Summation/Pipe Problems

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Adding dB-SPL

Two acoustic sources “a” and “b” of relative phase angles θ_a and θ_b

$$\Sigma_{\text{dB-SPL}a+b} =$$

$$20 \log_{10} [\text{sqrt} \{ (10^{\text{dB-SPLa}/20})^2 + (10^{\text{dB-SPLb}/20})^2$$

$$+ 2(10^{\text{dB-SPLa}/20}) (10^{\text{dB-SPLb}/20})(\cos(\theta_a-\theta_b))\}]$$

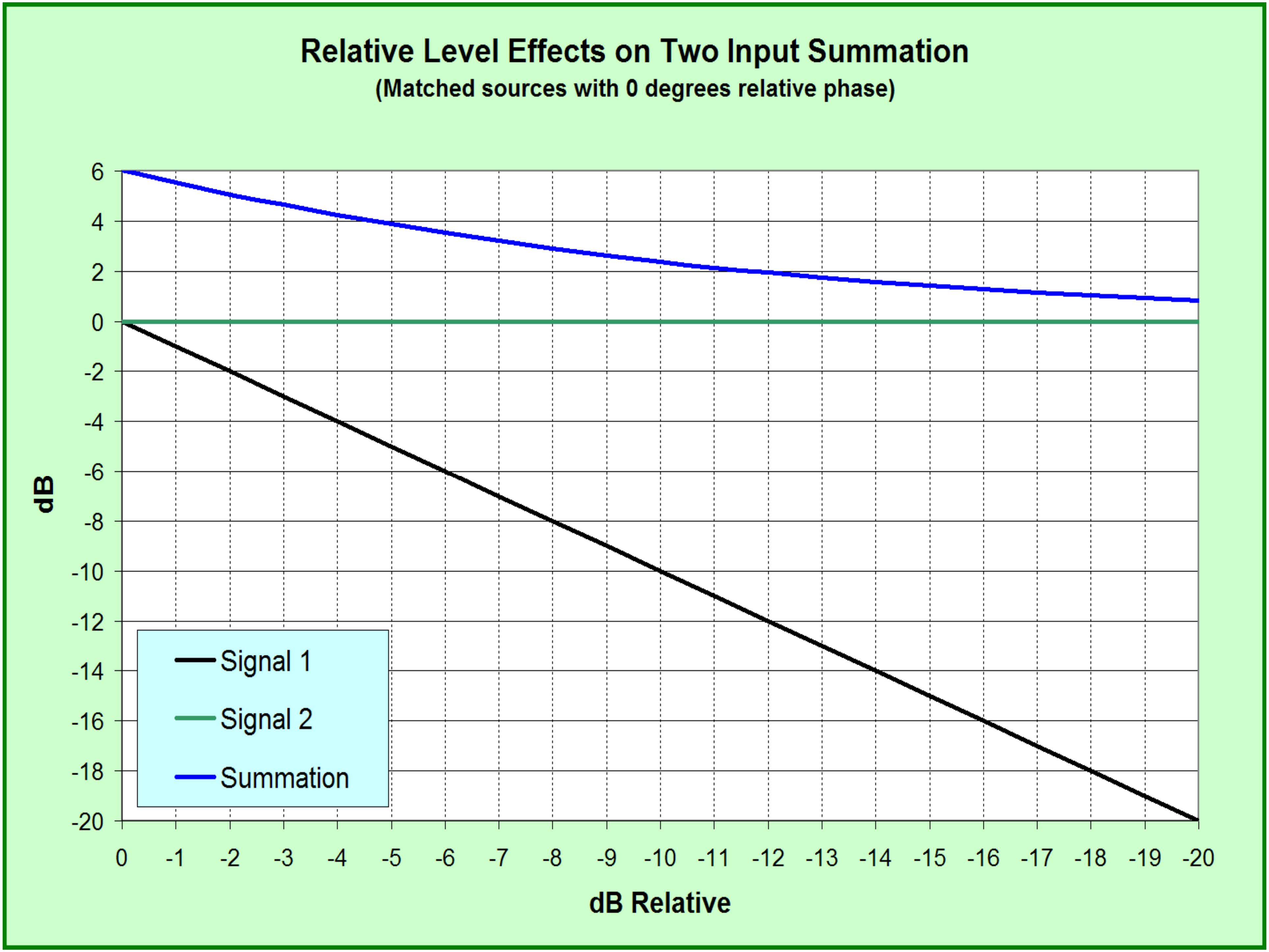
Adding dB-SPL – “Simplifications”

If both sources are in phase and only the relative level varies (where source “a” is 0 dB, simplifies to:

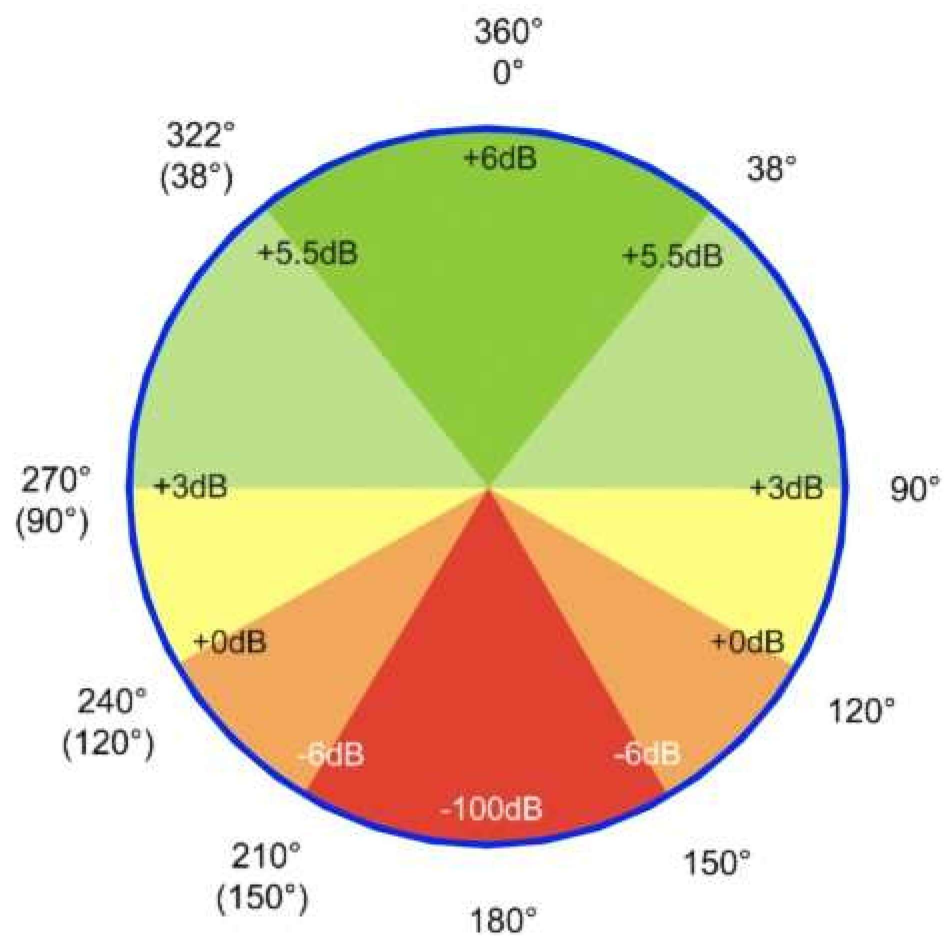
$$\Sigma_{dB\text{-}SPLa+b} = 20 \log_{10} \left[1 + 10^{dB\text{-}SPLb/20}\right]$$

If both sources are at 0 dB and phase angle $\theta_a = 0$ (i.e., same level, only relative phase angle varies), simplifies to :

$$\Sigma_{dB\text{-}SPLa+b} = 20 \log_{10} \left[\text{sqrt} \{ 2 + 2\cos(-\theta_b) \} \right]$$

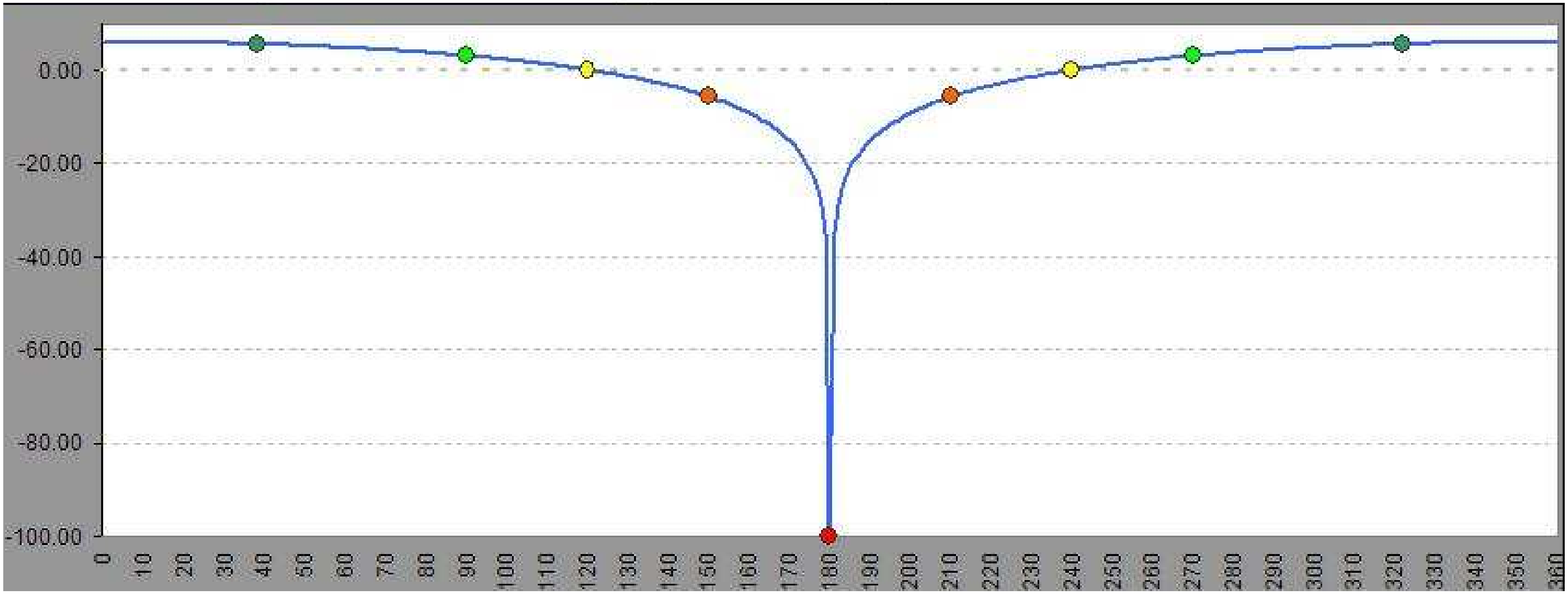

Relative Level Effects on Two Input Summation
(Matched sources with 0 degrees relative phase)
dB
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15 -16 -17 -18 -19 -20
dB Relative
Signal 1
Signal 2
Summation

Acoustic Addition & Subtraction: The Phase Wheel



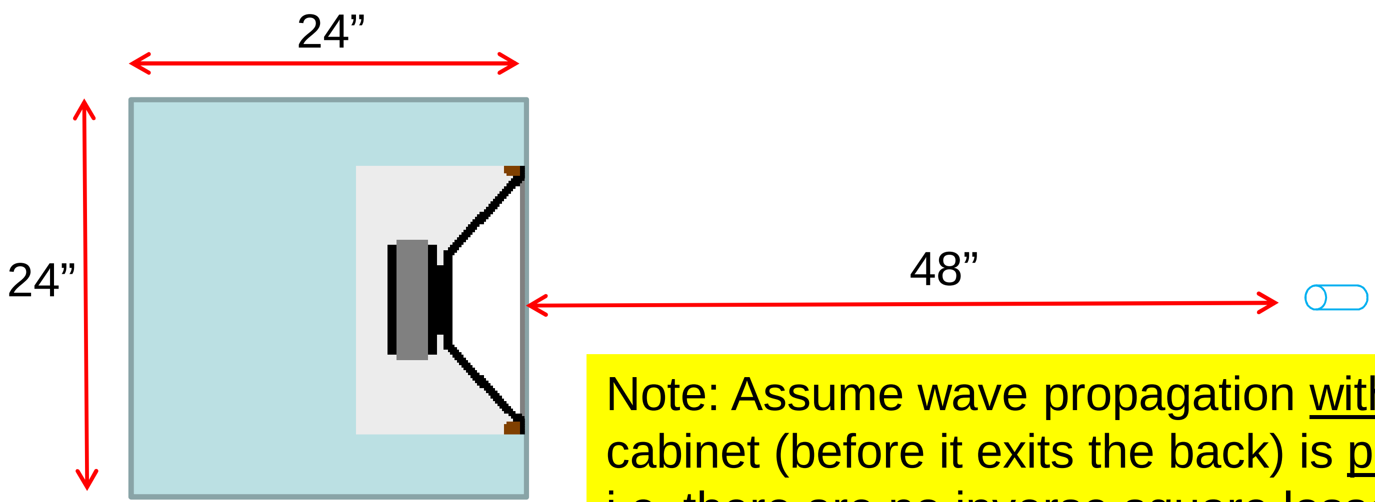
two identical signals summed at same level

Acoustic Addition & Subtraction: Level vs. Phase



two identical signals summed at same level

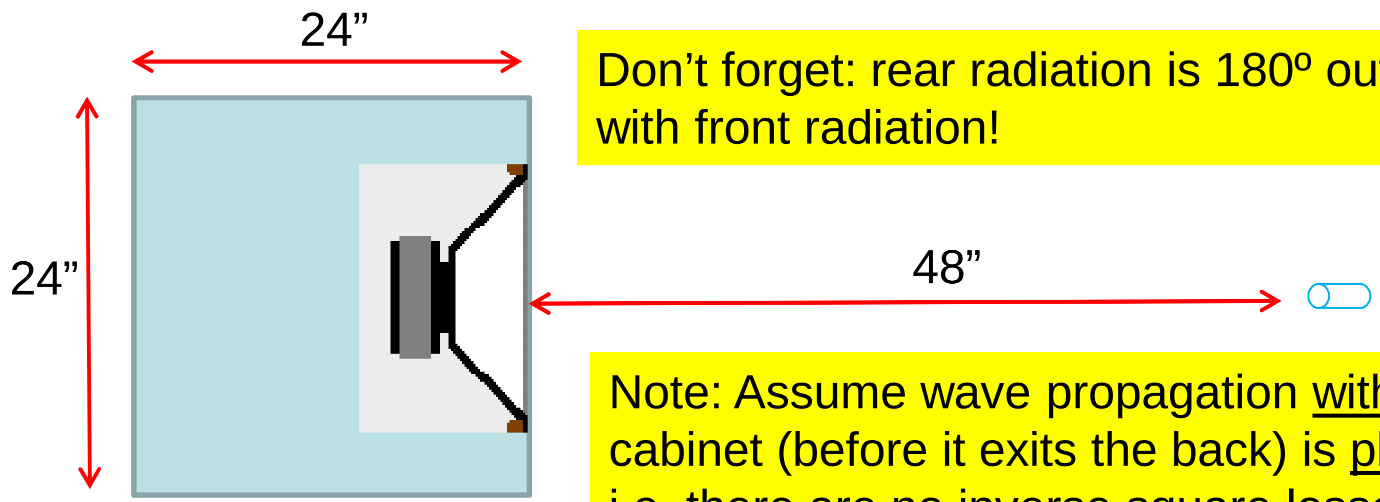
Open Back Cabinet



Note: Assume wave propagation <u>within</u> cabinet (before it exits the back) is <u>planar</u>, i.e. there are no inverse square losses.

A “full-range” loudspeaker is mounted in a free-standing 24” deep open-back cabinet. The loudspeaker is known to produce an SPL of 90 dB at a distance of 12” from the front radiating surface when operated at 100 Hz. Calculate the combined level (of the contributions from the loudspeaker’s front and rear radiating surfaces) at the measurement microphone location (48” from the front vibrating surface). Repeat the calculation if the frequency of operation is 50 Hz or 200 Hz (assume SPL is the same). HINT: Radiation from the rear of the cabinet can be assumed to be omnidirectional.

Solution for 100 Hz



Don’t forget: rear radiation is 180º out of phase with front radiation!

Note: Assume wave propagation within cabinet (before it exits the back) is planar, i.e. there are no inverse square losses.

Wavelength (λ) at 100 Hz = (1130x12)/100 = 135.6”

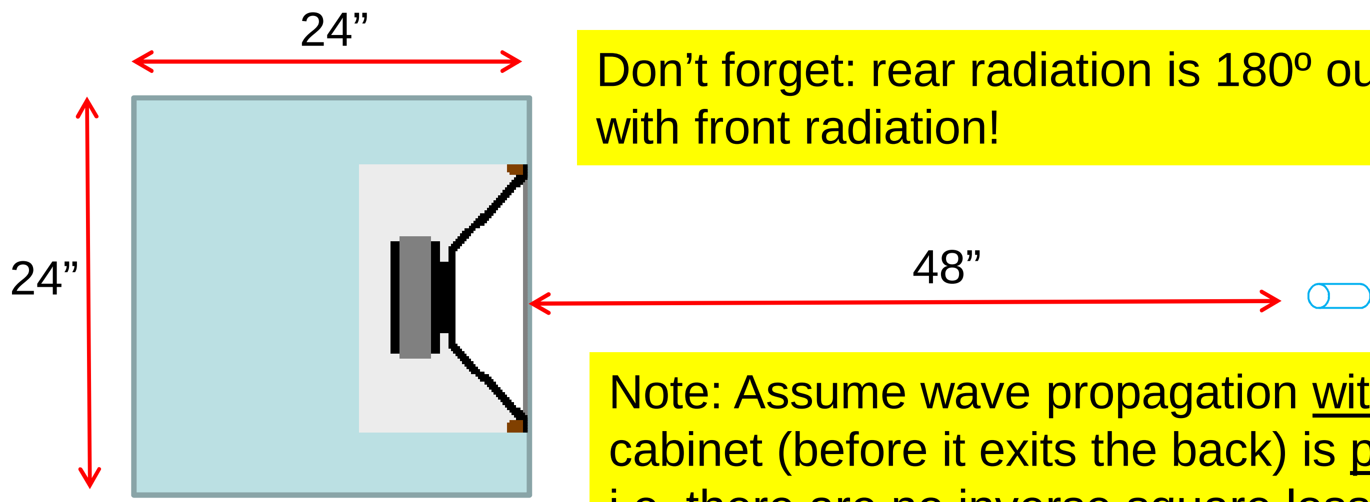
Phase shift of rear radiation relative to front radiation = 180 + (360x48)/135.6 = 307.4º

Given reference level of 90 dB at 12”, level from front vibrating surface at distance of 48” will be 90 - 12 = 78 dB

Level from rear vibrating surface will experience inverse square loss over a distance of 72” (recall that wave propagation for first 24” is planar); level at measurement microphone will be 90 - 15.6 = 74.4 dB

Plugging into general formula yields summed level = 81.5 dB

Solution for 50 Hz



Don’t forget: rear radiation is 180º out of phase with front radiation!

Note: Assume wave propagation <u>within</u> cabinet (before it exits the back) is <u>planar</u>, i.e. there are no inverse square losses.

Wavelength (λ) at 50 Hz = (1130x12)/50 = 271.2”

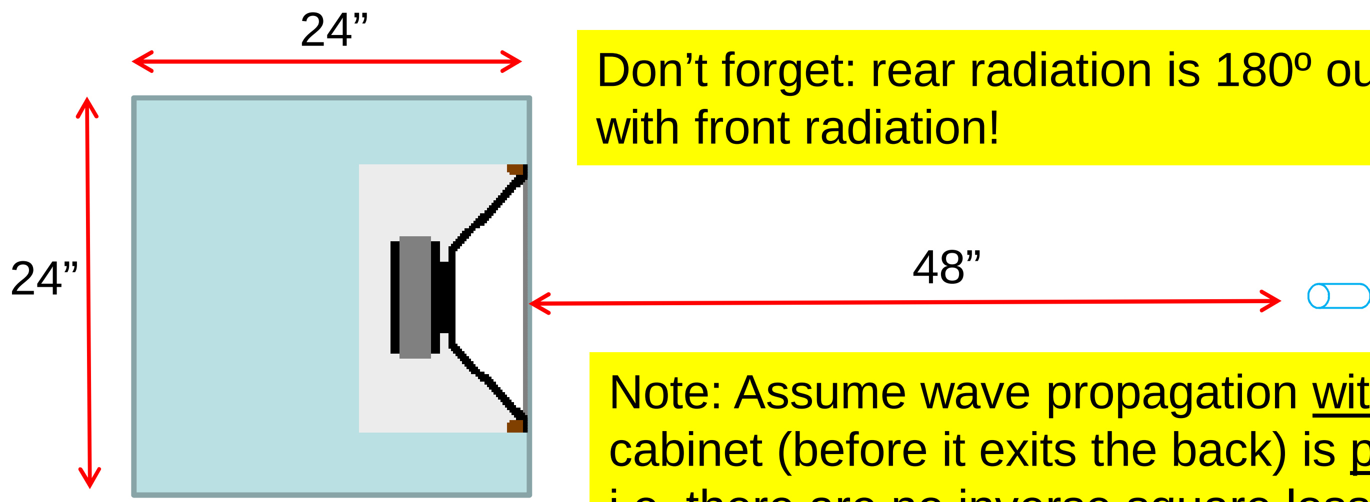
Phase shift of rear radiation relative to front radiation = 180 + (360x48)/271.2 = 243.7º

Given reference level of 90 dB at 12”, level from front vibrating surface at distance of 48” will be 90 - 12 = 78 dB

Level from rear vibrating surface will experience inverse square loss over a distance of 72” (recall that wave propagation for first 24” is planar); level at measurement microphone will be 90 - 15.6 = 74.4 dB

Plugging into general formula yields summed level = 77.3 dB

Solution for 200 Hz



Don’t forget: rear radiation is 180º out of phase with front radiation!

Note: Assume wave propagation <u>within</u> cabinet (before it exits the back) is <u>planar</u>, i.e. there are no inverse square losses.

Wavelength (λ) at 200 Hz = (1130x12)/200 = 67.8”

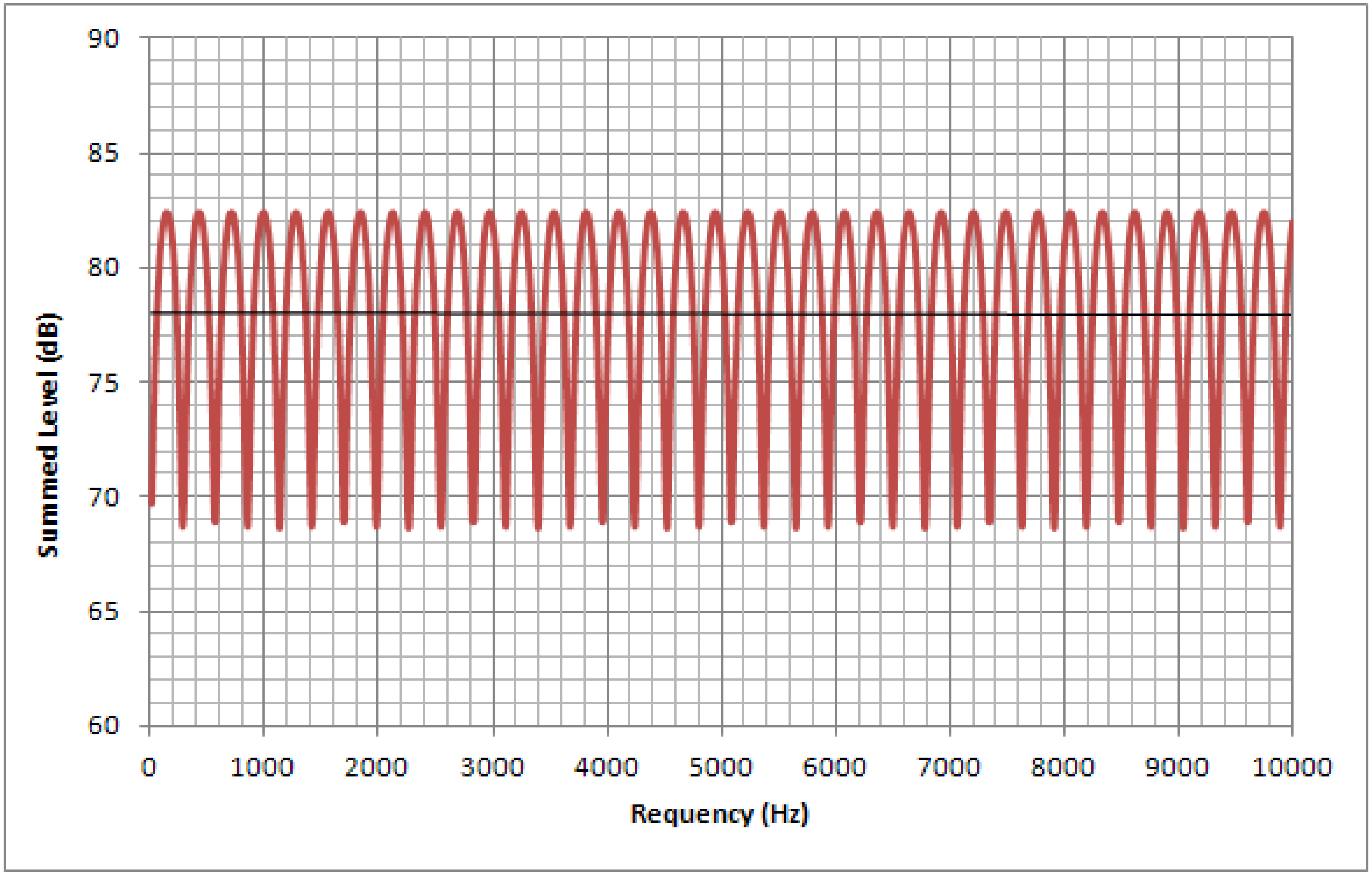
Phase shift of rear radiation relative to front radiation = 180 + (360x48)/67.8 = 434.9º

Given reference level of 90 dB at 12”, level from front vibrating surface at distance of 48” will be 90 - 12 = 78 dB

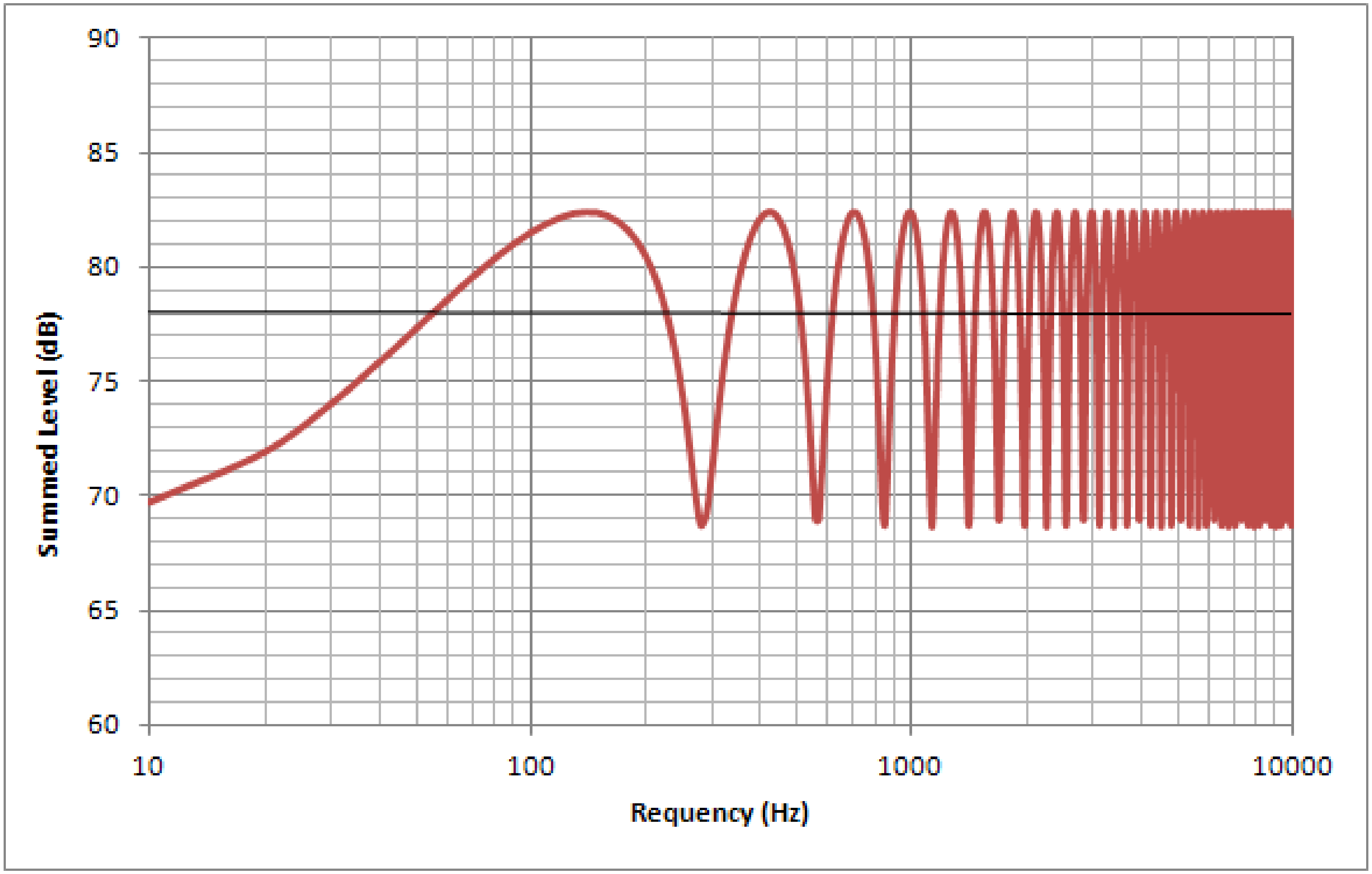
Level from rear vibrating surface will experience inverse square loss over a distance of 72” (recall that wave propagation for first 24” is planar); level at measurement microphone will be 90 - 15.6 = 74.4 dB

Plugging into general formula yields summed level = 80.5 dB

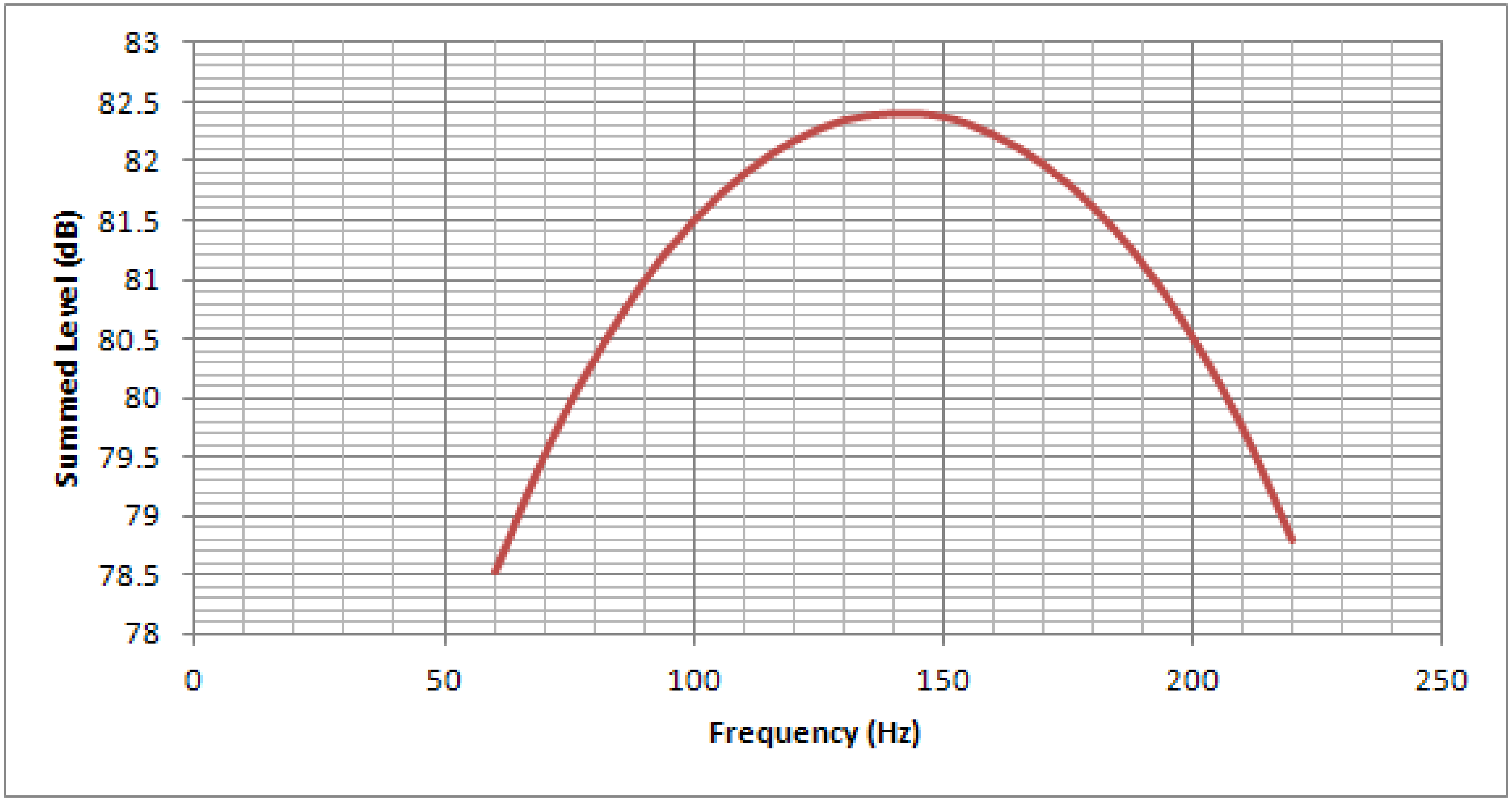
Plot for Range 10 Hz to 10,000 Hz (Linear Frequency Scale)



Plot for Range 10 Hz to 10,000 Hz (Log Frequency Scale)



Approximate Useful Range of Level Enhancement From Summation of Front and Rear Radiation

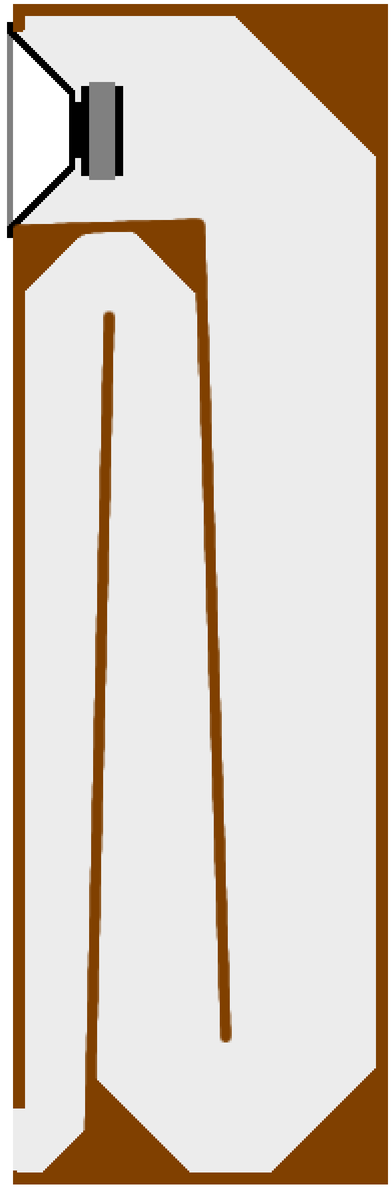


Problem: No way to “stop” (attenuate) rear radiation outside of this band (approx. 60 – 220 Hz) → leads to unacceptable amount of “combing”

Transmission Line / Labyrinth

- Transmission line – typically "heavily damped" (stuffed with acoustic material) to absorb energy from rear vibrating surface (or limit radiation from vent to low frequencies)
- Labyrinth – typically "lined" (with acoustic absorption material) but otherwise "substantially open" (radiation from vent limited to low frequencies)

Transmission Line / Labyrinth

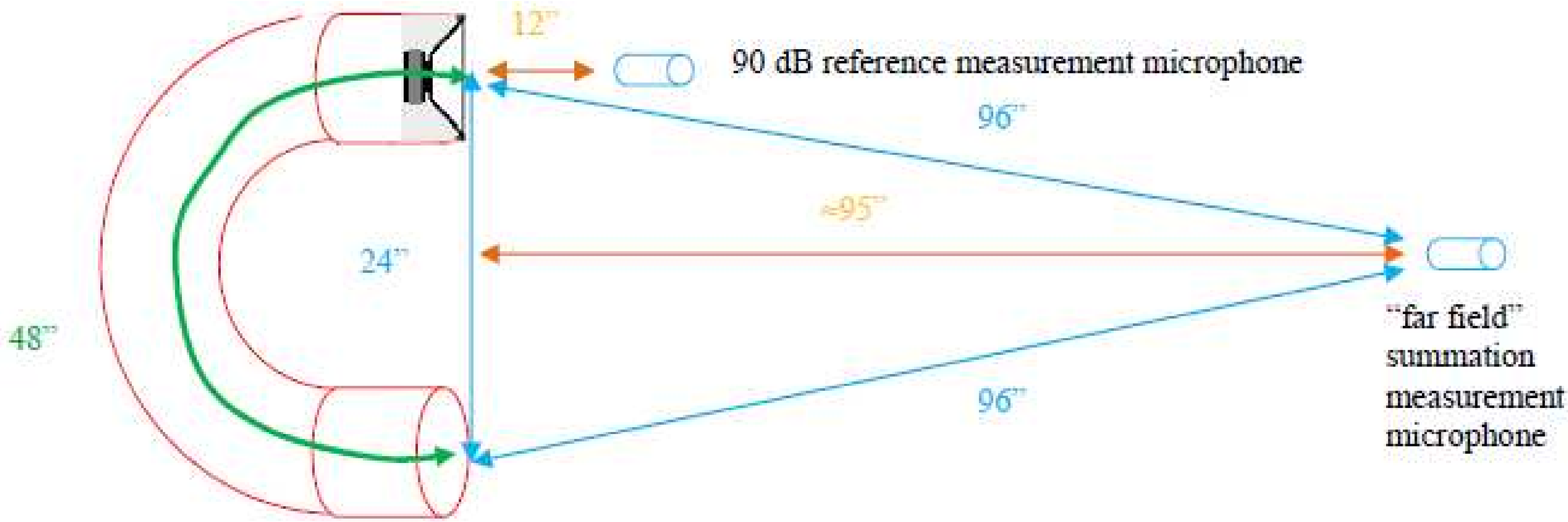


Length of transmission line has to be long enough to provide at least 90º of phase shift (1/4 of longest wavelength of interest)

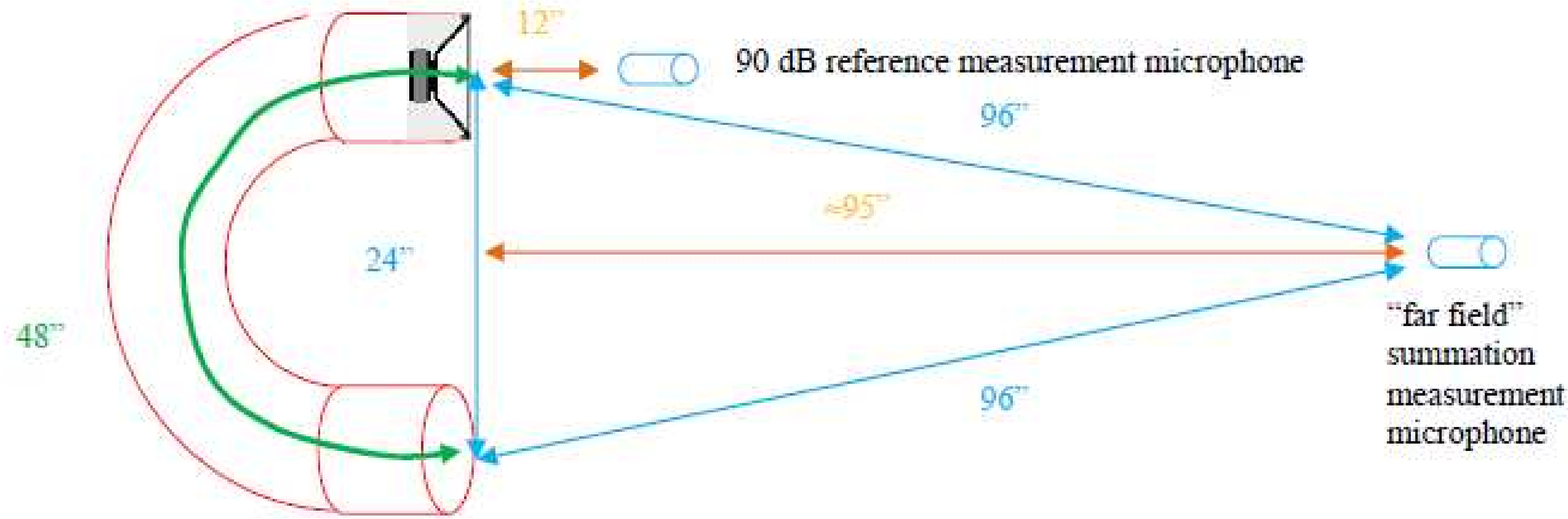
Phase shift (degrees) =
360 x L / (C/F)
where L is effective length of labyrinth, C is speed of sound, and F is frequency of operation (note – add 180 due to rear radiation)

Damped Pipe

- Pipe driven at one end and open at the other will resonate at a frequency of $F_{res} = C / 4L$, where C is the speed of sound (1130 ft/sec at 72º F) and L is the effective length of the pipe (F_{res} is called its “quarter-wavelength tuning” frequency)
- The effective (or “acoustic”) length of the pipe may be longer than its physical length
- Use of tapering and/or acoustic absorption material can increase the effective length



Assume a 4-inch loudspeaker is mounted in one end of a 48-inch long "C-shaped" section of 4-inch diameter PVC pipe (with the rear vibrating surfacing of the speaker facing the interior of the pipe). Assume the center-to-center distance between the two ends of the C-shaped pipe is 24 inches. Upon stuffing the pipe with acoustic absorption material and tightly capping the "rear" end of the pipe, a reference level of 90 dB SPL is measured on-axis (*with respect to the "speaker end"*) at a distance of 12 inches when operated at 50 Hz.



Determine the minimum frequency and the maximum frequency at which this undamped pipe should be allowed to operate (when observed in the "far field", e.g. at a distance of approx. 95" on-axis with respect to the midpoint of the two openings, with the stuffing material and end cap removed). Also determine the frequency at which the summation peak occurs due to phase shift. In each case determine the dB SPL measured at the "far field" summation point.

Finally, determine the fundamental resonance frequency of the undamped pipe, and sketch the expected frequency response of this loudspeaker system (observed as indicated above) as the operating frequency is swept from the minimum to maximum frequency calculated above. Discuss what happens outside the "useful operating range".

Solution

Determine the minimum frequency and the maximum frequency at which this undamped pipe should be allowed to operate phase shift, and the level measured at the "far field" summation point.

The minimum frequency (Fmin) is where the phase offset goes below 240º (rotating CCW on phase wheel), i.e. relative phase shift of acoustic path is 60º (180º + 60º = 240º total relative phase shift)

$60 = 360 \times 48/\lambda \rightarrow \lambda = 288" \rightarrow$ Fmin = 47 Hz

The level from front vibrating surface @ 96" = 90 dB – 18 dB = 72 dB; the phase-shifted level from rear vibrating surface is also 72 dB (assuming no losses through pipe)

$$\Sigma_{dB\text{-}SPLa+b} = 20 \log_{10} [\ \text{sqrt}\ \{ (10^{72/20})^2 + (10^{72/20})^2 + 2(10^{72/20})\ (10^{72/20})(\cos(240))\}\ ] = 72 \text{ dB}$$

Solution

Determine the minimum frequency and the maximum frequency at which this undamped pipe should be allowed to operate phase shift, and the level measured at the "far field" summation point.

The maximum frequency (Fmax) is where the phase offset goes above 120º (rotating CW on phase wheel), i.e. relative phase shift of acoustic path is (180º+300º=480º or 120º) following complete rotation

$300 = 360 \times 48/\lambda \rightarrow \lambda = 57.6" \rightarrow$ Fmax = 235 Hz

The level from front vibrating surface @ 96" = 90 dB – 18 dB = 72 dB; the phase-shifted level from rear vibrating surface is also 72 dB (assuming no losses through pipe)

$\Sigma_{dB\text{-}SPLa+b} = 20 \log_{10} [\ \mathrm{sqrt}\ \{ (10^{72/20})^2 + (10^{72/20})^2 + 2(10^{72/20})(10^{72/20})(\cos(120))\}\ ] = 72$ dB

Solution

Also determine the frequency at which the summation peak occurs due to phase shift, and the level measured at the "far field" summation point.

The response peak (Fpeak) based on phase shift is where a relative phase shift of 180º occurs:

$180 = 360 \times 48/\lambda \rightarrow \lambda = 96" \rightarrow$ Fpeak = 141 Hz

The level will be 72 dB + 72 dB = 78 dB SPL

Solution

Finally, determine the fundamental resonance frequency (F0) of the undamped pipe, and sketch the expected frequency response of this loudspeaker system (observed as indicated above) as the operating frequency is swept from the minimum to maximum operating frequency.

The fundamental resonance frequency of the undamped pipe will be:

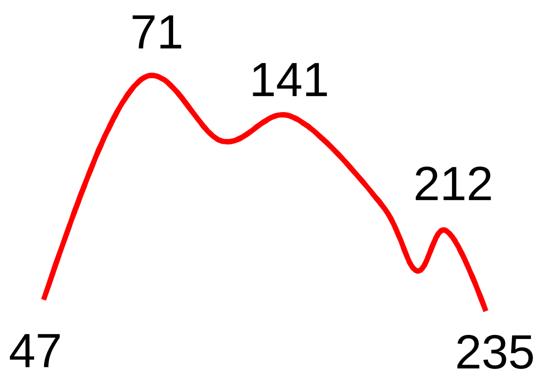
F0 = c/4L = 70.6 Hz (will expect response "bump" at this frequency)

Will also have resonance peaks at odd integer multiples (1,3,5…) of F0:

F1 = 3 x 70.6 = 211.8 Hz F2 = 5 x 70.6 = 353 Hz (beyond operating range)

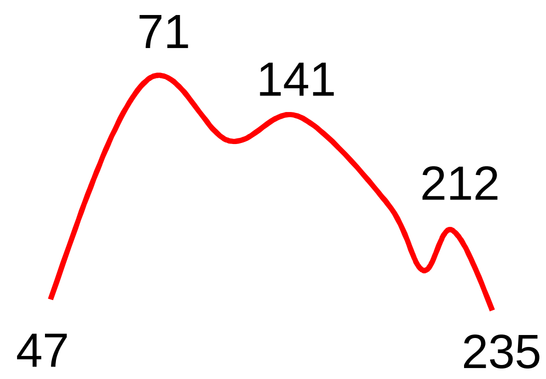
Summary:

- Fmin = 47 Hz, level = 72 dB
- F0 = 71 Hz (level unknown – depends on "Q" of pipe)
- Fpeak = 141 Hz, level = 78 dB
- F1 = 212 Hz (level unknown, but lower peak than F0)
- Fmax = 235 Hz



Solution

What happens <u>outside</u> the “useful operating range” of low frequency enhancement (due to resonance and summation effects)?



Will experience destructive summation effects due to phase offsets (will occur in a periodic, “combing” fashion).

Also, will experience successive resonance peaks at odd integer multiples of F0.

Conclusion: Need to “low pass filter” emissions from pipe end (probably best to cut off before F1, say around 200 Hz) – this can be accomplished by lining or stuffing the pipe with acoustic absorption material.

Also note: Turbulence noise will become more apparent at lower frequencies, so will want to “high pass filter” emissions from pipe end as well (probably best to cut off above Fmin) – this can be accomplished by restricting the cross sectional area of the opening.