Straight-Line Distance Calculation

Straight-line distance otherwise known as the crow flies is the simplest way of measuring distance between two destinations. However, this distance can be calculated using different formulas based on a variety of assumptions.

Pythagorean Theorem or Euclidean Distance

Distance = sqrt((X2 - X1)^2 + (Y2 - Y1)^2)

where X1, Y1 and X2, Y2 are the Cartesian coordinates of your destinations.

The above equation you may remember from your high school math. It is what you used to find the length of the hypotenuse for a right angle triangle given two points.

You can only use this equation when your coordinates are projected geographical coordinates and the distance between the two destinations is less than 20 kilometers (12 miles). The programs that convert postal codes (zip codes or post codes) into latitude and longitude values are using the spherical coordinates.

Spherical Distance

As you are aware, the earth is not a flat surface. The Pythagorean theorem does not consider the curvature of the earth in its calculation. We can use spherical trigonometry to determine the straight-line (curvature) distance between two destinations.

Earth's Radius

Firstly, you have to determine what radius of the earth you would like to use in your equation. The earth's radius is longest at the equator (6378km) and shortest at the north and south poles (6357km). The average radius of the earth is 6371km. This value is generally used. If a majority of your destinations are at the same latitude you may want to calculate the radius of the earth at that specific latitude.

radius(in km) = 6378 - 21 * sin(lat)
radius(in mi) = 3963 - 13 * sin(lat)

where lat is a latitude of the area.

A more accurate calculation of the earth's radius can be calculated as the following:

radius' = a * (1 - e^2) / (1 - e^2 * sin^2(lat))^(3/2)

where a is the equatorial radius, b is the polar radius, and e is the eccentricity of the ellipsoid = (1 - b^2/a^2)^(1/2). This radius takes into consideration that the earth is not really a sphere but rather an ellipsoid.

Conversion of Coordinates

Latitude and longitude must be in decimal coordinates (degrees). In order to convert degrees, minutes and seconds into decimal coordinates the following equation can be used:

Decimal degrees = degrees + minutes/60 + seconds/3600

This calculation can be found at the following url:

http://andrew.hedges.name/experiments/convert_lat_long/

Most computers use radians when calculating the trigonometry functions such as, sine, cosine, arccosine, etc. In order to convert the decimal degrees into radians:

Decimal degree = Pi/180 = 3.141592654…/180

Law of Cosines for Spherical Trigonometry

a = sin(lat1) * sin(lat2)
b = cos(lat1) * cos(lat2) * cos(lon2 - lon1)
c = arccos(a + b)
d = R * c

where R is the radius of the earth.

Haversine Formula

dlon = lon2 - lon1
dlat = lat2 - lat1
a = sin^2(dlat/2) + cos(lat1) * cos(lat2) * sin^2(dlon/2)
c = 2 * arcsin(min(1,sqrt(a)))
d = R * c

where c is the great circle distance in radians and R is the radius of the earth.

The Haversine Formula is more accurate than the law of cosines formula because of problems associated with small distances. For both methods, you need to have double precision in the decimal place (i.e. keep at least 14 digits after the decimal place). This will avoid any round-off errors and as well any problems that may occur when the computer calculates either the arccosine or the arcsine.

Websites

There are numerous websites that you can use to calculate the spherical distance by plugging in your decimal degrees coordinates or your degrees/minutes/seconds coordinates.

http:/www/movable-type.co.uk/scripts/LatLong.html

http://efficacy.net/experiments/haversine/

http://williams.best.vwh.net/gccalc.htm

http://www.wcrl.ars.usda.gov/cec/java/lat-long.htm

References:

Equations came from http:/op.gfz-potsdam.de/GMT-Help/Archive/msg00143.html

They can also be found at http://www.census.gov/cgi-bin/geo/gisfaq?Q5.1