$$z = f(x, y)$$

$$z = g(x) = f(x, c)$$

$$\frac{\partial f(x,y)}{\partial x} = \frac{dz}{dx}$$

$$\nabla f = \left[\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right]$$

$f(\vec{x})$ $\quad f: \mathbb{R}^n \to \mathbb{R}$

$\nabla f(\vec{x}_0)$ $\quad O(n)$

$$\left[\frac{\partial f(\vec{x}_0)}{\partial \vec{x}_0[0]}, \ldots, \frac{\partial f(\vec{x}_0)}{\partial \vec{x}_0[n-1]} \right]$$

$$f(x) = 0$$

x_0

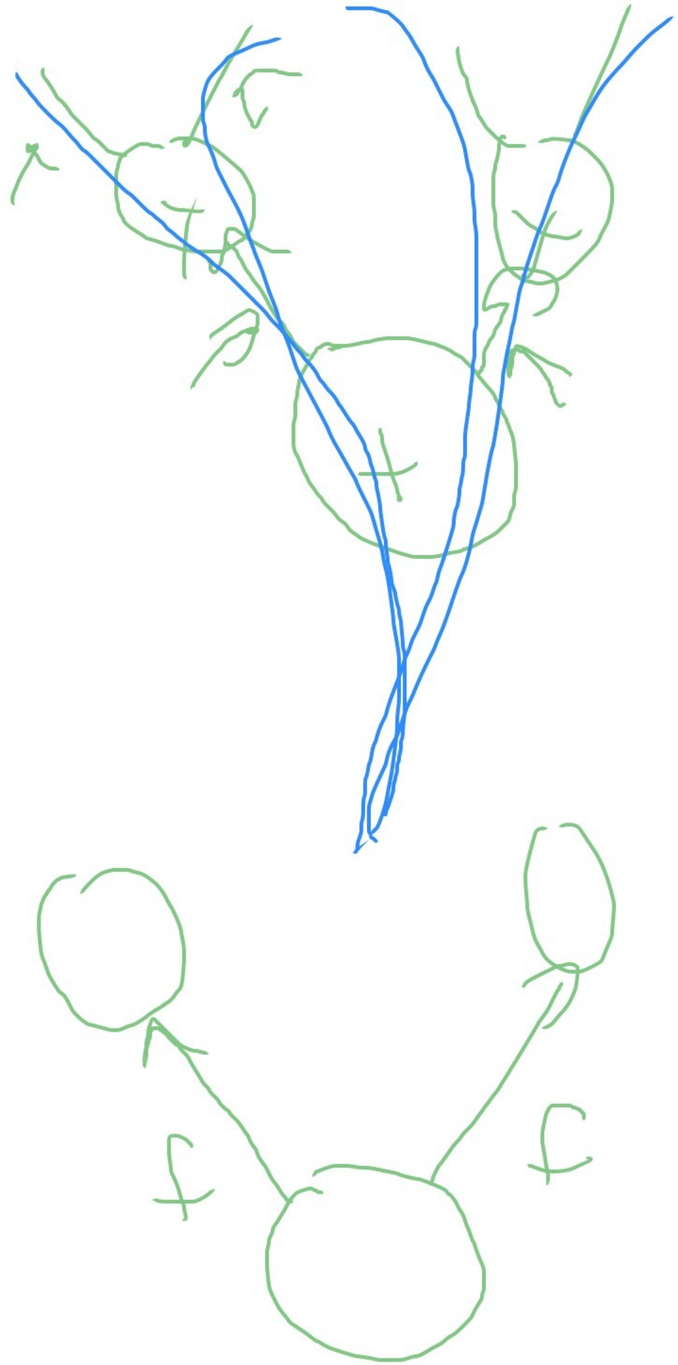
f'

$$x_{i+1} \leftarrow x_i - \frac{f(x_i)}{f'(x_i)}$$

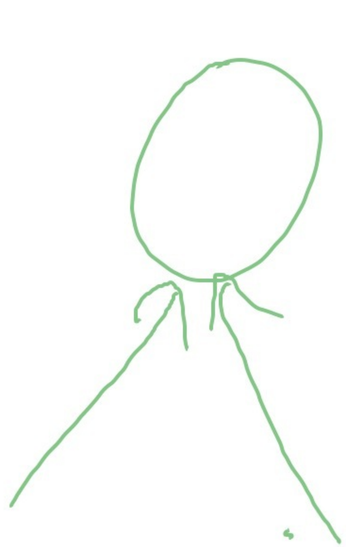
$$\arg\min_x f(x)$$

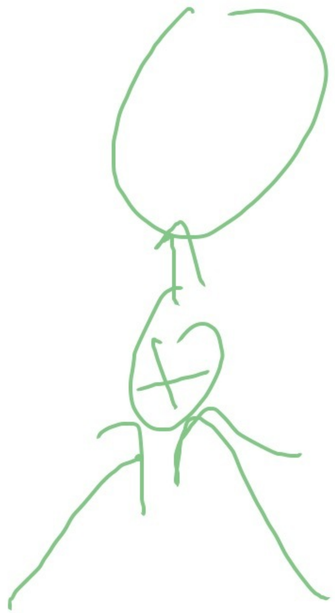
$$f'(x) = 0$$

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

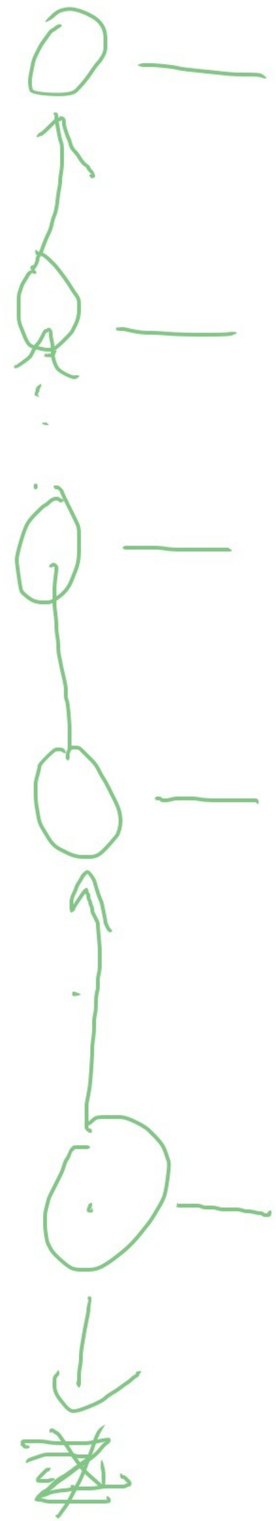


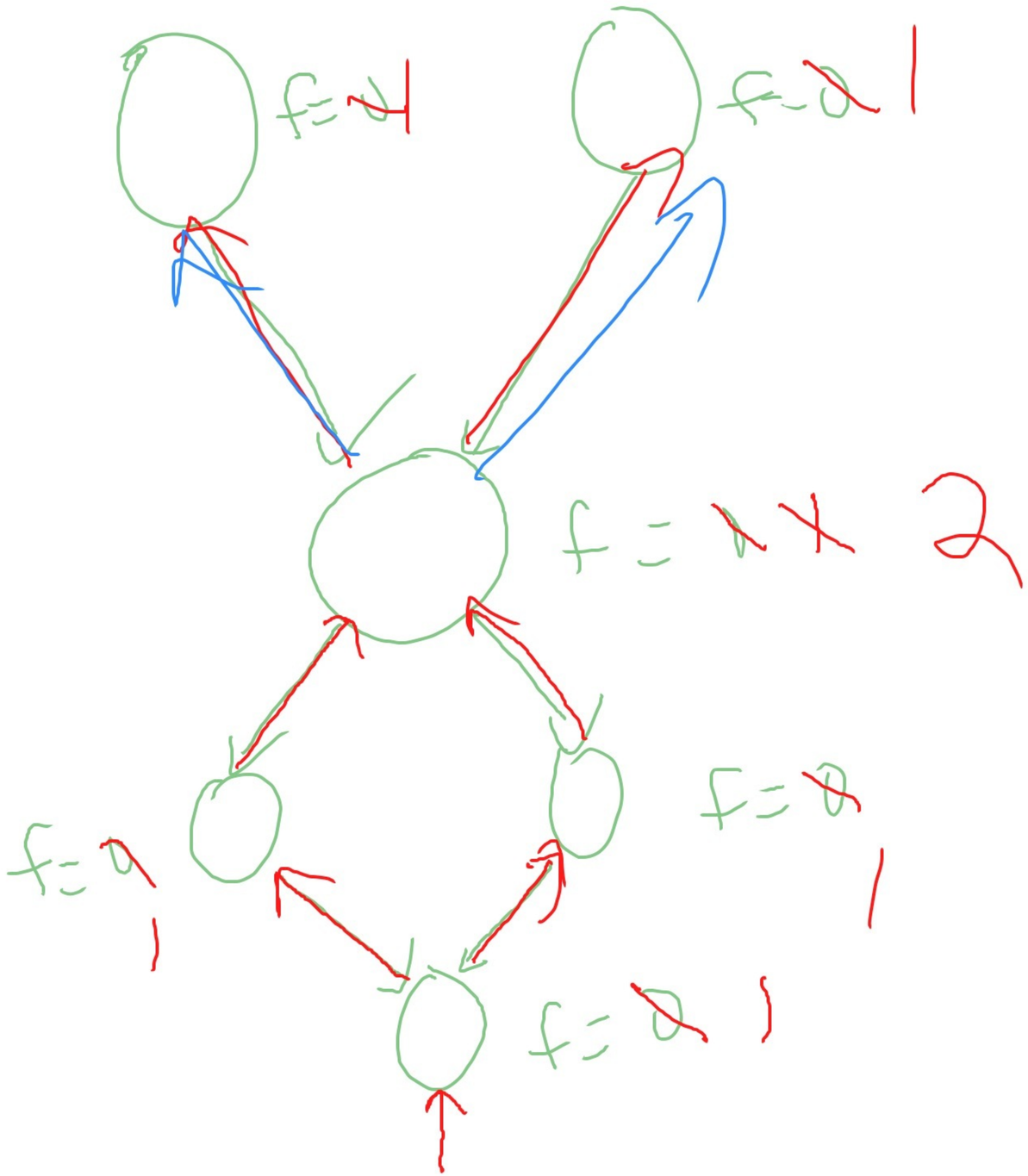
factory

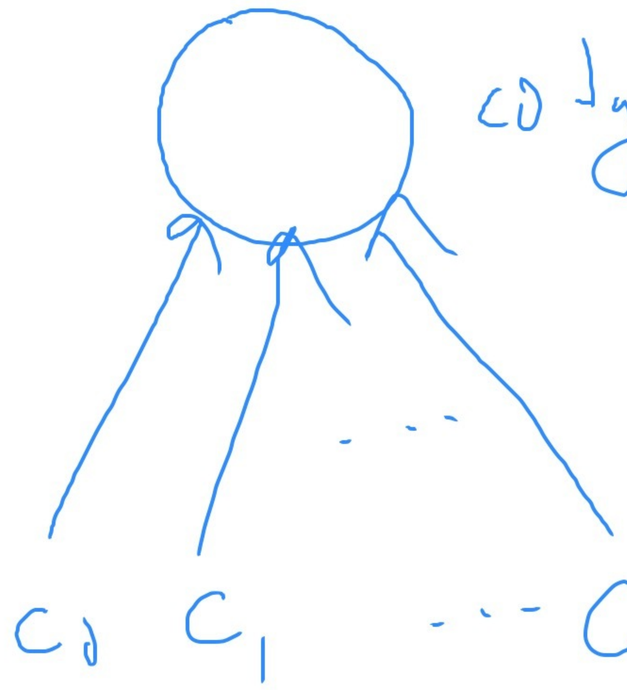




$\cot g$ —




f=4
f=0
f=2
f=0
f=0
1
f=0
1



$C_0 \quad C_1 \quad \cdots \quad C_{n-1}$

$\cot y = C_0 + C_1 + \cdots C_{n-1}$

$\cot y = 0$

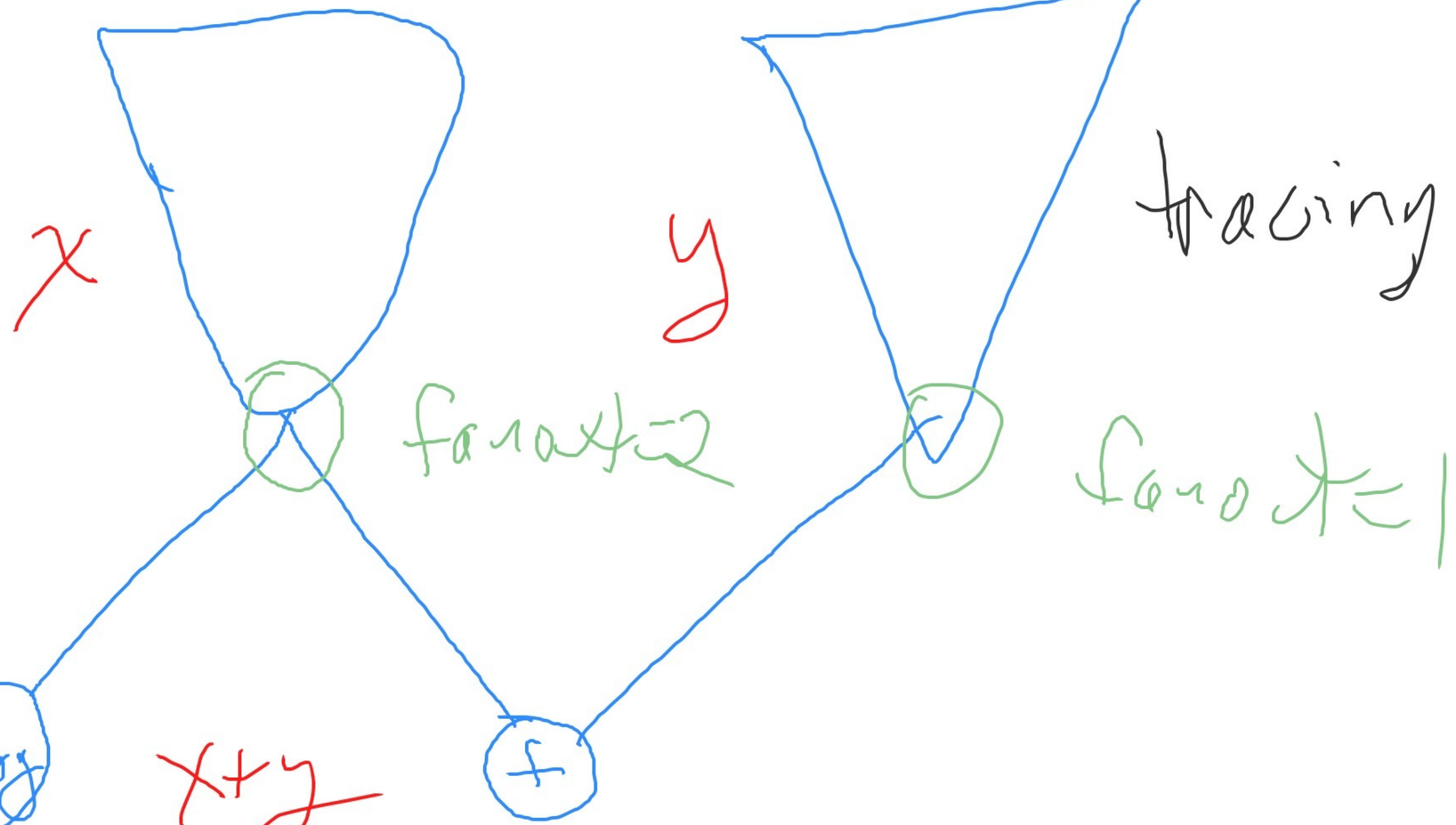
$\cot y \downarrow = C_0$

$\cot y \downarrow = C_1$

$\cot y \uparrow = C_n$

x, y

log(x)



log(x)

$$h(g(f(x)))$$

$$\frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$$

$$\frac{du}{dy} = \frac{du}{dz} \cdot \frac{dz}{dy}$$

$$\frac{du}{dx} = \frac{du}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

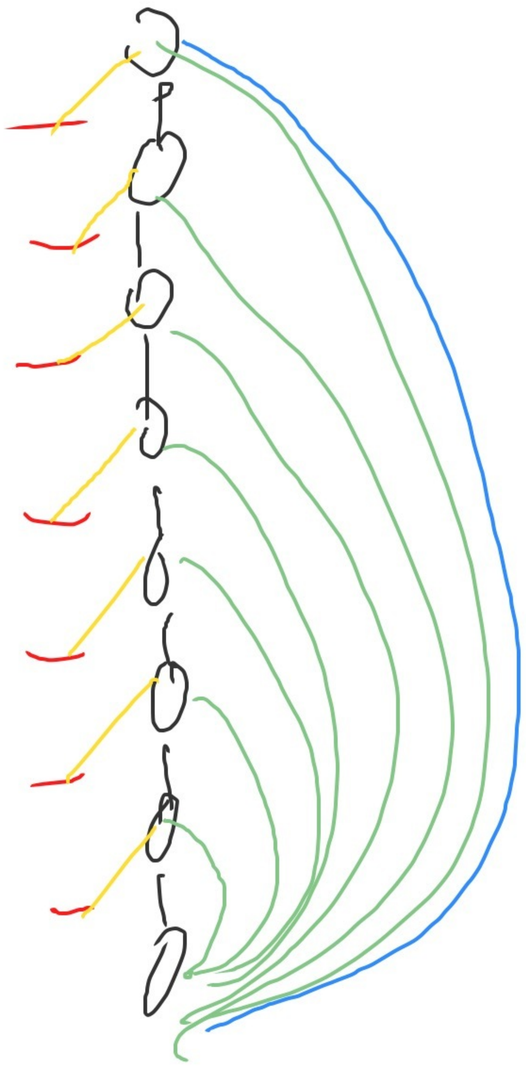
local derivatives factors

normal day division

cotangent

tg

ctg



local derivatives

multiplication

factor)

nonlocal derivatives

cotangents

primal

3

x

tape

cobundle

y

primal

4

tape

cobundle

factor

$\frac{\partial x+y}{\partial x}\Big|_{3,4}$

local derivative

$x+y$

$\frac{\partial x*z}{\partial y}\Big|_{3,4}$

factor

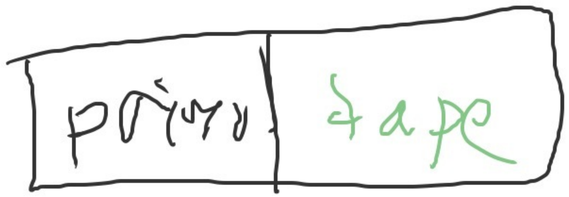
local derivative

7

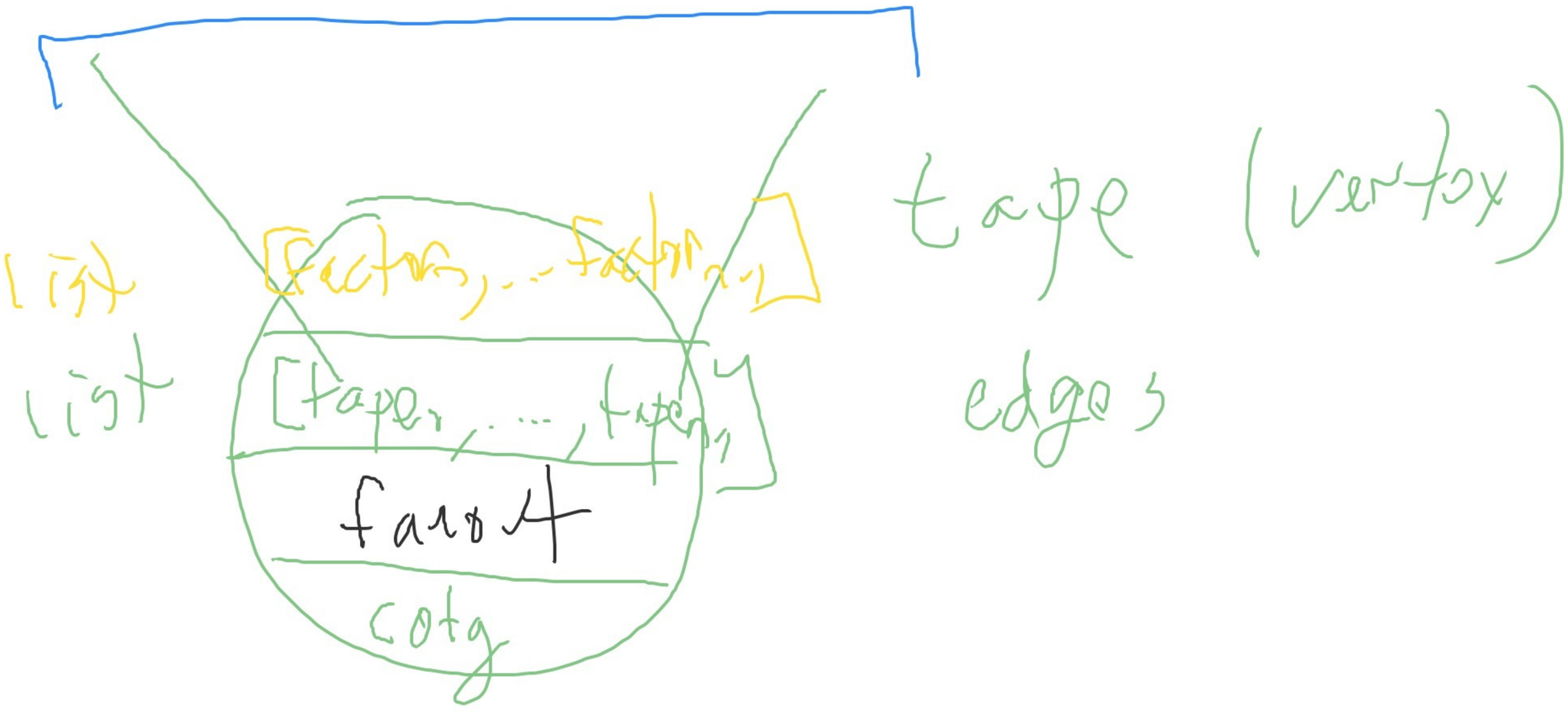
primal

tape

cobundles

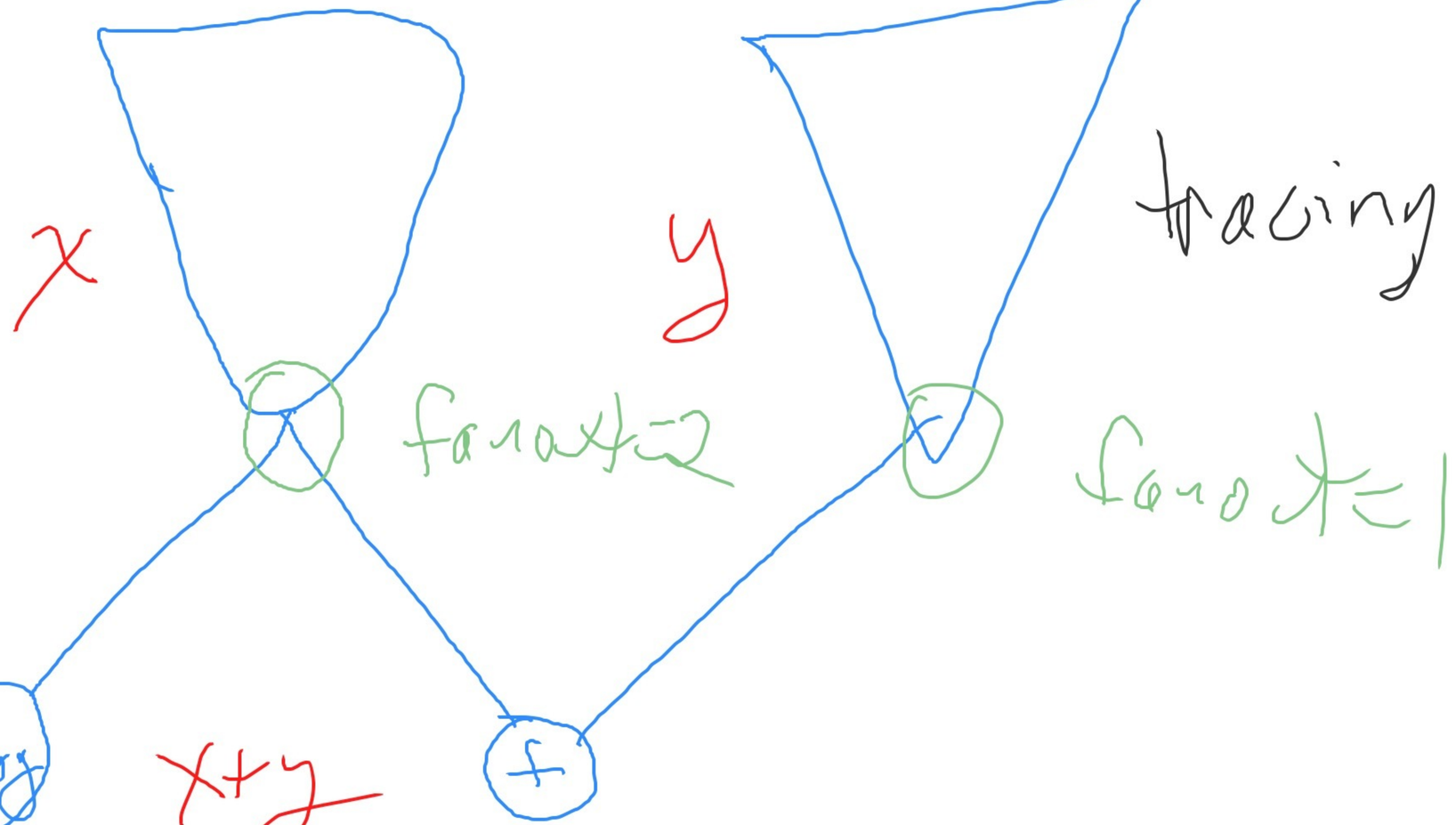

prim
tape

column


tape (vertex)
[fact0, ... fact2n-1]
int
list
[tape0, ... tape2n-1]
edges
fanout
cotg

x, y

log(x)



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local derivatives factors

normal day division

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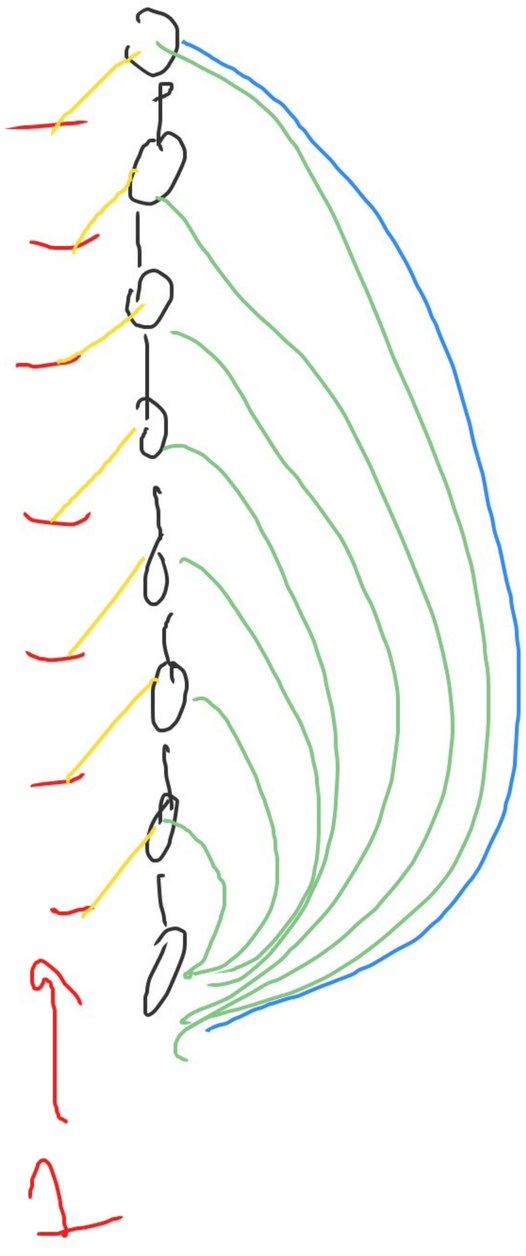
tg

ctg



multiplicity

local derivatives
(factor)

nonlocal derivatives
cotangents

primal

3

x

tape

cobundle

y

primal

4

tape

cobundle

factor

$\frac{\partial x+y}{\partial x}\Big|_{3,4}$

local derivative

$x+y$

$\frac{\partial x*z}{\partial y}\Big|_{3,4}$

factor

local derivative

7

primal

tape

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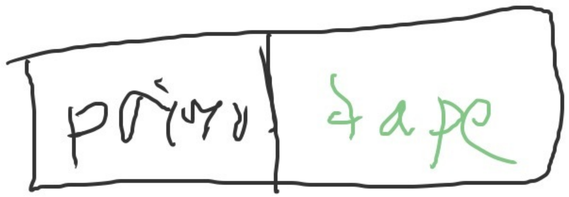
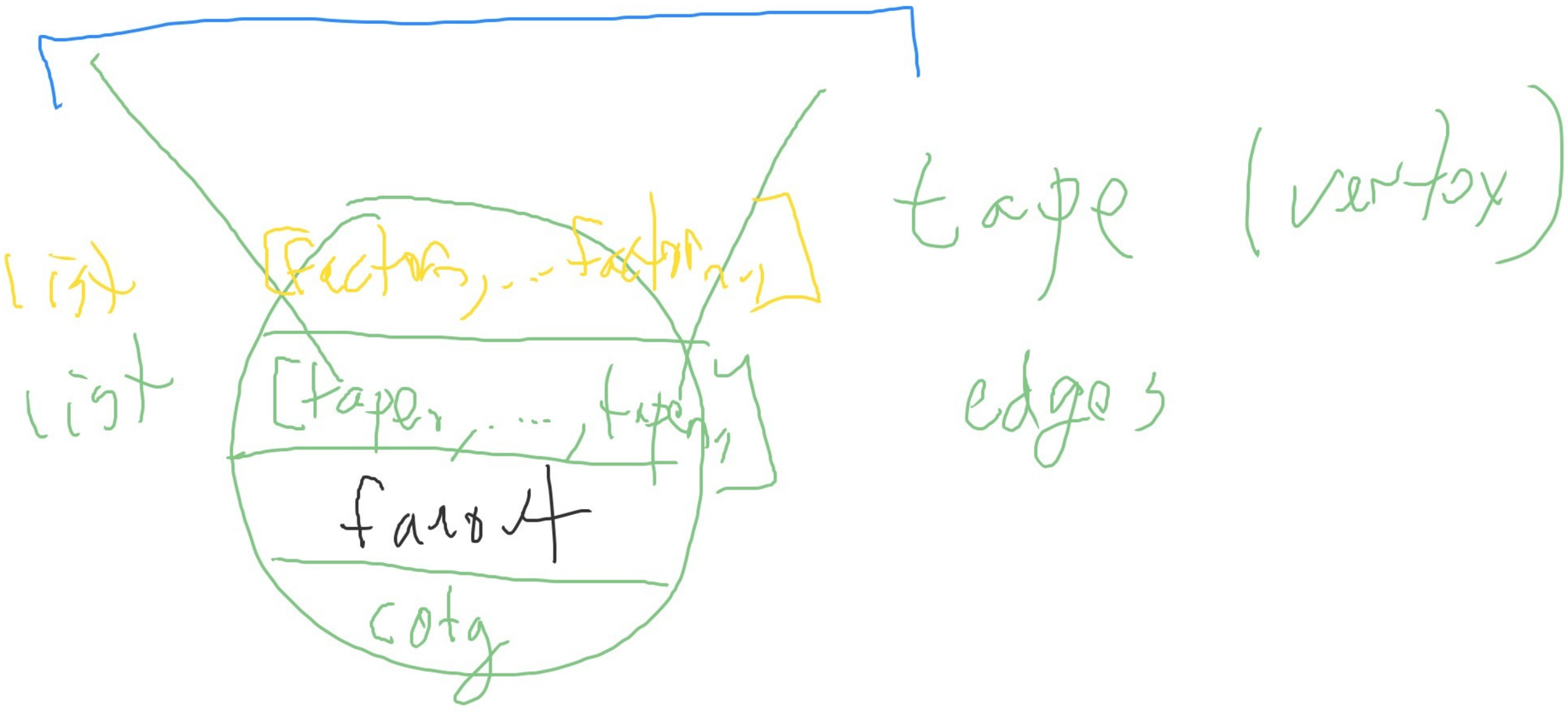

prim
tape

column


tape (vertex)
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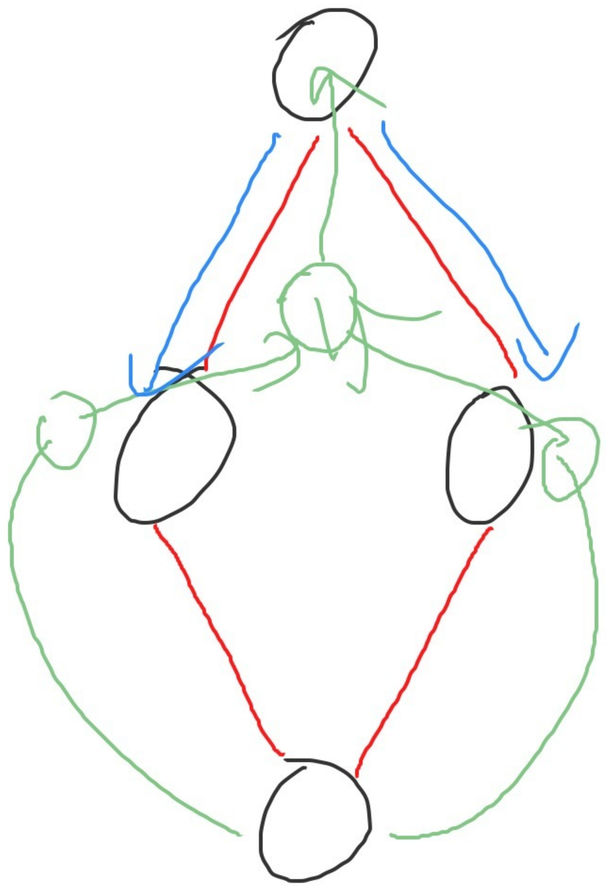
[tape0, ... tape2n-1]
edges
fanout
cotg

$\mathbb{R} \to \mathbb{R}$ exp, log, sin, cos

$\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ $+, -, \times, \div$

$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ $f(a,b,x) = ax+b$

$\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ $g(a,b,x,y) = ax+by$



fanout in prime!

addition in coty accumulations

$$- f_1 x_1 + f_2 x_2 + f_3 x_3 + f_4 x_4$$

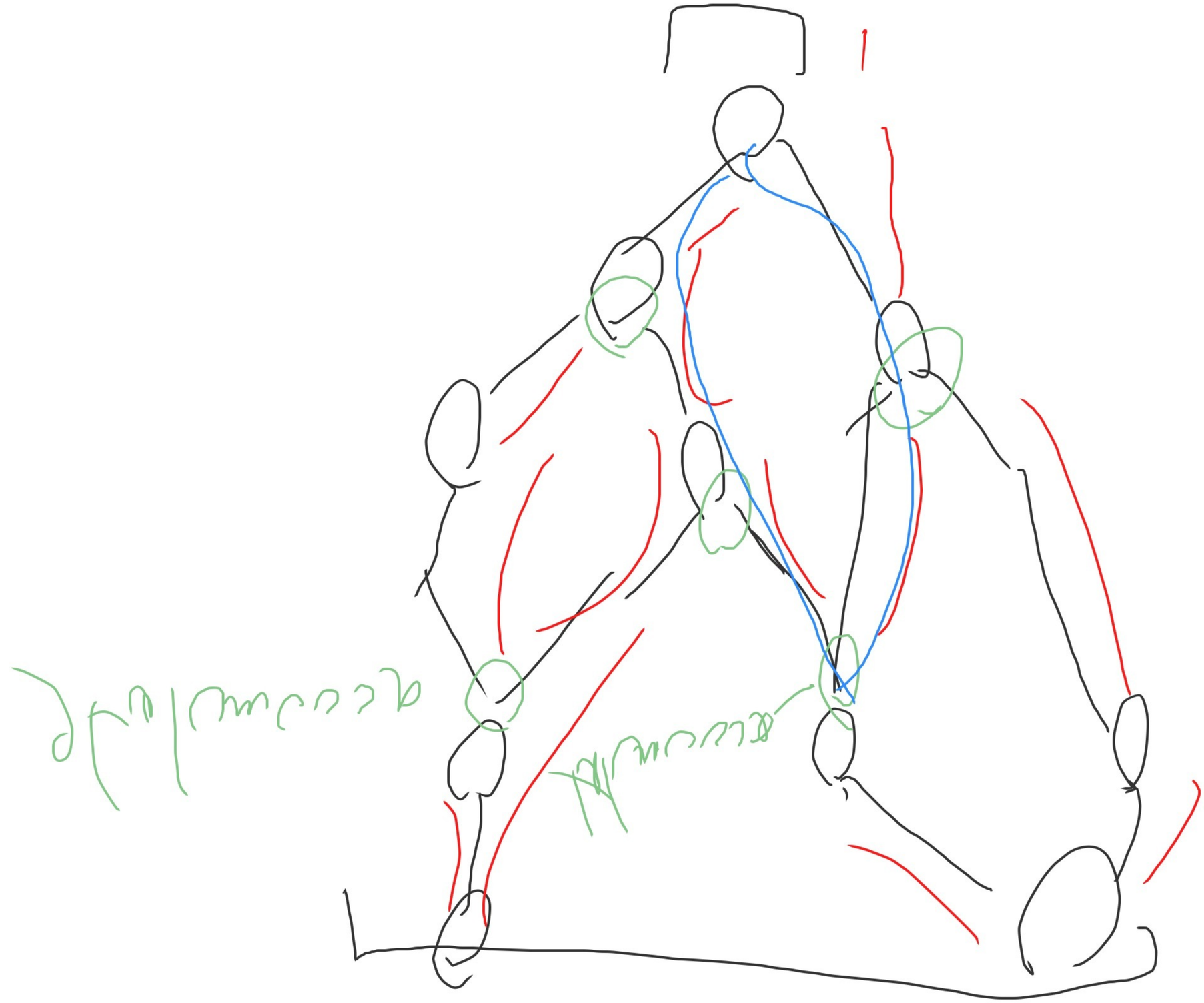
$$coty = 0$$

$$coty = coty + f_1 x_1$$

$$coty = coty + f_2 x_2$$

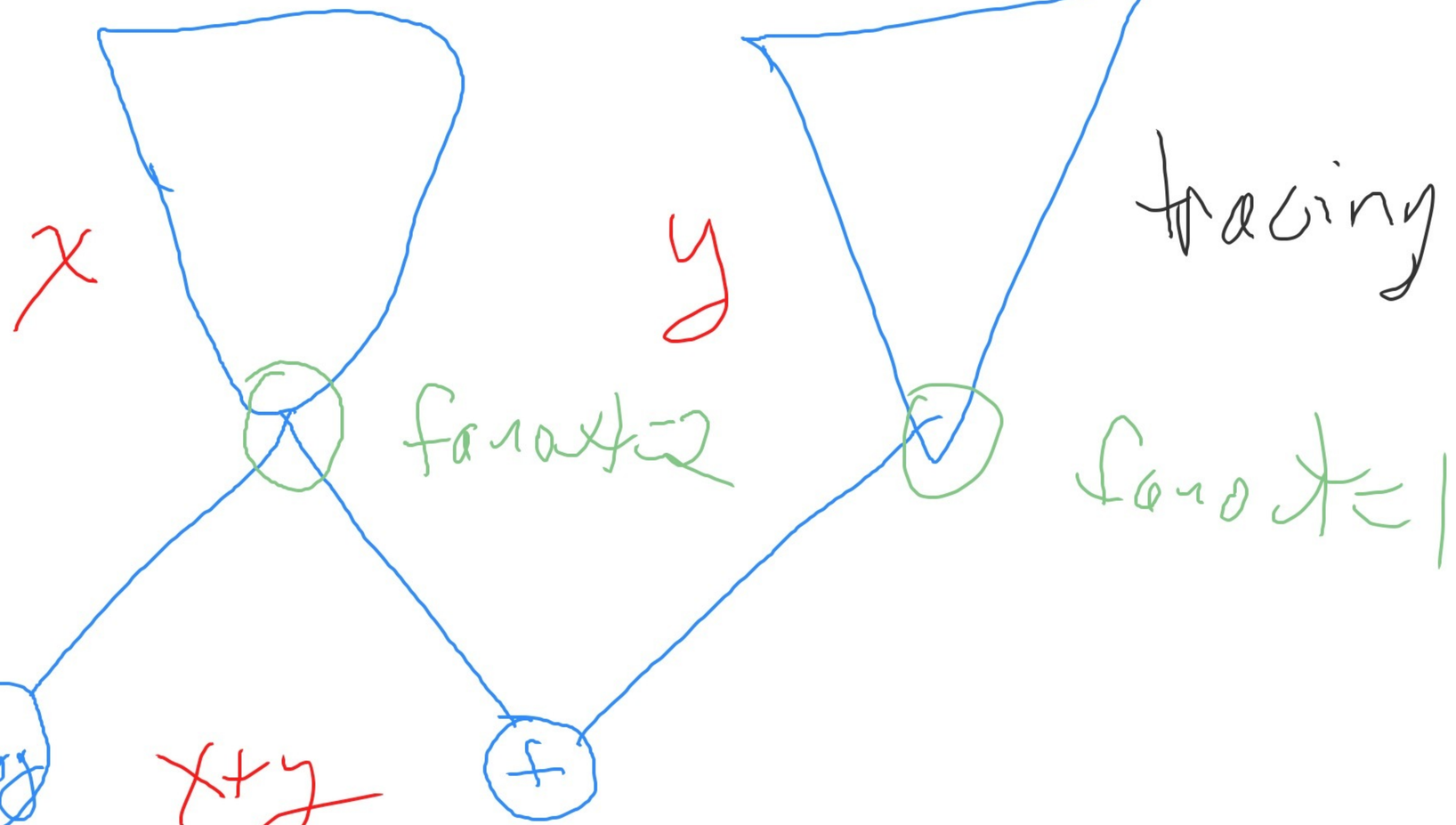
$$coty = coty + f_3 x_3$$

$$coty = coty + f_4 x_4$$



x, y

log(x)



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local derivatives factors

normal day division

cotangent

tg

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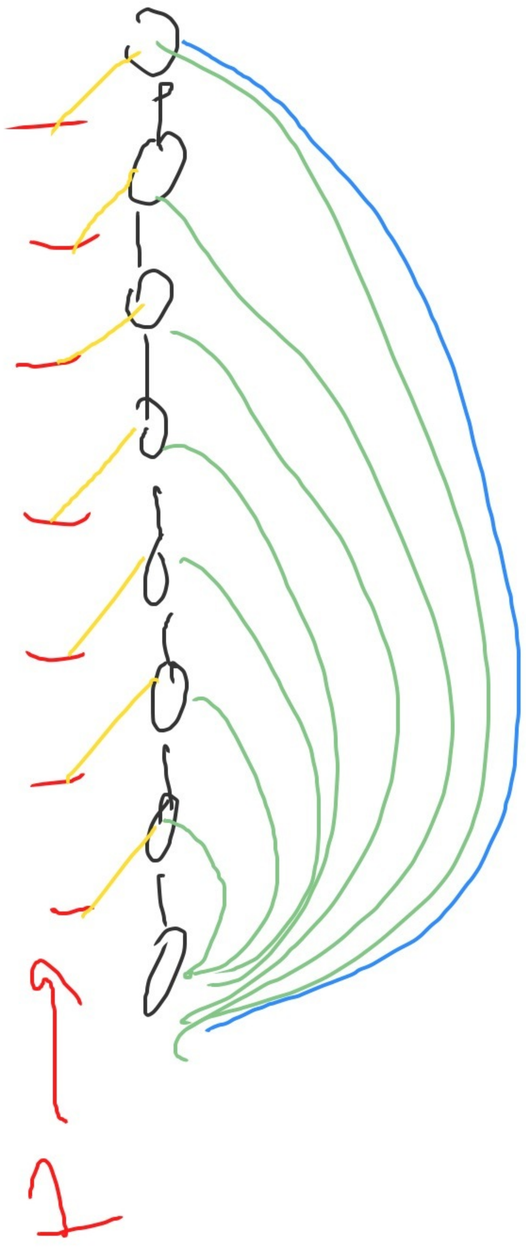


multiplicity

local derivatives
(factor)

nonlocal derivatives
cotangents

primal

3

x

tape

cobundle

y

primal

4

tape

cobundle

factor

$\frac{\partial x+y}{\partial x}\Big|_{3,4}$

local derivative

$x+y$

$\frac{\partial x*z}{\partial y}\Big|_{3,4}$

factor

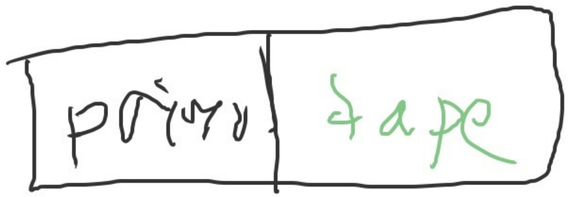
local derivative

7

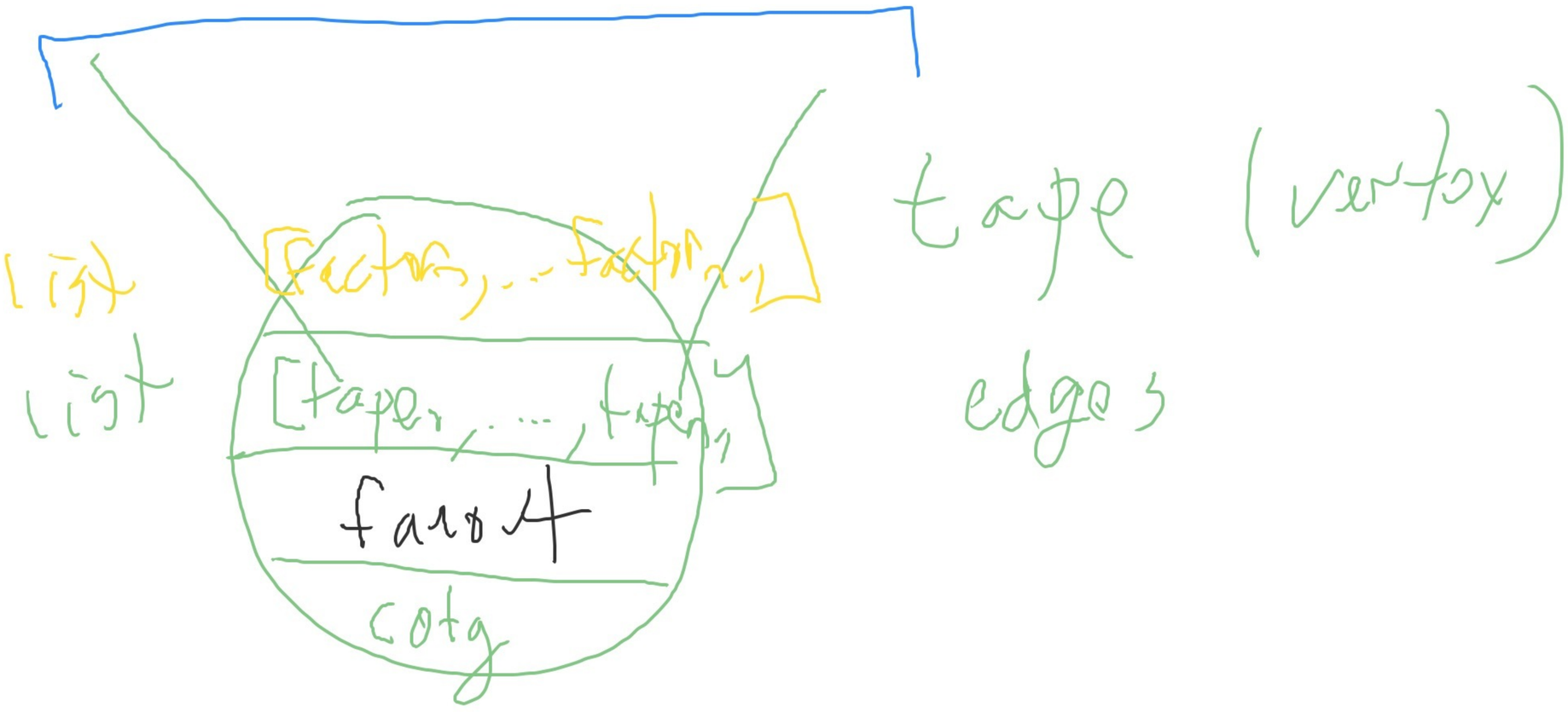
primal

tape

cobundles


prim
tape

column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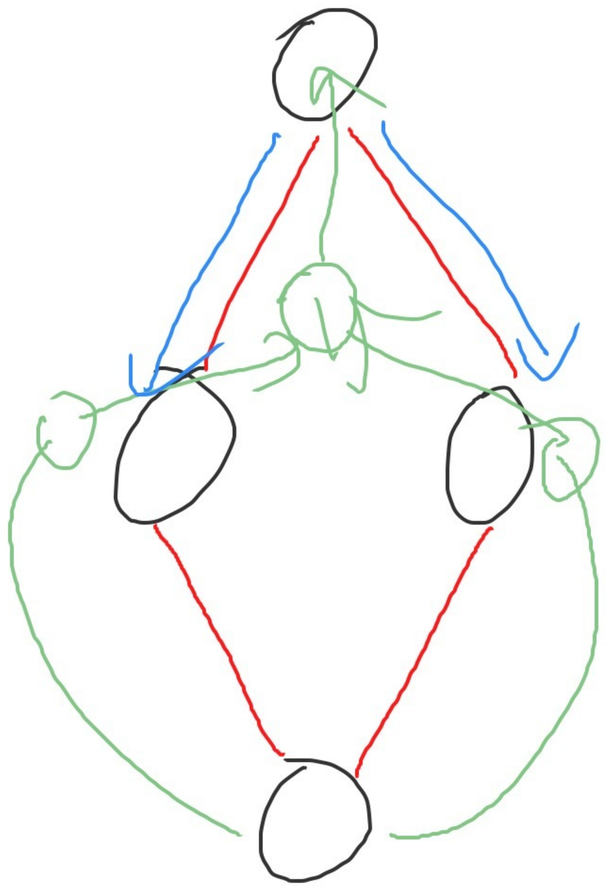


fanout in prime!

addition in coty accumulations

now to initialize coty = 0

— $f_1 x_1 + f_2 x_2 + f_3 x_3 + f_4 x_4$

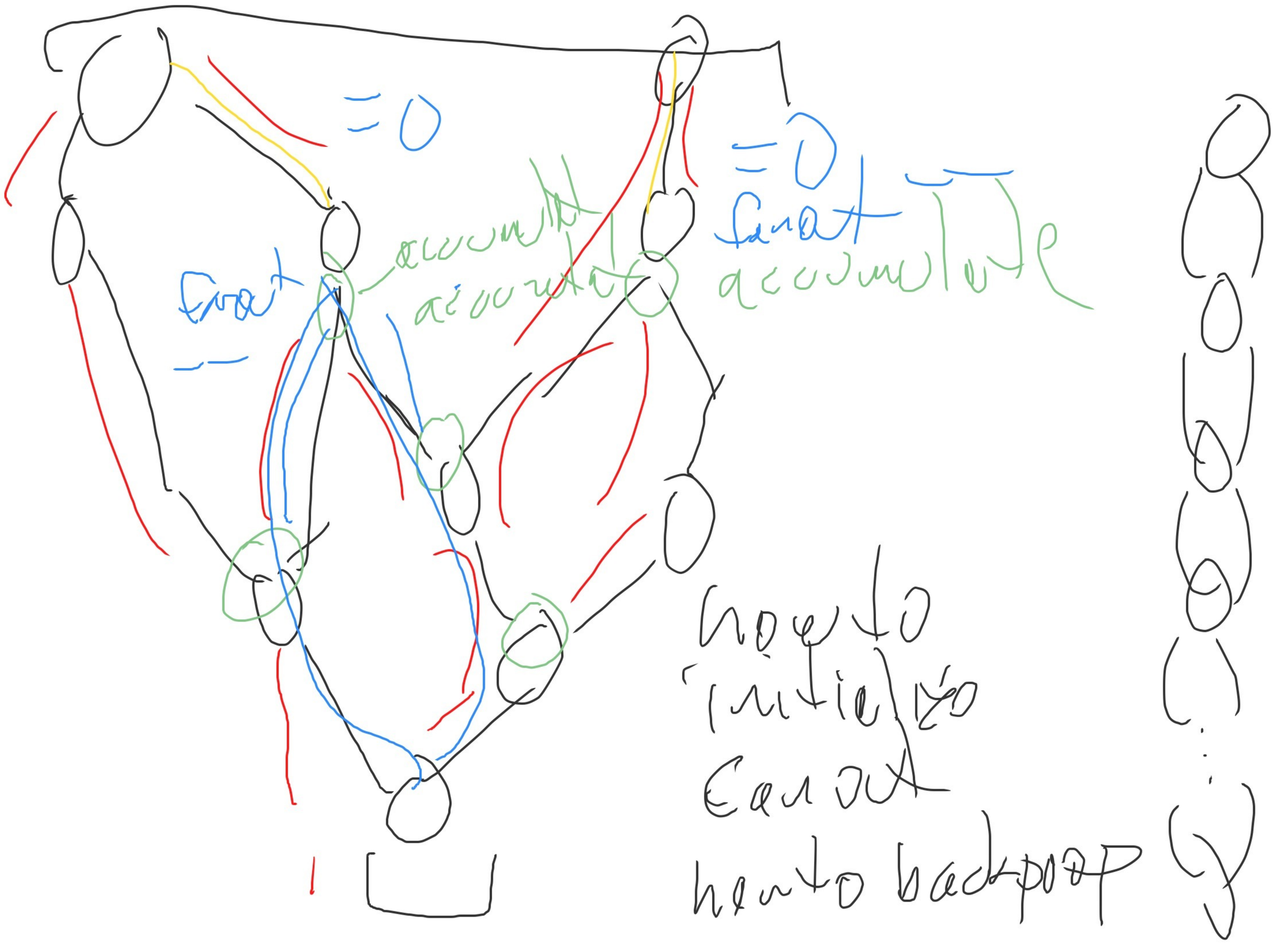
coty = 0

coty = coty + $f_1 x_1$

coty = coty + $f_2 x_2$

coty = coty + $f_3 x_3$

coty = coty + $f_4 x_4$


=0
=0
how to
initialize
Each
how to backprop