

# Location Estimation in Ad-Hoc Networks with Directional Antennas

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## Abstract

*With the development of location aware sensor applications, location determination has become an increasingly important middleware technology. Numerous current technologies for location determination of sensor nodes use the received signal strength from sensor nodes using omni-directional antennas. However, an increasing number of sensor systems are now deploying directional antennas due to their advantages like energy conservation and better bandwidth utilization. In this paper, we present techniques for location determination in a sensor network with directional antennas under different kinds of deployment of the nodes. We show how the location estimation problem can be solved by measuring the received signal strength from just one or two anchors in a 2D plane with directional antennas. We implement our technique using Berkeley MICA2 sensor motes and show that it is up to three times more accurate than triangulation using omni-directional antennas. We also perform Matlab simulations that show the accuracy of location determination with increasing node density.*

**Keywords:** location estimation, directional antenna, triangulation, received signal strength, received signal angle.

## 1 Introduction

Sensor networks provide a promising infrastructure for gathering information about parameters of the physical world. Tiny wireless nodes equipped with different kinds of sensors can be distributed over a field and can collect and transmit the data to a data aggregation point, such as a cluster head or a base station. In order to interpret the sensed data, it is often necessary to know the location of the node which is the source of the data. In addition, position information is valuable for optimizing the routing process, as shown in many position aware routing protocols [1][2]. In general, these strategies seek to avoid wasting valuable bandwidth by minimizing the control traffic for route determination. Most position-based routing schemes also remove the need to maintain routing tables at the nodes.

Also, the node's location may change. Mobile sensor networks are becoming an important class in which the nodes may move in a controlled manner or through passive mobility.

It is possible for a node to have up-to-date information of its location if it contains location determination hardware, such as a GPS receiver, mounted on it. There are different reasons why it may not be feasible to equip each sensor node with such hardware. A key design principle for many sensor network protocols is that they should be scalable to very large numbers, with up to hundreds of thousands of nodes. From an economic standpoint, the deployments should be cost-effective. The economic considerations have been driving the cost of the individual sensor nodes down to the point where sub-\$1 nodes are beginning to look achievable [4]. GPS hardware would increase the price of sensor networks substantially. Commercially available GPS receivers come in a wide price range of \$10-\$10,000. The receivers at the lowest end give poor accuracy, with inaccuracies of tens of meters possible [5]. Receivers that give sub-meter accuracy, which may be needed for many sensor applications, are more than \$5,000 in price. The hardware also adds to the weight of the unit with typical receivers ranging upwards of 5 oz. Finally, the battery lives' of the receivers are much shorter than that of the sensor nodes themselves, e.g., tens of hours for the typical GPS receivers compared to multiple months for a representative sensor node, the Berkeley mote. Thus, the combined unit of the sensor node and the GPS receiver will have to be replaced far too frequently for it to be practicable for a large class of deployments. More generally, the received signal strength for a GPS can be as low as -130 dBm, orders of magnitudes less than the strength of traditionally received signals in terrestrial applications and lower than the sensitivity of receivers on typical sensor nodes (-100 dBm for Berkeley motes). Therefore, expensive receivers would be needed. Also, since relatively unobstructed views are required for GPS localization, in many sensor network deployments, the GPS measurements would need to be supplemented with ranging data from the local network.

Though it may not be feasible for all the nodes to be equipped with special purpose location determination hardware, it may be possible to equip a small fraction of the

nodes in the network with such hardware. Such nodes, called “anchor nodes”, can act as reference points for location information and other sensor nodes, called “target nodes”, can use information from anchor nodes to estimate their location. In the most commonly used technique called *lateration* the distance measurements are required from  $(k+1)$  neighbors in a  $k$  dimensional plane. The example of lateration in a 2-dimensional plane is called *triangulation* in which the sensor node needs to know the distances from three neighboring nodes. Several approaches exist for estimating distance from a neighbor, e.g., signal attenuation and time of flight. In signal attenuation, the power of received signal is measured by the sensor node and knowing the signal strength emitted by the source node and the attenuation relationship with distance (such as,  $1/r^2$  where  $r$  is the separation distance), the relative distance can be calculated. Typically for indoor environments or large distances, the attenuation relationship becomes complex and difficult to represent concisely due to multi-path effects and reflection of the radio waves. Other techniques for measuring relative distances, such as time of flight ([12],[6]), are less useful in our environment since the radio signal travels at the speed of light and the distances traveled for signals by the sensor nodes are relatively short.

Directional antennas provide important benefits in sensor networks. Directionality can be used as a form of diversity built into the sensor node, which helps in coping with the variability in the communication channel and reduce the link error rate. The directionality provides increased transmission ranges compared to omni-directional antennas by focusing the transmission energy in the desired direction. They can also increase the security of communication by restricting the set of neighbors that can overhear a communication [15]. Directionality in expensive communication systems is commonly achieved through the creation of a phased array. However, this is extremely expensive and is used predominantly only in high cost military applications. In addition, it is required that the elements of the phased array be an appreciable fraction of a wavelength apart. This would not be possible in electrically small form factor sensor nodes. This precludes the use of a traditional array to provide the desired beam scanning. However limited directionality can be cheaply integrated into a small form factor sensor node. In this investigation, reduced size patch antennas have been developed using standard patch arrangements with high dielectric constant antennas. Multiple directional antennas are utilized and a simple switching network enables us to switch between polarization states ([7],[8]) and the direction of radiation.

The solution to location determination with omni directional antennas is *not* applicable to directional antennas since the radiation patterns are different and the received power is dependant on angle as well as distance. In this paper, we use a model for sensor nodes equipped with four directional antennas. We show that even if the nodes have sizes much smaller than the wave length of the RF wave, then a sensor node can estimate its location using information from a single anchor. The triangulation

method, in contrast, uses three anchor nodes in the ideal case of no errors for a two dimensional plane. The Error Resilient Triangulation (ERT) technique to cope with errors works as follows. Given a redundant number of anchors, a redundant set of linear equations is set up and solved to minimize the least square error of the position estimate. In our approach, the multiple distance measurements, one from each anchor node, are averaged to determine the target node’s location. We present the algorithms for position determination for different kinds of deployment – aligned and unaligned, single and multiple anchors.

The algorithms are implemented on Berkeley sensor motes commercially available through Crossbow Inc. of type Mica2 and experiments carried out with them in an outdoor environment in a real sensor network testbed. The experimental data underlines the fact that simple dipoles do not have a truly omni-directional radiation pattern and the pattern fluctuates with time. This opens to question results for protocols developed using ideal radiation patterns. The experimental results show that radial distance estimates are 11% better when one of the motes (anchor or target) have directional antennas compared to a completely dipole arrangement (i.e. all motes having a unidirectional radiator). The aggregate position error is minimized by employing two directional anchors and switching between the antennas of the anchors from which the transmitted signal is gathered. The average error over multiple anchor and target positions in this case is 11.6% while that for the omni-directional system with 3 anchors the error is 27.5%. The measurements with each node having directional antennas lead to the insight that even if traditional triangulation were to be used, it is useful to employ directional antennas to gather the radial distance data. Even by discarding the angle of transmission and reception data, the triangulation process is more accurate with directional antennas.

We also build a simulation model where the general case of anchor and target nodes being unaligned with respect to each other is simulated. Such orientations may result from some rapid deployment of the nodes. The simulation shows results for varying number of anchor node neighbors. We show through simulation that the aggregate error is reduced through the use of our technique compared to the ERT method by 2 to 3.5 times.

The rest of the paper is organized as follows. Section 2 presents background material on location determination in ad-hoc wireless networks. Section 3 sets up the mathematical model for our solution approach with directional antennas. Section 4 presents the simulation experiments and results. Section 5 concludes the paper with mention of future work.

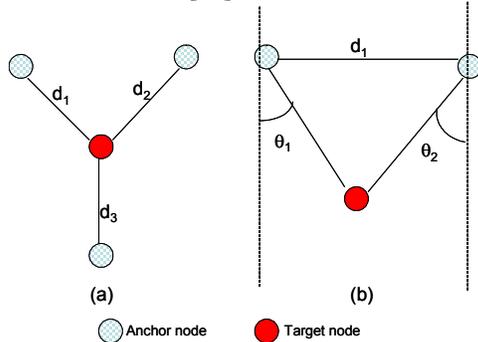
## 2 Background

Triangulation is a common method for locating objects using other objects which do know their position. This is an applicable model for our environment where the positions of some sensor nodes, possibly equipped with GPS

receivers, are known. A nice overview of triangulation based location determination techniques is to be found in [10] and [11]. The triangulation techniques can be subdivided into two categories – lateration, which uses distance measurements, and angulation, which uses angle measurements along with distance.

If individual distance measurements are completely accurate, lateration requires  $(n+1)$  neighbors with knowledge of location to pinpoint the target node in an  $n$  dimensional plane. An example of lateration in two dimensional space is shown in Figure 1(a) and is called triangulation. Example use is in the Active Bat Location System [12].

Different approaches exist for estimating the distance from a neighbor, for example time of flight, attenuation of signal strength, and directionality ([11],[13]). Measuring signal strength relies on the property that radio waves attenuate in their signal strength with increasing distance between the transmitter and the receiver. The receiver can calculate the distance if it knows the transmission power and the attenuation model [14].



**Figure 1. Location determination with neighboring anchor nodes. Lateration is in (a) and angulation in (b)**

The attenuation is often modeled as  $1/r^2$ , where  $r$  is a relatively short distance outdoors. Indoors, reflection, and multi-path fading make the model and hence, the location estimate, inaccurate. The third way of estimating location is to compute the angle of each reference point with respect to the sensing node in some reference frame. The position of the mobile node can then be computed using angulation.

In practice, the individual distance measurements are inaccurate because the exact relation between the measurement of physical properties, such as signal strength, and the inter-node distance is not known. Hence, information from greater than  $(n+1)$  nodes is needed for pinpointing a target node in an  $n$  dimensional plane. The work in [16] presents an approach for minimizing the aggregate error by considering measurements from a redundant number of anchor nodes. A redundant set of equations is linearized and solved to minimize the least square error.

In [17], Savarese *et al.* propose an iterative protocol that diffuses the location information gathered from nearby anchor nodes through the network. The protocol runs in two

phases – the Hop-Terrain phase where initial estimates are gathered from the anchor nodes and the Refinement phase where the estimates are refined with the nodes that have recently acquired rough location estimates themselves acting as anchor nodes. Using this technique, Bagchi *et al.* [18] show the relationship between the number of anchor nodes and the errors in the location determination, given a certain error in one hop neighbor distance estimation. For example, with 1/5th of the nodes having GPS and 5% error in distance measurement, 12 samples are required for a location estimation of 10% accuracy.

Angulation is an alternate method to lateration for computing location based on neighbor information, where angles are used in addition to distance. A schematic of the use of angulation is shown in Figure 1(b). Directional antennas are needed for the angle measurements. Previous work has used phased antenna arrays to use the angulation technique [10]. Multiple antennas with known separation measure the time of arrival of a signal. Given the differences in arrival times and the geometry of the receiving array, it is then possible to compute the angle from which the emission originated. If there are enough elements in the array and large enough separations, the angulation calculation can be performed.

There is a class of location determination techniques that do not rely on any property of the received signal. Instead, they rely on the connectivity measure, i.e., if a node  $\alpha$  is able to hear from another node  $\beta$ , then  $\alpha$  is connected to  $\beta$  and its location is constrained to be within the transmission range of  $\beta$  ([13],[19],[20]). This class of techniques based on connectivity measure provides location estimates which are quite coarse-grained. The granularity becomes coarser with larger transmission ranges of the reference nodes. An overhead of beacon or hello messages is also incurred and the convergence times of the algorithms are often sensitive to the frequency of these messages [20]. Also, some of the protocols ([19],[20]) require centralized processing which limits their scalability.

Römer proposes a technique geared to dust-sized sensor nodes which only have passive optical communication capability and do not have active RF communication capability. It relies on a powerful base station that sends a photo beam and rotates. Each sensor node has a photo beam detector and a clock and marks how long it sees the beam and the period of rotation and determines its location based on this. The method is only applicable if single hop communication is possible between all nodes and the base station. Also, as has been demonstrated in [16] and appears well accepted, distance measurements over large distances are very inaccurate.

### 3 Solution for Directional Antennas

#### 3.1 Directional Antenna Model

One of the simplest semi-directional antennas is the patch antenna. This antenna is used as a representative example of an antenna that may be used for localization and which will

still fit on a mobile form factor. The ideal patch radiation model is a hemispherical radiator which allows for semi-directional radiation. The typical gain of a patch antenna is on the order of 3.5 to 6 dBi, depending on the dielectric substrate used in the design. A representative angular variation of the gain for a typical microstrip antenna will be in the range of

$$\cos^2\left(\frac{\beta l}{2}\sin(\theta)\right) < G(\theta) < \cos\left(\frac{\beta l}{2}\sin(\theta)\right)$$

where  $\beta$  is the free-space constant and  $l$  is the longest length of patch, assuming the lowest order mode of operation [22]. The gain is defined as the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

In the E-plane cut, the antenna's radiated e-field from a standard patch radiator is ideally  $E = \cos\left(\frac{\beta l}{2}\sin(\theta)\right)$ .

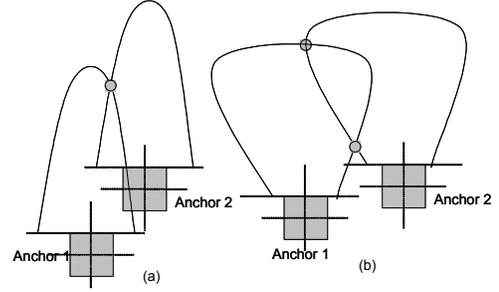
This pattern dependence is in relation to a coordinate system with the z-axis perpendicular to the microstrip patch radiator. This is the ideal solution for a patch antenna with an infinite ground plane and is only slightly altered using a finite size ground plane. The ground plane is used to shield the radiating field from the rest of the circuitry and the other radiators. Unshielded radiators, such as those that are standard with the notes, are susceptible to parasitic radiating currents which result in asymmetric patterns.

The received power at an antenna is given by  $P_r = \frac{P_t G(\Theta) H(\Theta)}{r^2} \left(\frac{\lambda}{4\pi}\right)^2$ , where  $\theta_t$  and  $\theta_r$  are the transmitting and the receiving angles, respectively, and  $r$  is the distance between the transmitter and the receiver.  $\lambda$  is the RF wavelength of the carrier frequency. Since  $(\lambda/4\pi)^2$  is a constant, we will exclude it from future expressions. It was however included in calculating the results.

A realistic antenna radiation pattern obtained from the design of patch antennas in the HFSS simulation package is used to model the gain for the experiments. The simulations were validated through actual experiments in an anechoic chamber. For the sensor network simulations in this paper we use an antenna model given by  $G(\Theta) = H(\Theta) = \cos\left(\frac{\beta l}{2}\sin(\Theta)\right)$ . This is the upper bound of the possible antenna gain given in Section 3 and is chosen in our analysis and simulation so that an anchor has a larger number of target nodes within its transmission range. However, the proposed techniques are equally valid for any other antenna model.

However, the model for the received power has implications for the uniqueness of the solution determined by our approach. Figure 2(a) shows the constant power contours from two anchors following the  $\cos(k\sin(\theta))$  directional gain model and (b) shows the  $\cos(k\theta)$  model. While the power contours of the former model intersect at just one point, that of the latter intersect at two points.

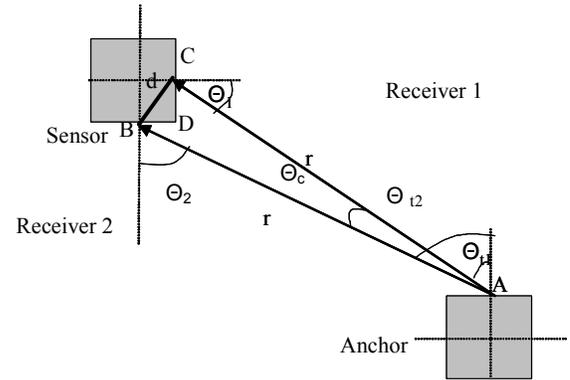
Therefore the position of the target node cannot be uniquely determined from the received power from two transmitters.



**Figure 2. Uniqueness of estimated location with two different power contours (a)  $\cos(k\sin(\theta))$  and (b)  $\cos(k\theta)$**

### 3.2 Aligned antennas

In a number of practical applications it is reasonable to expect that the sensors will be manually deployed. Sensors set up to monitor a bridge's health have to be placed by construction workers on the bridge for example. In such scenarios even though it may not be possible to know the precise location of the sensor, it is possible to place these sensors in a pre-determined orientation.



**Figure 3. Location determination with aligned nodes**

If the antennas of a target sensor node are aligned, then we can use the power received at multiple receiving antennas of the target from a single transmitting antenna on an anchor for position estimation. Without loss of generality consider that an anchor node is placed to the south-east of the target node as shown in Figure 3. Typically the size of the sensor would be much smaller than the transmission distance. So  $d/r = \Theta_c$ . Then received power at the two receiving antennas of the target node is given by

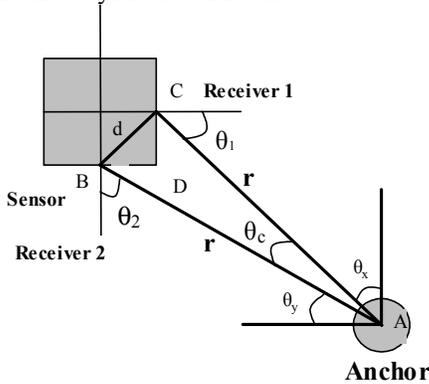
$$P_{r1} = \frac{P}{r^2} G(\Theta_{r1}) H(\Theta_1) = \frac{P}{r^2} G\left(\frac{\pi}{2} - \Theta_1\right) H(\Theta_1) \dots (1)$$

$$P_{r2} = \frac{P}{r^2} G(\Theta_{r2}) H(\Theta_2) = \frac{P}{r^2} G\left(\frac{\pi}{2} + \frac{d}{r} - \Theta_1\right) H\left(\frac{\pi}{2} + \frac{d}{r} - \Theta_1\right) \dots (2)$$

Thus, we have two equations (1) and (2) in two variables  $\Theta_1$  and  $r$ . Since, these are nonlinear equations it is difficult to get a closed form solution for  $\Theta_1$  and  $r$  in terms of the input variables  $P_{r1}$  and  $P_{r2}$ . However, these equations can be numerically solved by standard methods to obtain  $\Theta_1$  and  $r$ .

A baseline experiment for this is with the anchor node having omni-directional dipole antennas. In this case the gain of the transmitter,  $G(\cdot)$ , is constant over all and denoted  $G_{\text{omni}}$ . Figure 4 shows this configuration. Now, since we know the distance as well as the relative direction of the target with respect to the anchor, we can estimate its position. This estimate is based on measurements from just one neighboring anchor node whereas triangulation requires measurements from at least three anchors.

The estimates from multiple anchors can be averaged to obtain a better estimate of the position. Alternatively, the information about  $\Theta_1$  could be discarded and the range measurements ( $r$ ) can be used to triangulate the position of the sensor in a least squares manner. Both, these strategies have been evaluated in our simulations and the averaging strategy is found to yield better results.



**Figure 4. Location determination with an omni-directional transmitter and directional receiver**

### 3.3 Generalization to Unaligned Antennas

In cases where it is not possible to ensure a global orientation of all nodes of a network, additional measurements can be used to estimate position. Received power at two different antennas of the target node from two transmitting antennas of the anchor node is measured. Such an arrangement is shown in Figure 5.

Geometric relations between the various transmission and receiving angles can be derived from the figure.

$$\Theta_2 + \Theta_6 = \Theta_4 + \Theta_8 = \Theta_3 + \Theta_5 = \Theta_1 + \Theta_7 = \frac{\pi}{2} + \frac{d}{r}$$

$$\Theta_1 + \Theta_2 + \Theta_3 + \Theta_4 = \pi$$

Let  $P_{rj}$  denote the power received by antenna  $i$  on the target node when antenna  $j$  is transmitting on the anchor node. We can use these equations to simplify the received power equations as follows.

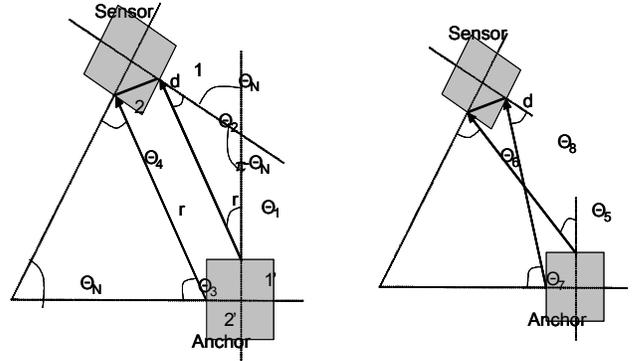
$$P_{r11'} = \frac{P_t * G(\pi - \Theta_2 - \Theta_3 - \Theta_4) * H(\Theta_2)}{r^2} \dots (3)$$

$$P_{r21'} = \frac{P_t * G\left(\frac{\pi}{2} + \frac{d}{r} - \Theta_3\right) * H\left(\frac{\pi}{2} + \frac{d}{r} - \Theta_2\right)}{r^2} \dots (4)$$

$$P_{r12'} = \frac{P_t * G\left(\frac{d}{r} + \Theta_2 + \Theta_3 + \Theta_4 - \frac{\pi}{2}\right) * H\left(\frac{\pi}{2} + \frac{d}{r} - \Theta_4\right)}{r^2} \dots (5)$$

$$P_{r22'} = \frac{P_t * G(\Theta_3) * H(\Theta_4)}{r^2} \dots (6)$$

Equations (3) through (6) in the four variables  $\Theta_2, \Theta_3, \Theta_4,$  and  $r$  can again be numerically solved to estimate the location of the target node.



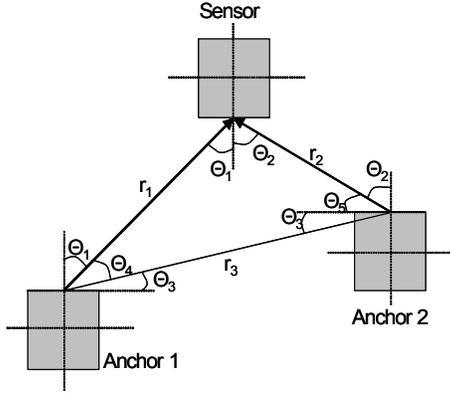
**Figure 5. Location determination for unaligned antennas**

This scheme requires that two target antennas be able to simultaneously receive transmissions from two anchor antennas. This would require a transmitter beam width of  $180^\circ$ . This is non-optimal for four antennas covering a  $360^\circ$  plane but is a tradeoff for increased degrees of freedom in the orientation of the nodes. Besides, the increased beam-width will lend greater fault tolerance to the system by providing greater redundancy in the areas reached by multiple transmitting antennas. It will also make the antenna design easier since high directionality, i.e. narrow beam width is not needed.

### 3.4 Aligned Antennas with Two Anchors

The two location determination methods described earlier rely on the difference in power received at two antennas of a sensor from the antennas on the same anchor node. The error in the power received can become correlated due to the proximity of the two antennas, even if

they are pointed in separate directions. In a real life scenario the correlation can significantly reduce the accuracy of the location estimate, especially for a very small sensor node. To investigate the performance of the location determination with increasingly uncorrelated channels, two transmitted signals were sent from two motes substantially removed from each other. This scheme is also useful in situations in which more than one directional antenna would not fit on a single mote. The arrangement is shown in Figure 6.



**Figure 6. Location determination using measurements from two anchors**

Since the location of the two anchors is known, the parameters  $r_1$  and  $r_2$  can be determined. Using geometric properties of the system we get the following relations between the various angles

$$\Theta_5 = \frac{\pi}{2} - \Theta_2 \quad \Theta_1 + \Theta_3 + \Theta_4 = \frac{\pi}{2}$$

The equations for the received power are given by

$$P_{r1} = \frac{P_t}{r^2} G(\Theta_1) H(\Theta_1) \dots (7) \quad P_{r2} = \frac{P_t}{r^2} G(\Theta_2) H(\Theta_2) \dots (8)$$

Using the law of sines along with relations between the angles derived earlier we get two more equations

$$\frac{r_2}{\cos(\Theta_1 + \Theta_3)} = \frac{r_3}{\sin(\Theta_1 + \Theta_2)} \dots (9)$$

$$\frac{r_1}{\cos(\Theta_2 - \Theta_3)} = \frac{r_3}{\sin(\Theta_1 + \Theta_2)} \dots (10)$$

This gives us four equations in four unknowns  $r_1$ ,  $r_2$ ,  $r_3$ , and  $\theta_1$ , which can be numerically solved. Thus, the distance and the angle with respect to each of the two anchors are determined. The sensor node's location can be estimated using either distance, angle pair and the final location estimated using averaging of each estimate.

In this section, we have provided the mathematical solution to the problem of location estimation with directional antennas in three different scenarios. The node specifications and the deployment conditions will determine which scenario is applicable.

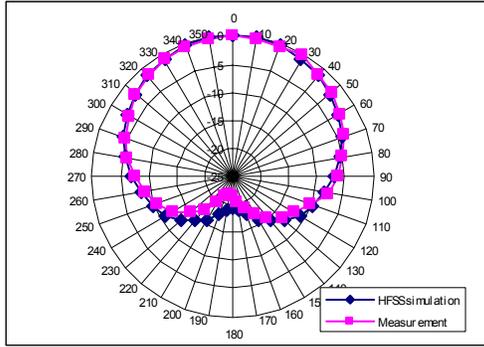
## 4 Experiments & Results

### 4.1 Experimental Setup

Crossbow MICA2 motes MPR400CB operating at 900 MHz and running TinyOS as the programming environment are employed as the sensor nodes for our testbed. Two kinds of omni-directional antennas are used - quarter wave whip antennas (MMA400CA) (on the transmitting anchor node, comes off-the-shelf with the motes) and quarter wavelength monopole antennas with an expanded ground plane (on the receiving target nodes, fabricated for easy interfacing and co-existence with the patch antennas). Directional antennas are fabricated and used on both the target sensor node, whose location is to be determined, and the anchor sensor nodes, whose location is assumed to be known. Patch antennas fabricated on duroid substrates, Rogers RO3010 whose dielectric constant is 10.2, are chosen as the semi-directional antennas in this testbed due to their simple fabrication, small size, and low-cost. The other candidate designs such as Yagi antennas, horn antennas, and antenna arrays, would have more directionality but are larger and more expensive and therefore not applicable. Sensor motes with four directional antennas controlled by a switching network according to the received signal strength indicator (RSSI) at the antennas are employed to be the target motes (functioning as receivers). The switching network is implemented by a GaAs MMIC SP4T switch. Software executing on the target motes monitors the received signal power on the four antennas and selects the requisite ones (the best, or the two best) for its location computation. The motes are tested in an outdoor environment to observe the performance of the location determination system with different kinds of wireless fading. The goal of the experiments is to determine the estimation error of the location determination system.

Three experiments are set up with transmitting anchor motes that initially have omni-directional (dipole) antennas (see Figure 7(a)). If necessary, aligned patch antennas are mounted on the anchor motes (see Figure 7(b) and (c)) to have a complete directional transmitting and receiving system. The target motes, functioning as receivers, are equipped with the switched directional antennas and collect the data which is forwarded to a laptop through a Universal Asynchronous Receiver/Transmitter (UART) Interface. Matlab programs are written to solve the equations given in Section 3 numerically. The software utilizes a gain pattern derived from the patch antenna from Ansoft's High Frequency Simulation Software (HFSS) package. HFSS, a full wave electromagnetic simulation commercial software, is the default industry standard RF design simulation tool [24][25]. The radiation pattern is simulated and compared with the measurement of the fabricated patch (Figure 7). It is observed that the two are in close agreement and hence the HFSS model is used for further analysis. The experiments in Section 4.2 have 150 samples for each position, taken in three separated time intervals. These are averaged for the reported number. Matlab simulation is

used to iteratively solve the equations to minimize the error in the radial distance and the angle of reception.



**Figure 7. Radiation pattern of the patch antenna from HFSS and the measurement in anechoic chamber room**

If the iterative Matlab simulation is considered too expensive to execute on the motes, a static lookup table of signal strength versus radial distance and angle of reception can be created and uploaded into the motes. This cuts down on the latency of the location estimation as well as the computational expense on the simple processors of the sensor nodes, at the expense of the accuracy of the solution. This approach will be further investigated in the future.

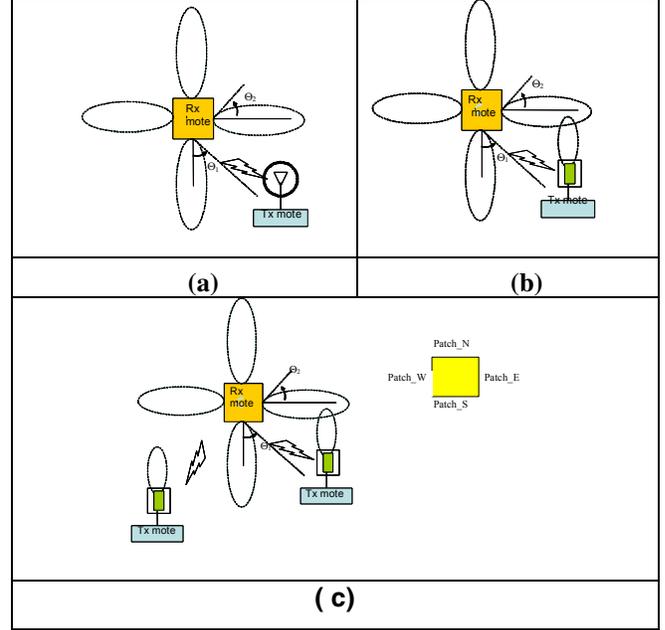
The location error is defined by the error distance from the estimated target position  $\hat{\vec{r}}_{target}$  to the actual target position  $\vec{r}_{target,0}$  divided by the known actual distance between the anchor mote and the target mote ( $R = |\vec{r}_{anchor,0} - \vec{r}_{target,0}|$ ) where  $\vec{r}$  is a two-dimensional position vector. Location error  $R_{error}$

$$\begin{aligned}
 &= \frac{|\hat{\vec{r}}_{target} - \vec{r}_{target,0}|}{|\vec{R}|} = \frac{|\hat{\vec{r}}_{target} - \vec{r}_{target,0}|}{|\vec{r}_{anchor,0} - \vec{r}_{target,0}|} \\
 &= \frac{\sqrt{(\hat{x}_{target} - x_{target,0})^2 + (\hat{y}_{target} - y_{target,0})^2}}{\sqrt{(\hat{x}_{anchor} - x_{target,0})^2 + (\hat{y}_{anchor} - y_{target,0})^2}} \quad (11)
 \end{aligned}$$

## 4.2 Experimental Results

### Experiment 1: Aligned case: Single omni-directional antenna anchor, dual patch antenna target

In Experiment 1, an anchor node is used with an omni-directional or dipole transmit antenna. The target node has a dipole antenna and two directional patch antennas. Test 1 is the case with a dipole antenna on the target node. Test 2 has the directional antennas on the target node. The target node is placed in the center of a circle of radius  $d = 8$  feet. Received power measurements are made with the anchor node at different points on the periphery of the circle.



**Figure 8. Antenna configurations for (a) Experiment 1 (b) Experiment 2 (c) Experiment 3**

The relative distance error is shown in Figure 9 for both tests. The x-axis is the angle of the anchor node with respect to the north-south axis drawn from the target node, with the anchor node being moved on the circumference of a circle with the target node at the center. When using the directional antennas, the target node selects the two strongest signals on its receiving antennas to execute the location estimation algorithm. When using the omni-directional antenna, the position cannot be determined since at least three anchor nodes would be needed and hence, the distance estimate is used as the basis for comparison. In this case the Friis' formula is used with an experimentally determined parameter for the exponent in the power loss,  $R^N$ ,  $N=1.89$  in the measurement environment.

This is classified as a line-of-sight and relatively multipath free environment and is applied to estimate  $|\hat{R}| = |\hat{\vec{r}}_{TX,0} - \vec{r}_{RX,0}|$ . The distance error in the entirely omni-directional antenna system is in the range of 0-105% with a mean of 34%, while this error is reduced to 0-90% with a mean of 23% in the case of the directional antennas.

This result can be explained by the fact that in omni-directional antennas, there are more multi-path effects leading to more interference and more fluctuation in received signal strength. Thus, even if triangulation is used based on the distance estimates with the patch antennas, directionality of the antennas is useful since it leads to more accurate estimates of individual distances.

However, by using the angular information that is gained from multiple receiving antennas, the location of the target node can be determined with a single mote without

the use of triangulation (see Section 3.2). The location error  $R_{error}$  of test 2 in Experiment 1 is shown in Figure 10. The error is larger than the radius error because the angle estimation error also contributes to the location error. On average, the location error is 43.15%. This error can be further mitigated by using other estimation anchors, as Experiment 3 shows.

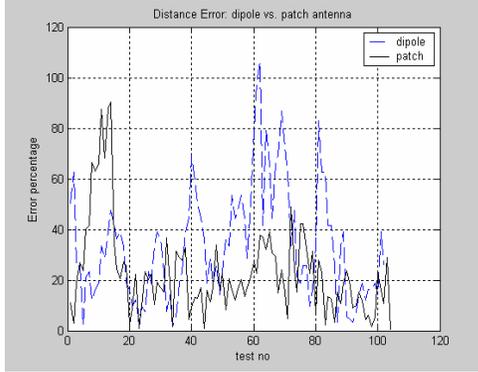


Figure 9. Relative error in distance estimation for Experiment 1

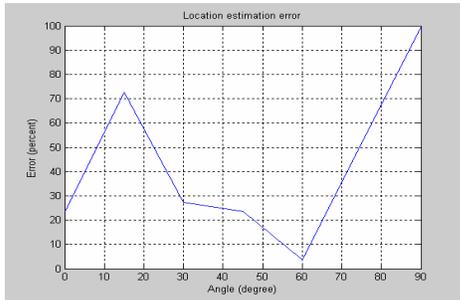


Figure 10. Location estimation error for Experiment 1, Test 2

### Experiment 2: Single patch antenna anchor with dual patch antenna target

In this experiment directional patch antennas are employed in the transmission nodes (see Figure 8(b)). The receiving antennas are all directional. The analytical model is as shown in Section 3.2. The errors in measured distance and angle are shown in Figure 11 and the maximum errors in the two metrics are seen to occur at different positions. The location estimation error calculated from actual measurements is shown in Figure 12. The location error grows larger as the angle  $\Theta_1$  increases because the transmission antenna is not oriented towards the target node. Compared to the experiment with omni-directional transmitting antenna, the location error is reduced when the angle is smaller than 45 degrees because the radiation pattern of directional transmission antenna overlaps with the receiving antennas. After this cross-over point, the patch #W on the anchor node which is oriented to the west can be used as the transmission antenna. Hence, the location error can be controlled to be less than 40% in a single anchor. The average error is 29.18%.

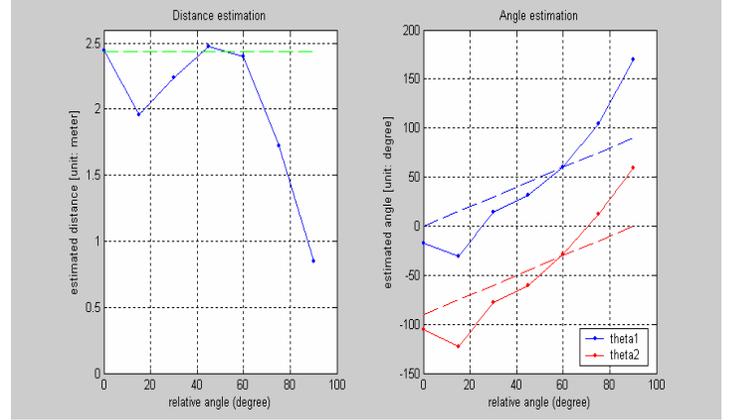


Figure 11. Distance and angle estimation error for Experiment 2. [The solid line is the estimation and the dashed line is actual measurement.]

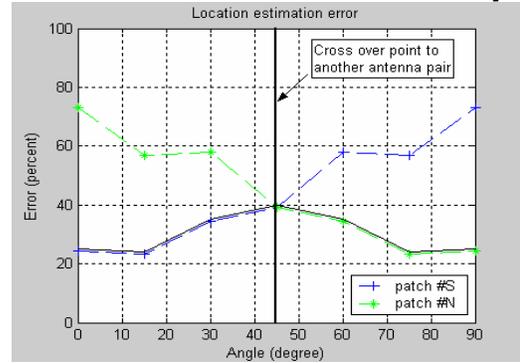
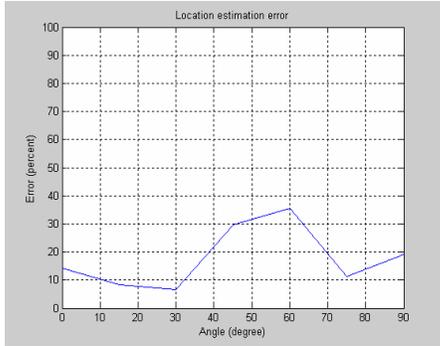


Figure 12. Location estimation error for Experiment 2

### Experiment 3: Two single patch antenna anchors and single patch antenna target

Two anchors can be used to provide additional data for location estimation, as shown in the analysis of Section 3.4. The configuration of Experiment 3 is shown in Figure 8(c). The outdoor measurement result is shown in Figure 13. For this experiment, measurements are taken individually from each directional anchor node and not at the same time. This does not affect the validity of the results since the stochastic model for the channel is time invariant. When an additional anchor provides another power measurement, the estimation error reduces from 29.18% in Experiment 2 to an average of 17.78% in Experiment 3. The first cause of this improvement is the increase in the separation of the two transmission patch antennas in Experiment 3 compared to the relatively small distance of the two patches in Experiment 2. This means that the error in the radial distance  $R$  and the reception angle  $\Theta$  are now uncorrelated. Second, the fading channels are more uncorrelated in Experiment 3 because the distance between the two anchors is much greater than the wavelength of the RF wave. Thus, we conclude that significant location estimation improvement can be obtained by using multiple anchors with directional antennas.



**Figure 13. Location estimation error for two anchor nodes with directional patch antennas**

We perform statistical analysis on the raw measurements to quantify the errors using information from three anchor nodes. For test 1 of experiment 1 (pure omni-directional), the error is 27.5%, for experiment 2 20%, and for experiment 3 11.6%. The improvement is more striking as the number of neighbor anchors increases as the simulation results show.

### 4.3 Simulation Results

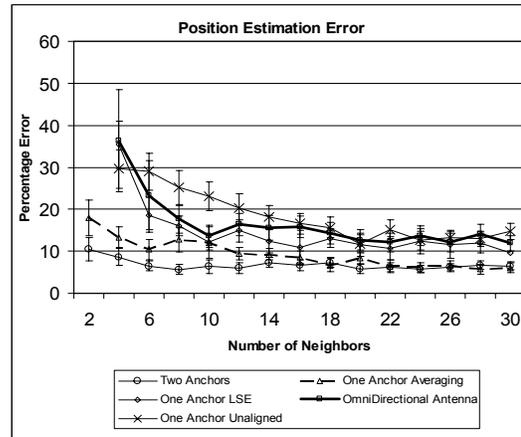
We are interested in evaluating the accuracy of the location determination protocols with varying number of anchor nodes. The accuracy is measured for between 2 and 30 anchors of different kinds: omni-directional, directional, unaligned, and the two anchor case. This range of experiment would be beyond the hardware resources of our testbed and hence a simulation methodology is used. The distortion in received power is simulated using a Rician fading channel. Each data point is based on the average of 50 different samples. For each sample, the anchor nodes are placed randomly within a 10x10 meter grid. The transmitted power is -10 dB, equivalent to that used in hardware, and the size of the entire sensor node is set at 10 cm.

Figure 14 is shown with respect to the increasing number of neighbors. From top, the different curves correspond to: single anchor unaligned, three omni-directional anchors and omni-directional target node, single anchor aligned antenna with least square error aggregation, single anchor aligned antenna with averaging for aggregation, two anchors with aligned antenna. Except for the omni-directional case, all the others have patch antennas on the target node. The first three form one group and the last two form another. The errors in the first group are noticeably higher than those in the second group. The estimates in both groups show a horizontal trend beyond 14 anchor neighbors indicating that higher density is not required for location estimation purposes. The generalized orientation, however, requires up to 20 neighboring anchor before stabilizing. The highest error is observed for the generalized orientation, indicating the importance of approximate alignment at the least. The next highest error is for the omni-directional anchor antenna, followed by the single anchor with least square error aggregation. This

indicates the value of directional antennas and averaging as the method for aggregation. The two anchor case with aligned antenna and using averaging as the aggregation technique outperforms the single anchor case at lower densities but is statistically equivalent at higher node densities. The error bars corresponding to 95% confidence interval are shown.

We observe that averaging as the method for aggregating gives much better results than least square estimation. One of the primary reasons for this phenomenon is that averaging cancels the errors in estimates in individual measurements while the effect of error in least squares estimation is additive.

The results shown in Figure 14 have slightly lower error rates than those found in hardware. This is most likely due to a limited ability to account for all multipath propagation, fading, and other environmental affects. Moreover, the aligned cases were always perfectly aligned in the simulation, while the experiments had relatively aligned antennas, but within some margin of error that is very difficult to determine.



**Figure 14. Evaluation of estimation error for varying number of neighboring anchors**

## 5. Conclusions

In this paper we have presented various techniques for location determination in ad-hoc networks using directional antennas. The combination of the schemes is designed to meet the variety of requirements and degrees of freedom for real life applications. The various tradeoffs between these schemes, such as freedom of orientation versus beam width and node size versus number of anchor nodes, have also been evaluated. The solution approach can form the foundation for location determination protocols for the increasingly popular directional antennas. Our results, shown through experiments on actual sensor nodes and simulations, bring out the fact that location estimation with omni-directional antennas requires at least three anchors and is less accurate than with directional antennas. Also, using uncorrelated communication channels through two geographically separated anchor nodes produces better

results than multiple antennas on the same anchor node. The error is reduced from 27.5% in an omni-directional system to 11.6% with two directional anchor nodes.

In future, we intend to experimentally determine the extent of correlation in channel noise at the different antennas of a sensor. This will be applied to a sensitivity analysis of our schemes. We also propose to analyze location estimates derived from measurements from multiple transmitting antennas to multiple receiving antennas. Finally, we will seek to solve the generalized antenna case with multiple anchor nodes.

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