Kaleidogami™: Multi-Primitive Reconfigurable Artistic Structures

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Abstract

In this paper we present our initial prototypical explorations as well as the associated transformative design concept called Kaleidogami™. This method is used for developing spatial objects that can be flattened, folded and reconfigured. We develop the metaphor and concept for a basic structural unit (BSU) such as using tetrahedral, cuboidal, prismatic, and pyramidal units to enable new forms of 3D folding. The fabrication is done using a single flat sheet of foldable substrate in 2D. We explore the diversity of structural polyhedral sculptures and movable constructs in a hierarchical architecture. More artistic constructions are contextualized with a Kaleido-Tangram like integration.

Introduction

Origami originally was a paper-craft from17th century AD that affords the diversity of representative 3D objects with individual unit arrangements and explicit folding processes from a 2D sheet of paper. Artistic origami designs reveal the rudimentary characteristics of paper folding, they are inexpensive, lightweight, compact and combinatorial. During the last 40 years, “Why’s, What’s and How’s” of different origami tessellations and structures have been geometrically and symbolically described by the underlying mathematical rules governing the creases, such as flat foldability [1] and folding any polygonal shape [2] . Recently, multidisciplinary developments in mathematics, engineering, architecture, and biology have inspired new ideas in the ancient art of origami such as programmable self-folding sheets [3] and biological self-assembly cells [4].

Our analysis of past work in origami and folding structures shows that its applications are limited by the following characteristics: (1) Most developments have a typical goal of achieving desired single folding state, i.e., the extended solar panel or the wrapped gift package, (2) In previous work, open skin-based (i.e. no enclosed volume) models and patterns were achieved by goal-oriented operations (Miura folding [5] as well as patterns represented in airbag [6], stent [7], and cartons [8]), and (3) Recent advances in modular origami [9] and polyhedral models use separate pieces of paper for each component or function. The designers still face the uncertainties of building combinatorial systems out of a single folded sheet. In this work, we strive to bridge these missing links from folding metamorphic structural units to manipulating kaleidoscopic objects from a single flat paper sheet.

In our work, the range of kaleidoscopic 3D structures encapsulated in the folding paradigm is based on (1) formation of an entire new class of basic structural units (BSU), (2) design of foldable reconfigurable structures by allowing BSUs to be connected to each other in a hierarchical manner, (3) transforming smaller structures into larger structures to achieve variable cell sizes without changing the form of the structure, and (4) exploration of artistic representations by implementing the idea of Kaleido-Tangram.

Basic Structural Units

The single ring with equal symmetric tetrahedra was first invented by Schatz [10] and known as Kaleidocycle [11] decorated by Schattschneider and Walker. Robert Byrnes later presented basic mathematical principles and play kits in his book Transforming Mathematical Surprises [12]. Here, we demonstrate four representative BSUs using tetrahedral, cuboidal, prismatic and pyramidal components (see Figure 1(A,B,C,D)) and concentrate on the hierarchical derivatives of various structural and movable constructs in 3D. Non-deformable paper sheet is used to construct the BSUs. We then model creases as hinges, facets that are not creased as structural surfaces and closed-form surfaces as rigid component bodies. In general, a BSU consists of a pair of mirror-shaped polyhedrons coupled with a common hinge (Figure 1). Further, the BSUs can be folded and automatically strung up from expanding crease patterns laterally and vertically on single flat paper. Structural formation rules for the BSU require no local or global self-overlaps in the 2D pattern and no facet penetration during 3D construction. In the Figure 1, the shaded areas are the gluing faces and the arrows represent a set of instructional folding orientations.

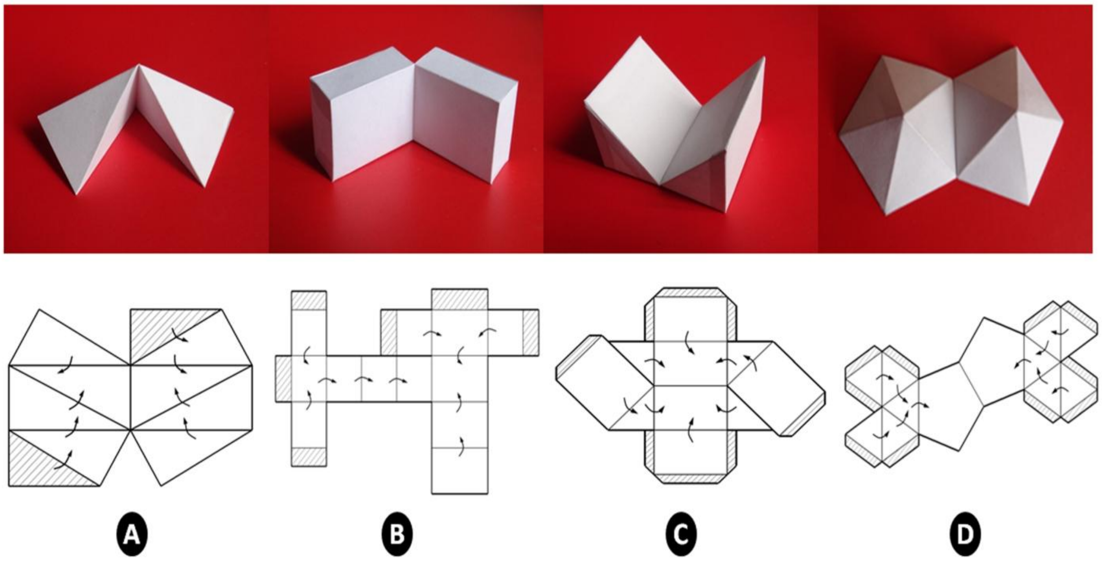


Figure 1: *4 representative BSUs.*
(A: tetrahedral; B: cuboidal; C: prismatic; D: pyramidal)

Kaleidoscopic Reconfigurable Sculptures

Kaleidoscope and Versatility: We demonstrate our first BSU with the tetrahedral unit. Special geometric features are allowed to be embedded on each of a tetrahedron's sides, such as right angles and equilateral edges, to start building kaleidoscopic symmetric structures, movable constructs and their transformations. Figure 2 shows a hierarchical evolution using skew tetrahedral BSU, where each side of the tetrahedron is a right triangle and tetrahedral edges are in the ratio of: $1: \sqrt{3} : 2 : \sqrt{5} : 2 : 1$.

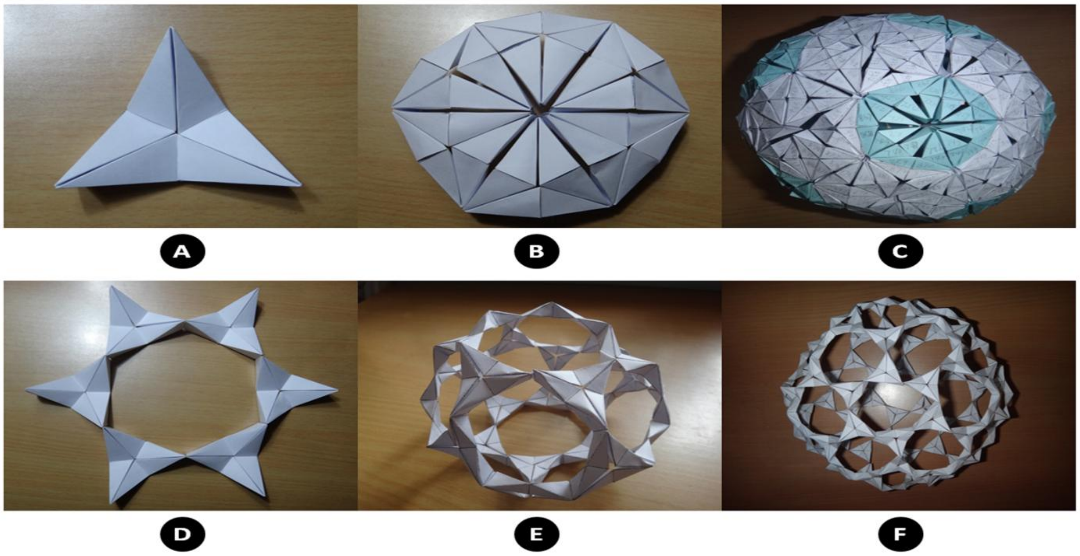


Figure 2: *Hierarchical derivatives using skew tetrahedral BSUs.*
From A to B to C: building truncated icosahedron; from A to D to E / F: building skeletonized ellipsoid.

Three skew tetrahedral BSUs (3stBSU) with serial connections (in Figure 2A) topologically form a closed-loop equilateral triangle. Furthermore, hinging six 3stBSU serially gives a closed-surface hexagon-like structure (see Figure 2B), and this hexagon structure is also able to self-reconfigure into a hexagram-like structure with a hollow center (see Figure 2D). By initiating the hexagon and hexagram structures with other pentagon and pentagram ones and applying techniques of folding polygons to convex polyhedral surfaces, we enable the formation of a closed-surface truncated icosahedron (overall 540 tetrahedral BSUs, as shown in Figure 2C) and skeletonized movable ellipsoid (overall 72 skew tetrahedral BSUs in Figure 2E; 216 skew tetrahedral BSUs in Figure 2F). We allow the joining of hinges of structural units for the spherical construction by gluing (see fabrication). Starting from a BSU, more complex relative motion between structures is enabled by using serial, parallel and multi-hybrid assemblies. Each structure and movable construct can be considered as the new BSU to cumulatively achieve more complex derivatives, while at each generation various reconfigurations can achieve multiple folding states.

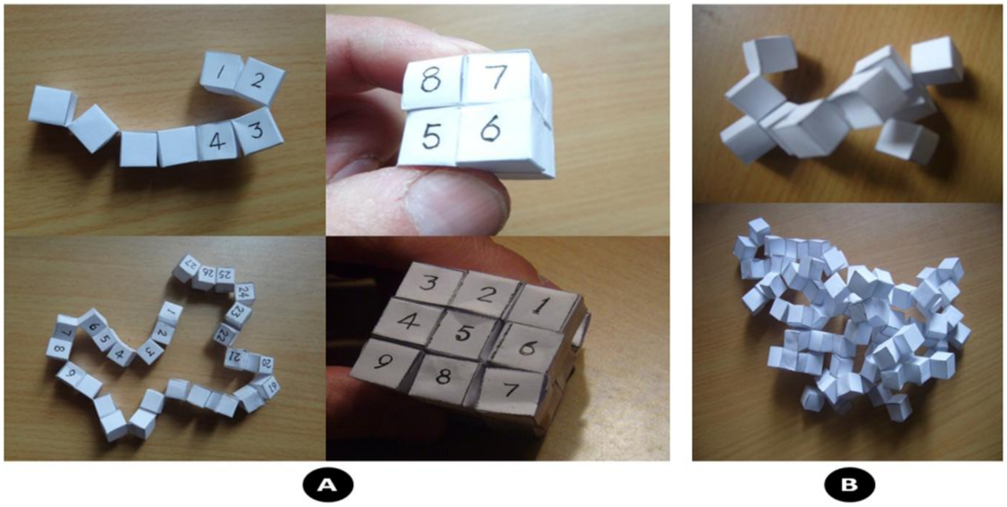


Figure 3: *Derivatives using cubic BSUs*

A number of recent architectural and engineering design practices are revisiting cubic structures, such as developing monumental headquarters, conceptual structures and self-reconfigurable robots [13]. Space utilization efficiency, kinematic performance, structural durability and aesthetic functions are ensured using the simple and combinatorial cubic structure. 2×2×2 and 3×3×3 Rubik's cubes are transformed from single chains as shown in Figure 3A. Random 10-cubic and 72-cubic chains are shown in Figure 3B.

Furthermore, prismatic and pyramidal structures, which are broadly implemented in architectural design such as vault roof and skylight [14], are also demonstrated in the representative family of BSU. Figure 4 reveals the movable chain made by 8 five-facets prismatic BSUs (Figure 4A) and a rigid spherical structure by six-facet pyramidal BSU (Figure 4B). Reconfiguration efficiently varies the 3D spatial structures and locking global or local zones embeds these ideas within architectural and engineering systems design.

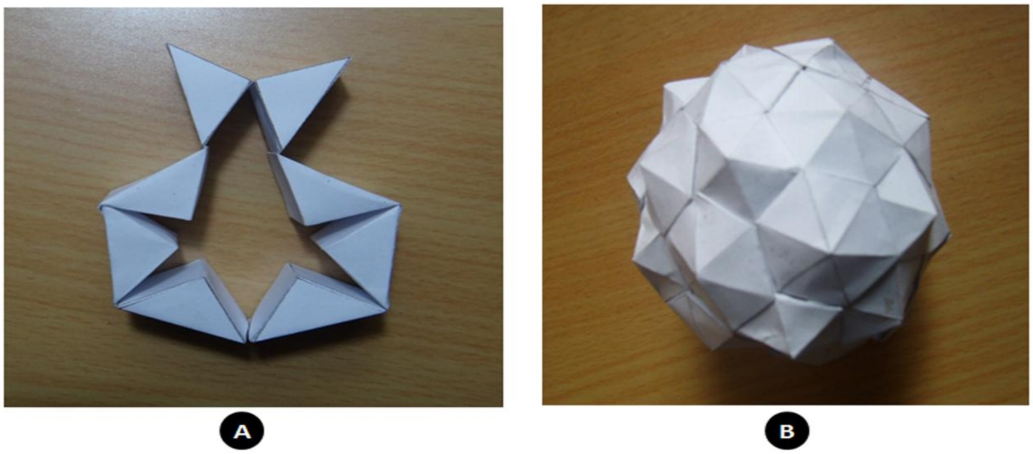


Figure 4: *2 Derivatives using prismatic and pyramidal BSUs*

Commonality and Interchangeability:

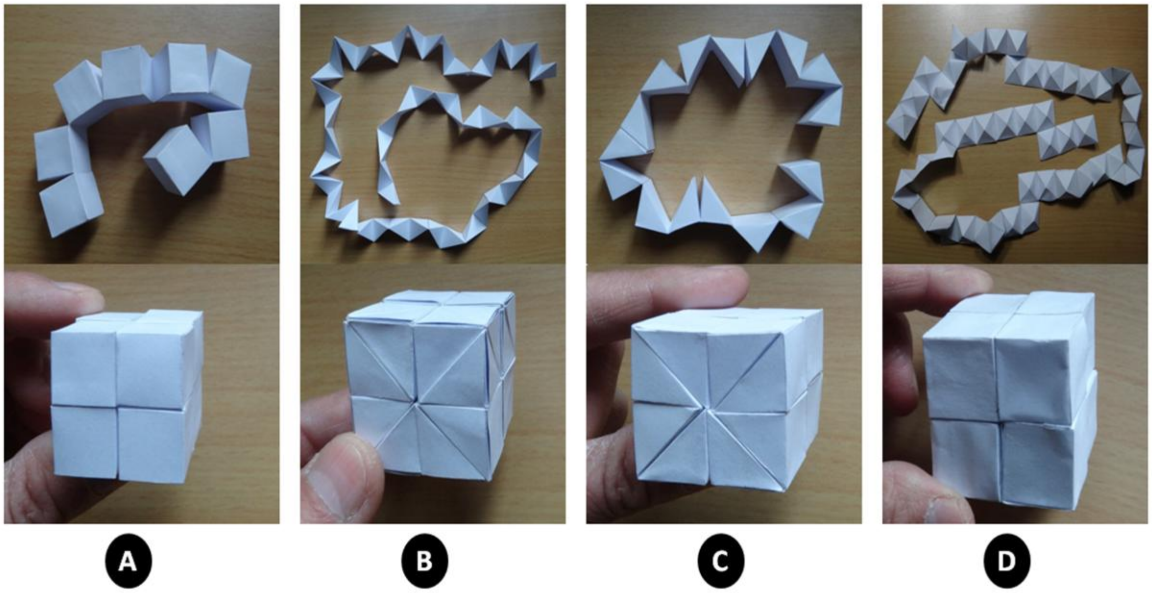


Figure 5: *2×2×2 Cube resulting from four different unit elements:*
(A) from cubic BSUs, (B) from tetrahedral BSUs, (C) from prismatic BSUs, (D) from pyramidal BSUs

Experiments also show that, by allowing smaller individual BSUs to be chained and transformed into other larger aggregate structures, one can achieve variable cell sizes without changing the form of the structure. For instance: 4 cubic BSUs (Figure 5A), 24 tetrahedral BSUs (Figure 5B), 8 prismatic BSUs (Figure 5C) and 24 pyramidal BSUs (Figure 5D), with a serial connection achieve the same 2×2×2 solid cube. The identical formation process and outcome give rise to possibilities for adapting rigidity and flexibility on the structures and also make each filial structure interchangeable under an integrated framework. The cubic configuration shown here is capable of converting back to single chain and rearranging into other different constructs.

Fabrication out of A Single Sheet: One of our goals of the work is to develop the rules of fabrication and construction while starting with a single flat paper sheet. Multi-folding procedures are exemplified in building a truncated icosahedron and illustrated using CATIA™. The multi-folding procedure consists of:

- Folding a chain of hinged tetrahedral units (Figure 6B) from a single long strip of paper (Figure 6A),
- Arranging the string of BSUs along a single-stroke traceable path that visits each skew tetrahedral BSU exactly once. The single-stroke traceable path is inspired by Hamiltonian paths [15]. Unfolding the convex polyhedra into planar polygons (pentagons and hexagons) is derived from Dürer's Nets [15] (shown in Figure 6C), and
- Hinging each of the neighboring pentagonal and hexagonal structures (Figure 6D) by gluing and folding them up into a truncated icosahedron (Figure 6E).

Using this strategy, we are able to construct many complex structures using a single paper strip. In the future we plan to optimize this construction strategy in a more compact and efficient way. By designating cuts-crease pattern on 2D sheets and cutting-folding-joining in 3D, the resulting derivative structures are lightweight and inexpensive enabling batch fabrication and economies of scale.

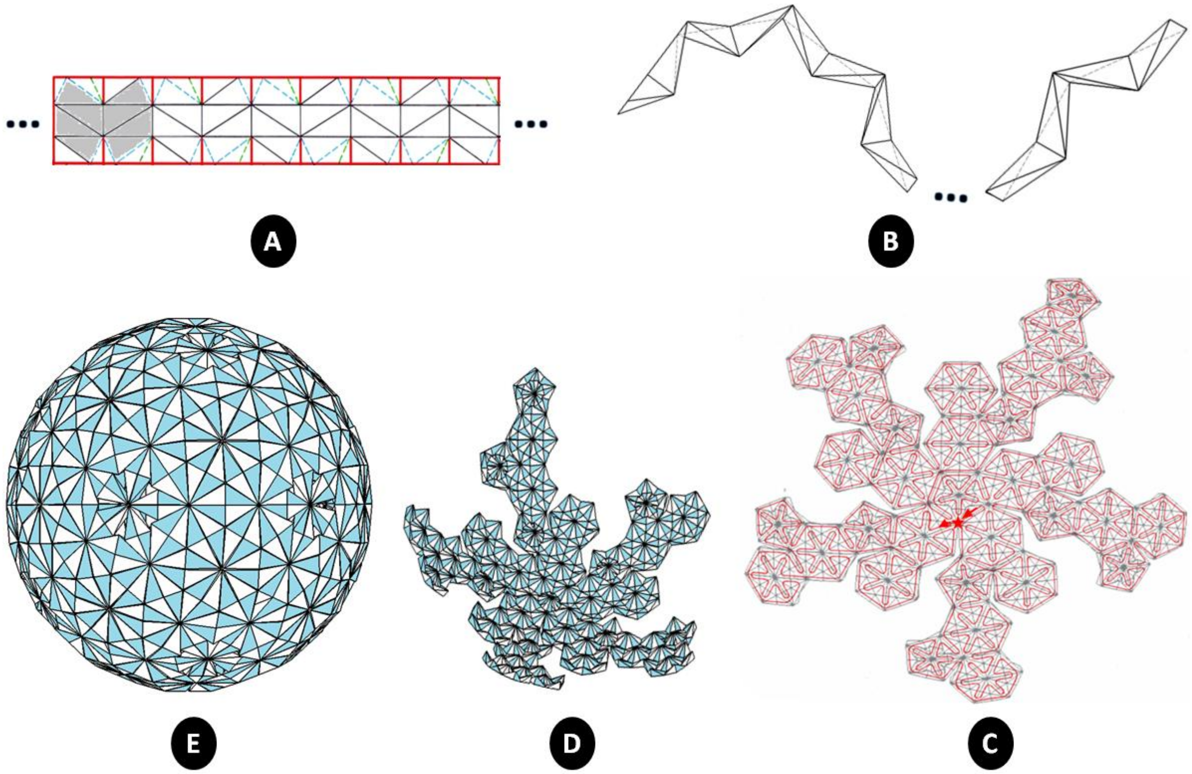
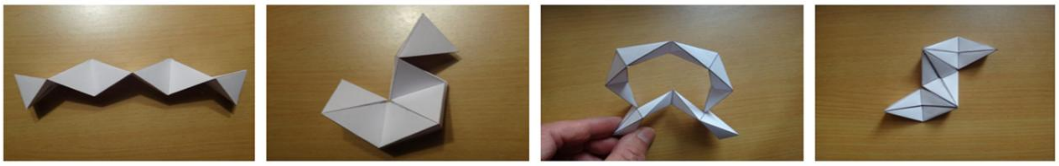


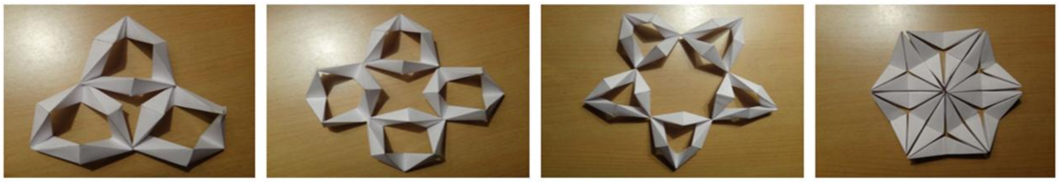
Figure 6: *Fabrication Processes of building truncated icosahedrons using skew tetrahedral BSUs*

Artistic Exploration: Kaleido-Tangram

Tangram, “the Fashionable Chinese Puzzle [16]” is a dissection puzzle which consists of seven pieces of flat shapes. A player manipulates the orientation and displacement of each piece by only translating, rotating and oscillating but without overlapping to create various 2D shapes. Analogous to the Tangram concept, the users can explore a new design space of 3D structures in Kaleidogami™ using a finite number of tetrahedral and cubic BSUs.



(A1) Goldfish **(A2) Hen** **(B1) Sparrow** **(B2) S shape**



(C1) Triplet **(C2) Quadruplet** **(C3) Pentaploid** **(C4) Snowflake**

(D1) Running Man **(D2) Marching Man** **(D3) Sitting Cat**

Figure 7: *Kaleido-Tangram Integration.*

In our work, BSUs are directly hinged as a single chain or tree-like structure after folding a single piece of paper. By rearranging, combining and reconfiguring, one can obtain different configurations with same amount of material (Figure 7 D1, D2, D3) or shape variation of BSUs. Versatile structural 3D animals and geometric models are exhibited in Figure 7. In a nutshell, Kaledogami™ inspires spatial and design thinking using sculpturing metaphor and fabrication as means to achieve artistic representation of 3D objects.

Conclusion

The main contribution of this work is the novel folding representation, called “Kaleidogami™”, encompassing multi-primitive and reconfigurable foldable units for assemblage of spatial structures and movable constructs. Our discoveries and fabrication rules enable concurrent design of the geometric structures that can be folded from a single flat sheet. Many scientists and engineers are motivated by the beauty of artistic representations while artists and architectural designers want to embed novel science-technology-engineering-mathematics (STEM) inspired concepts. We attempt to pursue the science and technology of reconfigurable structures and enable new adaptation of this geometry-inspired art form to artists, architects as well as origamists. Furthermore, by exploring our methods we enable an active exploration of Kaleidogami™ construction.

The following are our visions of the further Kaleidogamic study:

One can extend the computerized support and ease the realization of this new science-based art form. We demonstrate the foldable and reconfigurable structures using hands-on construction. Computational algorithms and tools can further help one unfamiliar with the geometric nuances and patience in folding to design, analyze and optimize Kaleidogamic structural and mobile forms, as well as construct and conduct the folding process plan. Besides paper, a variety of commercially available non-wovens also open possibilities for exploring the substrate selection and surface-structure function based on the needed properties such as wettability, creasability (wrinkle-resistance), adhesive properties, strength, and stiffness. Optimal selection of the material-manufacturing combinations and having the least environmental impact would provide a significant pathway of this research.

One can also explore numerous art and architecture applications. Responsive and interactive environments that mimic life forms have been explored by a number of artists, architects and designers [17]. Kaleidogamic study can span artistic and utilitarian possibilities, engage people, spark their imagination and enable creative interactions. Complexity of form, responsiveness and behavior could emerge from elegant simplicity of flat foldable surfaces that are creative and exhibit intelligence. The field of engineering and architectural design has long benefited from sophisticated geometrical possibilities from geodesic domes of Buck Minister Fuller to complexly double-curving surfaces of Frank Gehry. The reconfigurable characteristics of Kaleidogami™ to include considerations of structural systems make it a great approach to many artistic and architectural situations [18]. Through explorations at the intersections of art-science-geometry and digital information technologies we intend to promote imagination and critical thinking. Fundamental mathematical knowledge and learning skills such as spatial perception and logical thinking become accessible through this art form. We also naturally promote public engagement, while we strive to develop future collaborations at these intersections of traditional fields and create new fields for creative explorations without boundaries. We plan to explore such collaborations with local museums for example by incorporating new functions through kinetic art embedding special materials and digitally inspired technologies [19].

One of the benefits is also to broaden educational participation. More general mathematical rules in geometric combinatorics, structural combinations and decompositions will be developed so that a new breed of art-science-engineering students begins to engage in designing art with Kaleidogami™.

Acknowledgements

The authors of this paper would like to acknowledge the support of Professor Ramani by the Donald W. Feddersen Chair professorship that enabled his participation as well as teaching assistantship from the School of Mechanical Engineering to Wei Gao that allowed him to develop this area. We would also like to thank Professor Mahesh Daas for his insights and suggestion of strategies to explore art and architecture.

References

[1] E.M. Arkin, M.A. Bender, E.D. Demaine, M.L. Demaine, J.S.B. Mitchell, S. Sethia, and S.S. Skiena, *When Can You Fold a Map?*, Computational Geometry, 29, pp.23-46, 2004.

[2] E.D. Demaine, M.L. Demaine, and J.S.B. Mitchell, *Folding flat silhouettes and wrapping polyhedral packages: New results in computational origami*, Computational Geometry: Theory and Applications, volume 16, number 1, 2000.

[3] R. Nagpal, *Programmable Self-Assembly: Constructing Global Shape using Biologically-inspired Local Interactions and Origami Mathematics*, PhD thesis, MIT Department of Electrical Engineering and Computer Science, 2001.

[4] E. Hawkes, B. An, N. Benbernou, H. Tanaka, S. Kim, E.D. Demaine, D. Rus, and R.J. Wood, *Programmable matter by folding* Proc. Natl Acad. Sci. USA 107, pp. 12441-12445, 2010.

[5] K. Miura, *The science of Miura-ori: A review*, In 4th International Meeting of Origami Science, Mathematics, and Education, R. J. Lang, ed., A K Peters, pp.87–100, 2009.

[6] R. Hoffman. *Airbag folding: Origami design application to an engineering problem* (easi engineering gmgh, germany). In Third International Meeting of Origami Science Math and Education, Asilomar, CA, March, 2001.

[7] Z. You and K. Kuribayashi, *A Novel Origami Stent*, Summer Bioengineering Conference, 2003.

[8] G. Mullineux, J. Feldman, and J. Matthews, *Using Constraints at the Conceptual Stage of the Design of Carton Erection*, J Mech Mach Theory 45(12), pp.1897-1908 , 2010.

[9] L. Simon, B. Arnstein, and R. Gurkewitz, *Modular Origami Polyhedra*, Dover, Toronto, Canada, 1999.

[10] P. Schatz, *Rhythmusforschung und Technik*, Verlag Freies Geistesleben, Mathematics, 1975.

[11] D. Sehattsehneide and W.Walker, *M.C.Eseher Kalerdoeycels*, BallantineBooks, NewYork, 1977.

[12] R. Byrnes, *Metamorphs: Transforming Mathematical Surprises*, Tarquin Publications, 2008.

[13] Y. Meng, Y.C. Jin, *Morphogenetic Self-Reconfiguration of Modular Robots*, Bio-Inspired Self-Organizing Robotic Systems, 2011.

[14] Skylights design and construction, http://ktiriodesign.gr/skylights_heliolite_eng.html

[15] H.C. Reggini, *Regular polyhedra: random generation, Hamiltonian paths, and single chain nets*, Academia Nacional de Ciencias Exactas, Físicas y Naturales, 1991.

[16] J. Slocum, J. Boterman, D. Gebhardt, M. Ma, XH. Ma, H. Raizer, D. Sconneveld and C.V. Splunteren, *The Tangram Book*, Sterling, pp. 31, 2003.

[17] M.Senagala, *Kinetic and Responsive: A Complex-adaptive Approach to Smart Architecture*,Vision and Visualization, Proceedings of SIGRADI International Conference, Lima, Peru, 2005.

[18] M. Daas, (personal communication, January 8, 2012).

[19] R. Stein, *Desperately Seeking Innovation: Making Connections between Art and Science*, Dimensions Magazine: Association of Science and Technology Centers, March_April, 2012.