $j$ was equal to one or eight, then the range of the step size was increased, and the optimization procedure was repeated. The first estimate of the optimum parameter set was chosen that had the minimum objective function value.

The second optimization was conducted in a similar manner as the first one by further refining the parameter values. Refinement was accomplished using a much narrower range of parameters obtained from the

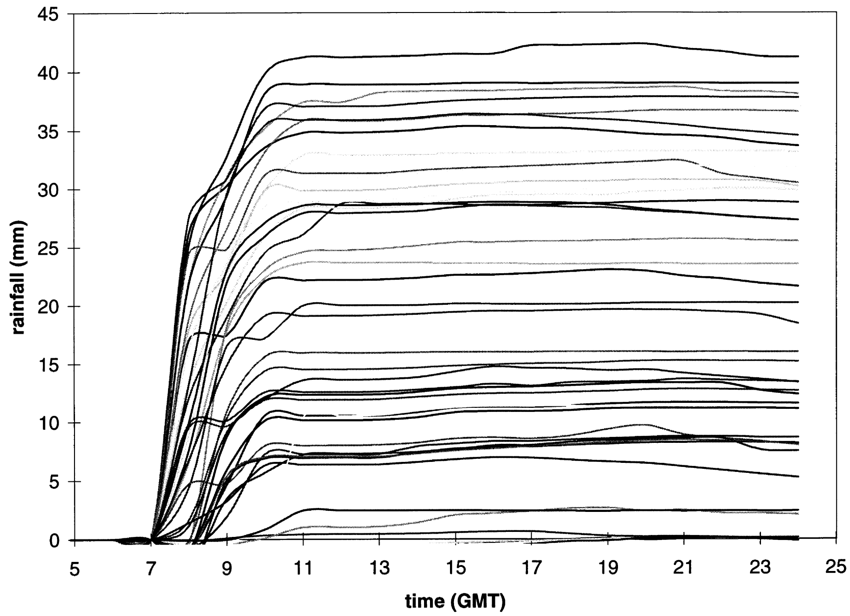


Fig. 2. Hourly distribution of rainfall on 8/3/96 over Little Washita basin.

first optimization. If the optimum parameter obtained by the first approximation was $(P_i)_j$, then the range of the parameters in the second optimization was $(P_i)_{j-1}$ to $(P_i)_{j+1}$. In some instances, more than one set of parameter values, very close to each other, were obtained that minimized the objective function. In that case, the range was set such that all the parameter estimates minimizing the objective function were bracketed. In the second optimization, a step size in the form of a fraction for each parameter was calculated that divided the range of the parameter into 10 evenly distributed values. Each parameter at the cell level was then increased or decreased by this fraction and model runs were performed. In this step also, the curve numbers were increased or decreased by a whole number. A total of 1000 model simulations were performed for each possible permutation of the parameter values. The set of parameters that minimized the objective function was considered as the final optimum parameter set. This "brute force" optimization procedure, although being computationally less efficient than other methods, has the advantage of not being sensitive to local minimums in the objective function (Allred and Haan, 1996).

For the watershed, 17 sets of parameters were obtained for each event corresponding to rainfall observed at each gauge location. Since the parameter values were different at each cell level, the values shown in the subsequent sections represent the average parameter values. The variability in the model parameters induced by the spatial variability of rainfall is termed the parameter uncertainty and is quantitatively described using average error (AE), relative error (RE), standard error (SE), and coefficient of variation (CV). These error statistics can be defined as

$$\mathrm{AE} = \frac{1}{n}\sum_{i}^{n}(|P_i - O|) \tag{4}$$

$$\mathrm{RE} = \frac{AE}{\mathbf{O}} \tag{5}$$

$$\mathrm{SE} = \sqrt{\frac{1}{n}\sum_{i}^{n}(P_i - O)^2} \tag{6}$$

$$\mathrm{CV} = \frac{\mathrm{SE}}{\mathbf{O}} \tag{7}$$

where $P_{\mathrm{i}}$ is the predicted value, $O$ is an observed parameter value, $\mathbf{O}$ is the mean of the observed data, and $n$ $(i = 1,2,3,\ldots,n)$ is the number of data pairs.

The variability in the rainfall amounts observed by

Table 2
Spatial variability of rainfall

| Statistics | Rainfall date | | | | |
|---|---|---|---|---|---|
| | 3/27/96 | 4/21/96 | 5/31/96 | 7/9/96 | 10/27/96 |
| Average (mm) | 32 | 26 | 78 | 69 | 19 |
| Area-weighted average (mm) | 33 | 25 | 83 | 64 | 23 |
| Range (mm) | 18–41 | 17–50 | 57–95 | 31–137 | 0–45 |
| Average error (mm) | 6.35 | 7.15 | 6.47 | 27 | 9.34 |
| Relative error | 0.2 | 0.27 | 0.08 | 0.39 | 0.51 |
| Standard error (mm) | 7.95 | 9.08 | 8.87 | 31.6 | 11.7 |
| CV | 0.25 | 0.35 | 0.11 | 0.46 | 0.64 |
| No. of gauges | 13 | 16 | 17 | 17 | 17 |

Table 3
Parameter variability induced by spatial variability of rainfall

| Statistic | Parameter values for the rainfall dates | | | | |
|---|---|---|---|---|---|
| | 3/27/96 | 4/21/96 | 5/31/96 | 7/9/96 | 10/27/96 |
| *CN* | | | | | |
| Average | 58 | 65 | 43 | 44 | 66 |
| Range | 51–70 | 47–72 | 36–52 | 23–64 | 47–76 |
| CV | 0.12 | 0.11 | 0.12 | 0.51 | 0.26 |
| Standard error | 6.83 | 6.98 | 4.85 | 16.9 | 14.2 |
| Average error | 4.38 | 5.81 | 3.75 | 13.8 | 12.7 |
| Relative error | 0.07 | 0.09 | 0.1 | 0.42 | 0.23 |
| *Slope (%)* | | | | | |
| Average | 3.96 | 5.69 | 3.93 | 4.12 | 3.62 |
| Range | 3.11–5.34 | 3.33–6.79 | 3.32–5.14 | 2.07–6.22 | 2.1–5.54 |
| CV | 0.2 | 0.58 | 0.12 | 0.33 | 0.26 |
| Standard error | 0.75 | 2.15 | 0.46 | 1.24 | 0.95 |
| Average error | 0.48 | 2.02 | 0.3 | 1.02 | 0.77 |
| Relative error | 0.13 | 0.55 | 0.09 | 0.28 | 0.21 |
| *K factor* | | | | | |
| Average | 0.35 | 0.44 | 0.32 | 0.51 | 0.36 |
| Range | 0.23–0.58 | 0.25–0.68 | 0.28–0.38 | 0.27–0.87 | 0.14–0.59 |
| CV | 0.38 | 0.54 | 0.08 | 0.85 | 0.5 |
| Standard error | 0.13 | 0.18 | 0.03 | 0.28 | 0.16 |
| Average error | 0.08 | 0.14 | 0.02 | 0.21 | 0.15 |
| Relative error | 0.23 | 0.43 | 0.08 | 0.62 | 0.45 |
| *Retention parameter (S) (mm)* | | | | | |
| Average | 188 | 144 | 351 | 388 | 139 |
| Range | 109–244 | 98.9–287 | 234–452 | 143–850 | 80.3–287 |
| CV | 0.23 | 0.33 | 0.17 | 0.49 | 0.44 |
| Standard error | 46.7 | 48.5 | 67.1 | 251 | 92.2 |
| Average error | 32.2 | 37.6 | 54.9 | 223 | 84.6 |
| Relative error | 0.16 | 0.25 | 0.14 | 0.43 | 0.41 |

17 raingauges for each event was quantified using Eqs. (4)–(7). Here $P_i$ is the rainfall observed at the gauge $i$, $O$ is the average rainfall for the area, and $n$ is the number of gauges used to capture the rainfall spatial variability.

## 4. Results and discussion

### 4.1. Spatial variability of rainfall

Consideration of spatial variability of rainfall is very important in studying the process of generation and transport of runoff, sediment, and nutrients from a watershed. Fig. 2 shows the hourly distribution of rainfall that occurred on 8/3/96 over the Little Washita basin as recorded at 42 Micronet stations. A large variation in the cumulative rainfall depth over the area is evident. The event rainfall depth varied from almost zero to 43 mm. Traditionally, rainfall is measured at a few gauges (possibly only one) scattered throughout the basin and these point measured values are used to determine the average rainfall depth for use in hydrologic/water quality (H/WQ) models. In an ideal condition, where the density and distribution of gauges are adequate, rainfall depth can be estimated with sufficient accuracy at any point in the basin by using a spatial interpolation technique. Unfortunately, this ideal condition rarely exists. In fact, it is not uncommon to have no rain gauge within the basin of interest. If each of the 42 gauges in Fig. 2 is assumed to be the representative gauge for the watershed, the result obtained using the rainfall recorded at each gauge location, one at a time, will have a large variability. A H/WQ model like AGNPS may not predict any significant output using the low rainfall values as compared to a larger rainfall depth ($>30$ mm) observed at some other gauge locations.

The characteristics of the rainfall observed by 17 gauges associated with the Cement watershed are shown in the Table 2. The average rainfall ranged from 19 to 78 mm for the five events analyzed. The CV ranged from 0.11 to 0.64. The smallest CV and relative error were associated with the rainfall on 5/31/96 and largest with the rainfall on 10/27/96. The standard error was smallest for the rainfall on 3/27/96. For the watershed, 13 rain gauges were used in the Thiessen polygon method to capture the true rainfall pattern and is shown as area-weighted rainfall in Table 2. The average rainfall was obtained from all of the 17 gauges. The average rainfall and the area-weighted rainfall were different for all events. Inclusion of additional gauges that were in the vicinity of the watershed but not a part of the Thiessen network introduced a bias in the average rainfall estimate. In actual conditions, it is not uncommon to have a rain gauge located outside the watershed of interest. As the number of rain gauges available to estimate the area-weighted rainfall increases, this bias can be expected to decrease.

### 4.2. Effect of rainfall spatial variability on model parameter uncertainty

Parameter variability induced by spatial variability of rainfall is shown in Table 3. AGNPS is a distributed parameter model. The model parameters vary from cell to cell. The parameter estimates discussed here represent the average parameter values. In AGNPS, land slope is used to calculate the amount of sediment and nutrients eroded within each cell and the subsequent routing of the sediment and nutrients from each cell to the watershed outlet. The $K$ factor is used in Universal Soil Loss Equation (USLE) to calculate the amount of sediment and nutrients eroded within each cell. CN indicates the runoff potential of an area. The CV in CN ranged from 0.11 to 0.51 for the five events considered. The SE ranged from 4.85 to 16.85. Here the largest SE in the rainfall was associated with the largest SE in CN. Coefficient of variation and SE are numerical representations of the variability in the data. It means that a rainfall with a large variation in observed depth will produce a higher variability in CN. This can be expected since for a fixed runoff, there is a one-to-one correspondence between rainfall and CN. For a small observed rainfall value, CN must be higher to produce a volume of runoff equal to the measured runoff and vice-versa. In general, the standard error in CN decreased with a decrease in the SE for rainfall depth.

The CV in the estimated slope ranged from 0.12 to 0.58 for the watershed. The range of SE were 0.46 to 2.15%. Although the largest CV and SE in the estimated slope were not associated with the rainfall having largest CV and SE, in general a higher variability in rainfall resulted in a higher variability in

Table 4
Relative errors in estimated parameters due to rainfall spatial variability

| Rainfall date | Parameter | Relative error | |
|---|---|---|---|
| | | Maximum | Minimum |
| 3/27/96 | CN | 0.23 | 0.02 |
| | *S* | 0.46 | 0.04 |
| | Slope | 0.41 | 0.01 |
| | *K* factor | 0.75 | 0 |
| 4/21/96 | CN | 0.25 | 0 |
| | *S* | 0.92 | 0 |
| | Slope | 0.83 | 0.1 |
| | *K* factor | 1.06 | 0.03 |
| 5/31/96 | CN | 0.33 | 0 |
| | *S* | 0.4 | 0 |
| | Slope | 0.39 | 0.02 |
| | *K* factor | 0.15 | 0.03 |
| 7/9/96 | CN | 0.94 | 0.03 |
| | *S* | 0.72 | 0.04 |
| | Slope | 0.67 | 0.01 |
| | *K* factor | 1.63 | 0 |
| 10/27/96 | CN | 0.38 | 0.03 |
| | *S* | 0.61 | 0.08 |
| | Slope | 0.49 | 0 |
| | *K* factor | 0.78 | 0.03 |

estimated slope. The rainfall on 5/31/96 was the most homogeneous in nature. This resulted in the smallest CV in the slope estimates.

Coefficient of variation and SE in the *K* factor ranged from 0.08 to 0.85, and 0.03 to 0.28, respectively. The corresponding ranges in CV and SE for the retention parameter (*S*) were 0.17 to 0.49, and 46.7 to 251 mm, respectively. Similar to the CN, the smallest variation in *S* resulted from the rainfall most homogeneous in nature (5/31/96).

In general, a wide range in estimated parameters resulted when the rainfall measured at each gauge location was used individually, one at a time. None of the parameters can be considered unlikely when viewed individually for each event. Together the sets of parameter values obtained illustrate the possible range depending upon the rainfall spatial variability.

A larger range in the rainfall values within a single event resulted in a higher range in all estimated parameters. When compared to the true parameter values, the variation was very large for all events. For slope, *K*, and *S* the range varied by several factors for some events. Parameter uncertainty comes into play when developing and testing a model. One might have several observed events and use each to estimate model parameters. The result may be quite inconsistent estimates. Usually this uncertainty in the model parameters is attributed to the errors in the structure of the models. Results of this study indicate that even in the case of physically-based distributed-parameter models, uncertainty in the parameter estimates would be observed because of the input error coming from the spatial variability of rainfall.

One of the sources of the uncertainty in the model parameters/outputs could be inappropriate algorithms used to model the processes. Since we used the AGNPS model to get the true situations for the model outputs, this eliminates the possibility that the variability in the model parameters were due to the model shortcomings. In other words, the variability in the AGNPS parameters were not due to algorithmic problems, but due to rainfall spatial variability. The size of the watershed on which the model is applied is also not a source of this uncertainty because AGNPS model has been validated for relatively larger watersheds (Young et al., 1989).

When the model is calibrated against a number of ‘true’ rainfall events, i.e. when all stations are used to compute the most correct rainfall pattern from different events of different magnitude and spatial patterns, some uncertainty in the model parameters/outputs may still remain due to other sources defined in the Section 2.

Table 5
Relative errors in rainfall values

| Rainfall date | Relative error | |
|---|---|---|
| | Maximum | Minimum |
| 3/27/96 | 0.45 | 0.01 |
| 4/21/96 | 1.04 | 0 |
| 5/31/96 | 0.31 | 0.02 |
| 7/9/96 | 1.13 | 0.05 |
| 10/27/96 | 0.9 | 0.13 |

### *4.3. Relative errors in estimated parameters due to rainfall spatial variability*

Errors for estimated parameters relative to the

Table 6
Correlation among the estimated parameters

| | Rainfall | Slope | CN | *K* factor |
|---|---|---|---|---|
| 3/27/96 | | | | |
| Rainfall | 1 | | | |
| Slope | − 0.96 | 1 | | |
| CN | − 1 | 0.96 | 1 | |
| *K* factor | − 0.99 | 0.97 | 0.99 | 1 |
| 4/21/96 | | | | |
| Rainfall | 1 | | | |
| Slope | − 0.98 | 1 | | |
| CN | − 1 | 0.98 | 1 | |
| *K* factor | − 0.86 | 0.87 | 0.89 | 1 |
| 5/31/96 | | | | |
| Rainfall | 1 | | | |
| Slope | − 0.98 | 1 | | |
| CN | − 1 | 0.98 | 1 | |
| *K* factor | − 0.91 | 0.86 | 0.91 | 1 |
| 7/9/96 | | | | |
| Rainfall | 1 | | | |
| Slope | − 0.97 | 1 | | |
| CN | − 0.97 | 0.98 | 1 | |
| *K* factor | − 0.89 | 0.91 | 0.96 | 1 |
| 10/27/96 | | | | |
| Rainfall | 1 | | | |
| Slope | − 0.94 | 1 | | |
| CN | − 1 | 0.95 | 1 | |
| *K* factor | − 0.87 | 0.92 | 0.89 | 1 |
| *Overall* | | | | |
| Rainfall | 1 | | | |
| Slope | − 0.54 | 1 | | |
| CN | − 0.97 | 0.64 | 1 | |
| *K* factor | − 0.38 | 0.73 | 0.48 | 1 |

calibrated parameter values were calculated. Table 4 shows the errors in estimated parameters relative to the calibrated parameter values. The maximum and minimum relative errors in rainfall as compared to the area-weighted average rainfall for all events analyzed are shown in Table 5. Rainfall observed at each gauge location gave a different set of parameters that minimized the objective function.

The minimum and maximum relative errors for the parameters were derived from 13 to 17 sets of parameters for each event. The maximum relative error in CN, *S*, slope, and *K* factor were 0.94, 0.92, 0.83, and 1.63, respectively, for all events considered. The corresponding rainfall relative errors were 0.52, 1.04, 0.32, and 0.52, respectively. Maximum relative error in CN was obtained at the gauge 161 for rainfall on 7/9/96. The rainfall at this gauge location was minimum for this event. For *S*, the maximum relative error occurred at gauge 163 on 4/21/96. For this event, rainfall relative error and rainfall depths were maximum at this gauge location. Maximum relative error in slope estimate was at the gauges 132 and 150 on 4/21/96. The rainfall observed by these gauges was the minimum for the event. The maximum relative error in estimated *K* factor was associated with the minimum rainfall observed at the gauge 161 on 7/9/96.

The minimum relative errors for CN, *S*, slope, and *K* factor were zero. The corresponding rainfall relative errors were 0.09, 0.26, 0.09, and 0.15, respectively. The minimum relative error was close to zero for all events for all parameters. Here it should be noted that the minimum relative errors in the parameters were not associated with the rainfall minimum relative errors. For example, event on 4/21/96 had the rainfall measured at gauge 147 very similar to the area-weighted rainfall, but the relative errors in the parameters were not minimum at this location.

Maximum relative errors in CN and S were associated with the minimum rainfall observed at a gauge location for all events, except on 4/21/96. For the rainfall on 4/21/96, the maximum rainfall observed at the gauge 163 produced the maximum relative error in CN, and *S*. For estimated slope and *K* factor, the maximum relative error resulted from the gauges that observed the minimum rainfall. Since AGNPS is designed to predict erosion events, not low flow events, the relative errors in parameter estimates can be expected to be large for smaller rainfall events.

### 4.4. Correlation structure among the parameters

The correlation among the parameters and the input rainfall was calculated for all events (Table 6). The correlation of *S* with other parameters is not shown because *S* is derived from CN and its correlation will be similar, but opposite in sign, to that shown for CN.

The correlation analysis shows that for a particular event, the model parameters are highly correlated. High parameter correlation contribute to the difficulty of finding optimum parameter values for models having several parameters. When calculated over all events, the correlations are reduced as would be expected, but still indicate that the optimum parameter values are not independent of each other ($\alpha = 0.05$).

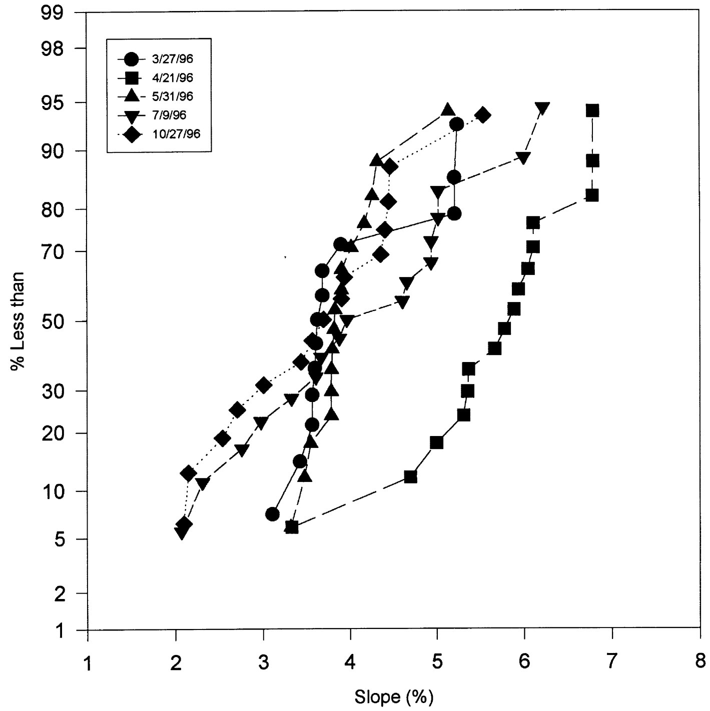


Fig. 3. Cumulative probability plot of estimated slope.

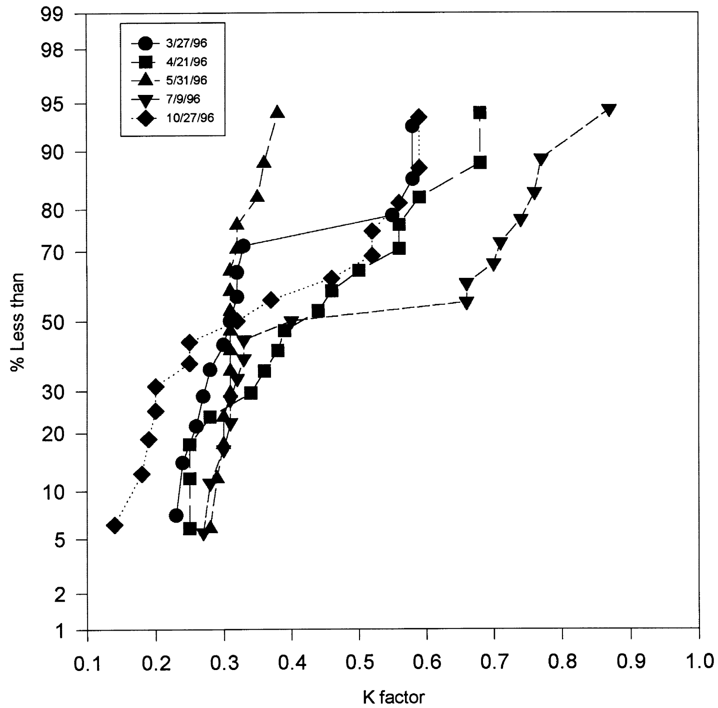


Fig. 4. Cumulative probability plot of estimated *K* factor.

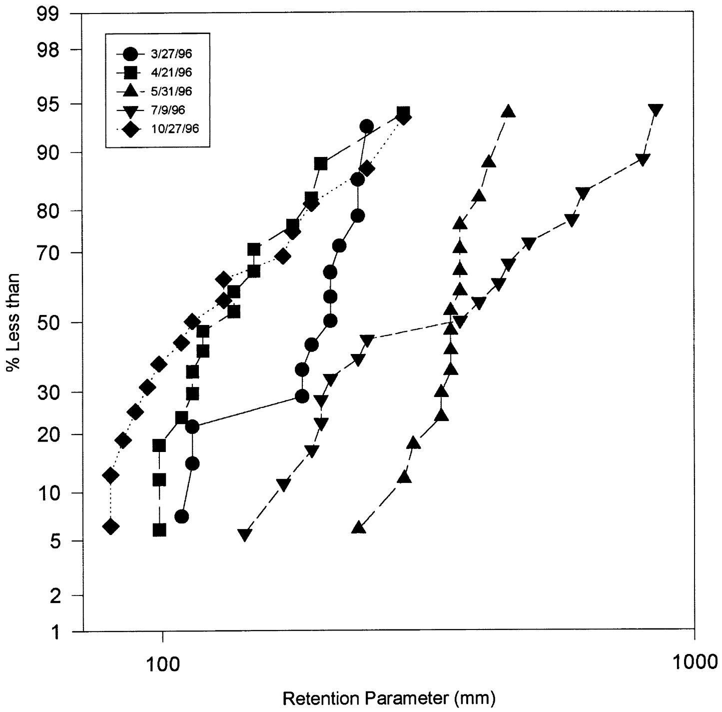


Fig. 5. Cumulative probability plot of estimated retention parameter.

A high correlation between optimized CN and rainfall was expected. In AGNPS, land slope is used to calculate the amount of sediment and nutrients eroded within each cell and the subsequent routing from each cell to the watershed outlet. A high significant negative correlation between slope-rainfall means for a given amount of sediment and nutrient transported at the watershed outlet, if the rainfall is higher, slope should be lower, and vice-versa to predict the sediment/nutrient transport equal to the observed output. The *K* factor is used in the Universal Soil Loss Equation (USLE) to calculate the amount of sediment and nutrients eroded at each cell. A high correlation between slope-*K* factor can be expected.

### 4.5. Probability structure of the estimated parameters

Cumulative probability plots of estimated parameters for all events are shown in Figs. 3–5. A chi-square test was conducted to see if slope and *K* factor estimates were normally distributed. Slope was found to be normally distributed for the rainfall events on 4/21/96, 7/9/96, and 10/27/96 ($\alpha = 0.05$). USLE *K* factor was normally distributed for the events on 4/21/96 and 10/27/96 ($\alpha = 0.05$). Retention parameter (*S*) is plotted on a log normal scale (Fig. 5) because *S* is assumed to have a log normal distribution. A chi-square test showed that the estimated *S* followed a log–normal distribution for the events on 4/21/96, 7/9/96, and 10/27/96 ($\alpha = 0.05$). Here, the total number of data points available for all events ranged from 13 to 17. These number of data points may be relatively small for determining the probability distribution function of the parameters.

These probability plots should be considered as marginal distributions. The true probability structure of the parameters taken as a group is multivariate since the parameters do have a correlation structure.

Fig. 3 shows that the probability of estimating a slope less than the true slope (3.71%) is greater than 0.50 for the rainfalls on 3/27/96 and 10/27/96. It means that, for a rainfall pattern like this, slope will be underestimated using the rainfall observed at majority of the gauges. For the other events analyzed, Fig. 3 shows that the probability of estimating a slope less than the true slope is less than 0.50 and the slope is overestimated using the rainfall observed at the majority of the gauges.

The true parameter value for the *K* factor was 0.33. Fig. 4 shows that *K* factor is overestimated for the rainfalls on 4/21/96 and 7/9/96 using the rainfall at a majority of the gauges and is underestimated for other events. The base values for the retention parameter were 188, 144, 351, 410, and 139 mm, respectively, for the rainfalls on 3/27/96, 4/21/96, 5/31/96, 7/9/96, and 10/27/96. The retention parameter was overestimated for the rainfall on 3/27/96, and underestimated for the rainfalls on the other events when the rainfall at each gauge location was used, one at a time, to estimate the parameter (Fig. 5).

## 5. Summary and conclusions

In general, a wide range in estimated parameters resulted when the rainfall measured at each gauge location was used individually, one at a time, to estimate the model parameters. A larger range in the rainfall values within a single event resulted in a higher range in all estimated parameters. The smallest parameter uncertainty resulted from the rainfall that was most spatially homogeneous in nature. The variations were very large when compared to the true parameter values. For slope, *K* factor, and retention parameter the range varied by several factors for some events. Traditionally, variability in the estimated parameters is considered as the model uncertainty because the models are simplified descriptions of the processes occurring in the field. Results of this study indicate that even in the case of physically-based distributed-parameter models, uncertainty in the parameter estimates would be observed because of the input error coming from the spatial variability of rainfall.

The correlation analysis among input rainfall and calibrated parameters showed that rainfall, CN, slope, and *K* factors were highly correlated. This correlation is a major contributor to the difficulty of estimating parameters in H/WQ models. A high correlation between two parameters means that one parameter cannot be estimated without adjusting the value of the other.

Spatial variability of rainfall must be captured and used in H/WQ models in order to accurately predict the hydrologic and water quality responses of watersheds. Since rainfall is a driving force behind many kinds of pollutant release and subsequent transport and spread mechanisms, ignoring this property of rainfall in the application of H/WQ models limits the accuracy of the model results. O'Connell and Todini (1996) have stressed the use of radar and dense network of rain gauge data to gain a better understanding of the hydrologic importance of rainfall spatial variability. Rainfall spatial pattern can be better captured using a network of rain gauges and radar rainfall data. A radar data, when calibrated with raingauges can give an accurate estimate of rainfall that are continuous in space and provide information on the spatial variability of rainfall. Ease of availability of radar rainfall data has provided the model developers and users an opportunity to capture and use spatially rainfall pattern to eliminate the errors due to input rainfall data.

## Acknowledgements

This research was supported by the Oklahoma Agricultural Experiment Station as a contribution to the Southern Regional Research Project S-273. Appreciation is expressed to Dr. J.D. Garbrecht and Dr. P.J. Starks of the USDA–ARS at El Reno, Oklahoma, for giving permission to use data from the Little Washita basin and the Micronet.

## References

Agricultural Research Service (ARS), 1991. Hydrology of the Little Washita River Watershed, USDA Agricultural Research Service Press, El Reno, OK.

Allred, B., Haan, C.T., 1996. Small watershed monthly hydrologic modeling system. Users Manual, Biosystems and Agricultural Engineering Department, Oklahoma State University, Stillwater, OK.

Beven, K.J., Hornberger, G.M., 1982. Assessing the effect of rainfall pattern of precipitation on modeling stream flow hydrographs. Water Resour. Bull. 18 (5), 823–829.

Corradini, C., Singh, V.P., 1985. Effect of spatial variability of effective rainfall on direct runoff by geomorphologic approach. J. Hydrol. 81, 27–43.

Dawdy, D.R., Bergman, J.M., 1969. Effect of rainfall variability on stream flow simulation. Water Resour. Res. 5 (5), 958–969.

Faures, J., Goodrich, D.C., Woolhiser, D.A., Sorooshian, S., 1995. Impact of small-scale spatial variability on runoff modeling. J. Hydrol. 173, 309–326.

Goodrich, D.C., Faures, J., Woolhiser, D.A., Lane, L.J., Sorooshian, S., 1995. Measurement and analysis of small-scale convective storm rainfall variability. J. Hydrol. 173, 283–308.

Grunwald, S., Frede, H.-G. 1997. Using AGNPSm in German watersheds, GCTE-Conference (Global Change and Terrestrial Ecosystems), Utrecht, Catena, 14–17 April, 1997.

Haan, C.T., 1989. Parametric uncertainty in hydrologic modeling. Trans. ASAE 32 (1), 137–146.

Hamlin, M.J., 1983. The significance of rainfall in the study of hydrological processes at basin scale. J. Hydrol. 65, 73–94.

Luzio, M.D., Lenzi, M.A. 1995. The importance of proper rainfall inputs for the applicability of the AGNPS model integrated with geographic information system at watershed scale, In the proceedings of the International Symposium on Water Quality Modeling, Orlando, FL, April 2–5.

Ma, S. 1993. Integrating GIS and remote sensing approaches with NEXRAD to investigate the impacts of spatial scaling of hydrologic parameters on storm-runoff modeling, PhD dissertation, University of Oklahoma Library, Norman, OK.

O'Connell, P.E., Todini, E., 1996. Modeling of rainfall, flow and mass transport in hydrological systems: an overview. J. Hydrol. 175, 3–16.

Osmond, D.L., Gannon, R.W., Gale, J.A., Line, D.E., Knott, C.B., Phillips, K.A., Turner, M.H., Foster, M.A., Lehning, D.E., Coffey, S.W., Spooner, J., 1997. WATERSHEDSS: A decision support system for watershed—scale nonpoint source water quality problems. J. American Water Resource Association 33 (2), 327–341.

Seliga, T.A., Aron, G., Aydin, K., White, E. 1992. Storm runoff simulation using radar rainfall rates and a unit hydrograph model (SYN–HYD) applied to GREVE watershed, In: American Meteorological Society, 25th International Conference on Radar Hydrology, pp. 587–590.

Shah, S.M.S., O'Connell, P.E., Hosking, J.R.M., 1996. Modeling the effects of spatial variability in rainfall on catchment response. 1. Formulation and calibration of a stochastic rainfall field model. J. Hydrol. 175, 67–88.

Troutman, B.M., 1985. Errors and parameter estimation in precipitation-runoff modeling. 1 Theory. Water Resour. Res. 21 (8), 1195–1213.

Troutman, B.M., 1983. Runoff predictions errors and bias in parameter estimation induced by spatial variability of precipitation. Water Resour. Res. 19 (3), 791–810.

Troutman, B.M., 1982. An analysis of input errors in precipitation-runoff models using regression with errors in the independent variables. Water Resour. Res. 18 (4), 947–964.

Wilson, C.B., Valdes, J.B., Rodriguez-Iturbe, I., 1979. On the influence of the spatial distribution of rainfall on storm runoff. Water Resour. Res. 15 (2), 321–328.

Young, R.A., Allesi, R.S., Needham, S.E., 1992. Application of a distributed parameter model for watershed assessment. Managing Water Resources During Global Change, American Water Resources Association, Reno, NV, November 1–5, pp. 107–115.

Young, R.A., Onstad, C.A., Bosch, D.D., Anderson, W.P., 1989. AGNPS: A nonpoint-source pollution model for evaluating agricultural watersheds. J. Soil and Water Conser. 44 (2), 168–173.