Music unfolds on a vast playing field whose complex mathematical structure has never been well understood. This talk will illustrate applications of several different branches of mathematics in describing musical phenomena. Emphasis will be on applications of graph theory, particularly in describing various versions of a diagram called a Tonnetz in which certain musical pitch relationships can be graphed and certain kinds of chord progressions can be traced. Some Tonnetz graphs are embedded in a torus; one version, for example, illustrates a toroidal embedding of the complete graph $K_7$. In some cases relationships between musical structures may be described algebraically using transformation groups, which range from familiar small cyclic groups to complex constructions such as wreath products. Relationships between scales of different cardinalities, such as diatonic (major or minor) scales with seven notes in each octave and chromatic scales with twelve, exhibit many mathematically interesting properties. Other ways of conceiving of chordal relationships lead to topological descriptions in which, for example, all possible two-note chords define a Möbius strip while larger chords lie in more complex spaces (orbifolds) in higher dimensions.

(Host: Birgit Kaufmann)