DETC2011-47137

ADDRESSING UNCERTAINTIES WITHIN PRODUCT REDESIGN FOR SUSTAINABILITY: A FUNCTION BASED FRAMEWORK

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ABSTRACT

The Function Impact Method (FIM) is a semiquantitative eco-design methodology that is targeted specifically towards the early stages of the design process. The FIM allows a designer to predict the environmental impacts associated with a new functional embodiment by extrapolating knowledge from Life cycle assessment (LCA) of similar existing designs. LCA however, is associated with substantial sources of uncertainty. Furthermore, the FIM uses a subjective weighting scheme for representing function-structure affinities. In the authors' previous work, a Monte-Carlo variation analysis was used to estimate sensitivity of the input data and estimating the preferred redesign strategy. This paper proposes a method to formalize the input uncertainties in the FIM by modeling the uncertainties present in the results of the LCA's and the involved function-structure affinities using Info-gap decision theory. The desirability of redesigning a particular function based on the magnitude of its functionconnectivity and eco-impact is estimated, and a decision making methodology based on robust satisficing is discussed. This method is applied for making robust redesign decisions with regards to re-designing a pneumatic impact wrench for sustainability.

Keywords: Function Impact Method, Life Cycle Analysis, Info-gap decision theory, Decision making, Eco-design

INTRODUCTION

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Growing environmental concerns, coupled with public pressure and stricter regulations, are fundamentally impacting the way companies design and launch new products across the world [1]. Therefore, companies are confronted with the responsibility of designing/redesigning products to make them environmentally friendly. Among the various methods for assessing the environmental impact of a product, Life cycle assessment (LCA) has emerged as the most objective methodology. LCA is intended to be a comparative assessment methodology [2] whose significant application is to provide information that is useful in making design/redesign decisions regarding a product or a process. LCA however, is associated with substantial sources of uncertainty. They can include input data with varying degrees of quality as well as possible simplifications within inventory analysis and assessment models. [3, 4]. Often, there is very little knowledge about the nature of these uncertainties and characterizing them is usually impractical. The impact assessment phase of the LCA, in addition, presents significant uncertainties due to spatial and temporal variations. Several modeling techniques have been proposed to deal with the uncertainties present in LCA [5, 6, 7]. Most of the above literature however, is aimed directly at LCA experts. Very few techniques exist which integrate uncertainties in LCA with sustainable design decision making development [8, 9, 10, 11]. Moreover, their focus is not on facilitating robust decisions in context to the product designer.

Design tools like the Function Impact Method (FIM) are aimed at bridging this gap between LCA experts and product designers' [12]. The FIM combines raw LCA data with existing product knowledge in the form of design structure and creates representations of environmental impact that are easily understood and effective for making redesign decisions. Thus, while using the FIM it is necessary that the designer has information about (1) the uncertainties involved in the FIM and (2) the robustness of his/her design decisions with respect to these uncertainties. On the other hand, the designer cannot be expected to be an LCA expert. Easy-to-use models that allow for the incorporation of uncertainties with sparse information (only nominal values of the LCA output) are necessary. Among the methods available for decision making under such severe uncertainty, information-gap decision theory (IGDT) is a promising approach because IGDT 1) does not require additional assumptions about the nature of the uncertain data and 2) allows the designer to make tradeoffs between robustness of each redesign decision and its corresponding uncertainty [13].

This manuscript is aimed at establishing an uncertainty schema for the FIM by addressing uncertainties in the LCA and function-structure affinities using informationgap decision theory. IGDT provides a framework based on satisficing rather that optimizing, which is applicable under high risk scenarios such as designing to prevent environmental damage. The paper will first review the general idea of the Function Impact Method (FIM), and then introduce a measure for estimating the desirability of redesigning a particular product function. This proposed measure is dependent on function impact as well as function-coupling characteristics within a design. Next, a brief introduction to information-gap decision theory is presented. Finally, the proposed methodology is tested through the redesign of a C.H. ½ inch impact wrench with regards to environmental sustainability.

The Function Impact Method (FIM)

In the author's previous work [14], the FIM was presented as a novel eco-design methodology that facilitates the use of LCA data to support the integration of sustainability concepts during the early design phase. The core idea behind the FIM is to distribute life cycle environmental impacts across product functions. The main goal of the FIM is to identify the environmental impact of each function with respect to the overall system performance, as well as to reveal potential areas for redesign. The mathematical representation of environmental impacts attributed to each function is given in Eq. 1:

$$(1) \ \ FI = \left[\beta_{i,j,n}\right] = \left[\left\{\sum_k \! \left(M_{i,j,k} + \, \sum_m P_{i,j,k,m}\right) \cdot \, \alpha_{k,n}\right\} + \, U_{i,j} \, \cdot \, \gamma_n\right]$$

where $\beta_{i,j,n}$ is the environmental impact of category j due to function n for benchmark product i, and γ_n is the percentage of function n contributes to the overall functionality (i.e. the use) of the product. Furthermore, $M_{i,j,k}$ is the environmental impact of category j associated with component k due to material, Pi,j,k,m is the environmental impact of category j associated with component k due to the m-th manufacturing step, and U_{i,j} is the environmental impact of category j during the use of the product. For example, if a product included a motor to perform a specific function, the environmental impact associated with powering the motor would carry some percentage (γ_n) of the total impact during the product's use phase. In general, γ_n allows the designer to trace functions back to a component level from a use phase perspective, while $\alpha_{k,n}$ indicates the percentage distribution of each component to a given function during all other significant phases of a product's life cycle.

To summarize, in order to use the methodology for product development, LCA must first be conducted on market leading designs of existing products (e.g., staplers, coffee makers, compressors). The environmental impacts can then be distributed among product functions to establish function-impact correlations, which will be used to support both novel concept generation as well as redesign decisions. A tabular representation of the FIM is shown in Figure 1.

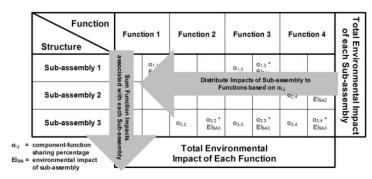


Figure 1: REPRESENTATION OF THE FUNCTION IMPACT METHOD (FIM)

The function-structure affinities $(\alpha_{k,n})$ carry significant uncertainties due to the subjective nature of their elicitation. In past work, a sensitivity analysis was performed by perturbing the chosen affinities by ± 10 percent of their mean value. A Monte Carlo variation

analysis (MCVA) was then performed and the function-impacts were ranked as per their magnitude. A redesign option was chosen if it carried the largest probability of having the highest function-impact rank. The MCVA is useful in that it assigns a probabilistic confidence to the chosen redesign decision. However, it is limited by the facts that 1) an assumption about the probabilistic distribution of the uncertain data has to be made and 2) it offers no information about the nature of the uncertainties involved, or the robustness of the chosen decision unless additional information in terms of confidence intervals are specified.

Information Gap decision theory

Info-gap decision theory, developed by Yakov Ben-Haim in 2001 [13] is an approach suited for making decisions under sparse information. Its core objective is to organize information and the lack of it in terms of families of clusters or nested sets. Info-gap decision theory has been successfully applied to several interdisciplinary fields including ecological conservation [15], electricity procurement [16], remanufacturing process selection and product re-design [17]. Within this paper, the focus is limited to decision making in early design using interval bound info-gap models, as they are best suited for analysis of design decisions. An interval bound info-gap model is characterized by the following parameters:

- u: the uncertainty variable whose nominal value
 (ũ) is known
- α : the level of nesting, i.e. the horizon of uncertainty
- r_c: a critical value of performance that must be achieved
- d:a set of design options
- R(d,u): a reward model for the system under consideration
- α(d, r_{cr}): the info-gap robustness function, which details the largest info-gap uncertainty tolerable to deliver the minimum acceptable performance (r_c) for a specific design option

The corresponding info-gap model $U(\alpha, \tilde{u})$ is as per Eq. 2:

(2)
$$U(\alpha, \tilde{u}) = \{ u : |u - \tilde{u}| \le \alpha \}, \alpha \ge 0$$

In cases where the maximal variation is proportional to the nominal value of the uncertainty variable, the info-gap can be modeled as in Eq. 3:

(3)
$$U(\alpha, \tilde{u}) = \left\{ u : \left| \frac{u - \tilde{u}}{\tilde{u}} \right| \le \alpha \right\}, \quad \alpha \ge 0$$

The robustness function in info-gap decision theory is formulated as an optimization problem with the objective of maximizing α whilst satisfying the critical performance constraint, r_c . In cases of larger the better, it can be mathematically represented as in Eq. 4:

(4)
$$\hat{\alpha}(d, r_{cr}) = \max \{ \alpha : (\min_{u \in U(\alpha, \widetilde{u})} R(d, u)) \ge r_{cr} \}$$

If smaller performance is better, the robustness function is mathematically defined as in Eq. 5:

(5)
$$\hat{\alpha}(d, r_{cr}) = \max \{\alpha : (\max_{u \in U(\alpha, \widetilde{u})} R(d, u)) \le r_{cr} \}$$

The design option that yields the greatest magnitude of robustness for a specified critical performance is preferred as per the robust satisficing model. Robust satisficing unlike many other uncertainty models does not yield a design optimized for performance. Instead, a design option is selected based on its likelihood of surviving failure. This analogy is appropriate for situations such as environmental sustainability, as the penalty of failure is very high. In the section below, the FIM coupled with IGDT is applied for the re-design of a C.H. ½ impact wrench. By analyzing the uncertainties present in the function-impacts, the designer can select the re-design strategy which is most robust with regards to desirability for sustainable redesign.

METHODOLOGY

To establish a measure for function-coupling, the product is represented as a bipartite graph FS = (F, S, E), where $\{F_i, ..., F_m\}$ represent product functions and $\{S_i, ..., S_n\}$ represent product structures. This bipartite graph can be represented by a binary matrix, whose elements establish a correlation between the i^{th} product function and the j^{th} product structure. The matrix can be mathematically represented as shown in Eq. 6.

(6)
$$FS = \begin{bmatrix} c_{ij} \end{bmatrix}$$
 $i = 1, ..., m$; $j = 1, ... n$
where: $c_{ij} = 1$, if $F_i \rightarrow S_j$ (\exists an edge)
 $c_{ij} = 0$ otherwise

To establish function-function correlations, a function adjacency matrix is constructed as given in Eq. 7. The coupling, or the connectivity, of a particular product function to all other functions is obtained as the row sum of the function adjacency matrix as given by Eq. 8.

$$(7) \quad FF = FS * FS^T$$

(8)
$$conn_i = \sum_{\substack{j=1\\ i \neq i}}^n f f_{ij}$$

FS: function–structure matrix representing the design FF: function-function adjacency matrix

 $conn_i$: the connectivity of the i^{th} product function to all other product functions. It should be noted that the diagonal elements of FF is omitted for the calculation of the connectivity metric because it represents the total number of connections between the i^{th} product function and the j design structures.

The desirability (or opportuneness) of redesigning a particular product function, is determined by Eq. 9. The measure depends on 1) the normalized magnitude of function-coupling and 2) the normalized function-impact.

Axiomatic design, defines design complexity/information as a logarithmic function of the probability of achieving the specified Functional Requirements (FRs) [18]. Thus, an exponential scale is used in Eq. 9 to linearize the measure of function-coupling, which in this case is a measure of the complexity within a given design. The desirability measure indicates that a function is preferred for redesign if it has a high function impact and if it is relatively uncoupled. The coupling measure is critical for redesign as it identifies functions that are easier to rework from a modularity perspective [19]. Thus, the best possible case for redesign is when a function is fully uncoupled and has a high value of function impact. To account for the difference in scales between the values of the function-impact and the function-coupling, these values are normalized among themselves before calculating the function's redesign desirability measure. Thus, the magnitude of desirability has a theoretical maximum of 1 + k and a minimum tending to zero. The scaling factor in Eq. 9 is a fraction which signifies the

preference one wishes to allocate to function-coupling as compared to function-impact for redesign.

$$(9) \quad D_i = k * e^{-conn_i} + FI_i$$

 \boldsymbol{D}_{i} : desirability measure for redesign of the ith product function

k : preference factor that establishes the relative redesign preference between function coupling and function-impact

FI_i: the function-impact of the ith product function

The calculation of function impact is given by Eq. 10. In this calculation, both eco-impact (I_j) and the function-structure affinity w_{ij} (same as $\alpha_{k,n}$ in the FIM description) are treated as uncertain variables whose nominal values are known. The info-gap models for these variables are given by Eq. 11 and Eq. 12, respectively. The info-gap models are such that the maximal variation is proportional to the nominal value of the uncertainty variable.

(10)
$$FI_i = \sum_{j=1}^n w_{ij} I_j$$

$$(11) \quad U_I(\alpha_I, \tilde{I}) = \left\{ I : \left| \frac{I - \tilde{I}}{\tilde{I}} \right| \le \alpha_I \right\}, \alpha_I \ge 0$$

$$(12) \quad U_w(\alpha_w,\widetilde{w}) = \left\{ w: \, \left| \frac{w-\widetilde{w}}{\widetilde{w}} \right| \, \leq \, \alpha_w \right\}, \alpha_w \geq 0$$

The reward or utility function for the present model is represented by the desirability to redesign a function as given in Eq. 13:

(13)
$$R(d, u) = D_i = k * e^{-conn_i} + FI_i$$

The objective of this formulation is to maximize the robustness function given by Eq. 14:

$$(14) \quad \hat{\alpha}(i, d_{cr}) = \max \left\{ \alpha : \left(\min_{\substack{I \in U_I(\alpha_I, \tilde{I}) \\ w \in U_w(\alpha_w, \tilde{w})}} [d_i] \right) \ge d_{cr} \right\}$$

where:

	FUNCTION	Tran Mot			nvert sure to	Impo	rt Air	Ho		_	ulate tput		vent ear	_	vent page		vent kage		ngage tion		cate olt	E E
STRUCTURE (Subassemblies)		Percent	Impact	Percent	Impact		Impact	Percent	Impact	Percent	Impact	Percent	Impact	Percent	Impact	Percent	Impact	Percent	Impact	Percent	Impact	STRUCTURE IMPACT (Pt)
Housing Assembly	Housing, Back Plate, Paper back, casing crews, gasket, set					0.05	0.264	0.80	4.231					0.05	0.264	0.10	0.529					5.2893
Grip	grip													1.00	0.006							0.0064
Extender Assembly	replacement choke, extension	0.50	0.451																	0.50	0.451	0.9028
Rotor Assembly	rotor, housing, rotor Fins	0.50	0.669	0.50	0.669																	1.3374
Chuck	washer, thin washer, chuck	0.80	0.370															0.20	0.093			0.4630
Valve Assembly	hose connector, valve ball, rod, spring					0.75	0.166									0.25	0.055					0.2218
Regulator Assembly	knob, switch, spring, set screw					0.25	0.057			0.75	0.170											0.2266
Trigger	trigger, washer, spring, cover, screws, trigger rod, rod					0.10	0.003			0.90	0.023											0.0253
Hammer	hammer cage, hammer, dowel pin, gear	0.40	0.480					,										0.60	0.720			1.2005
Bearing Block	thin bushing plate, thick bushing plate			0.25	0.243							0.75	0.729									0.9725
FUNCTION IMPACT (Pt)		18.5%	1.971	8.6%	0.912	4.6%	0.490	39.7%	4.231	1.8%	0.193	6.9%	0.729	2.5%	0.271	5.5%	0.584	7.6%	0.813	4.2%	0.451	10.6457

Figure 2: FUNCTION IMPACT MATRIX OF THE C.H 1/2 INCH IMPACT WRENCH

 d_{cr} : the critical or the minimum allowable value of the desirability measure $% \left\{ 1\right\} =\left\{ 1\right\}$

 w_{ij} : function-structure affinity of the ith product function to the jth component

 $\widetilde{\mathbf{w}}$: the nominal value of the corresponding function-structure allocation

I_i: environmental impact of the jth component

 $\tilde{\mathbf{l}}$: the nominal value of the corresponding environmental impact

 α_{I} : the horizon of uncertainty for the corresponding environmental impact

 α_w : the horizon of uncertainty for the corresponding function-structure allocation

CASE STUDY

The pre-mentioned methodology is applied to a redesign project for a pneumatically-powered Campbell Hausfeld (C.H.) ½ in. impact wrench. The impact wrench is disassembled and a bill of materials (BOM) is constructed, including each component, its weight, its material and any processing steps necessary to produce the part. The BOM is essential for conducting a life cycle analysis of the product. For each component, material and manufacturing processes were estimated based on queries within CES EdupackTM 2010 and availability within SimaProTM 7.1. The LCA was conducted via SimaProTM 7.1 and the Ecoinvent 2.0 database. A full functional analysis was completed to understand the interstructural component relationships. Extracting design knowledge from the product through disassembly helps construct the function-structure matrix (FSM). having the environmental impact of each component and the FSM, the FIM can be completed by assigning affinities based on structure to function. In this case, two design experts independently assigned affinity weights to each function-structure relationship, and then concurred on the final value. The FIM is shown below in Figure 2 with the highest contributors to eco-impact highlighted.

To incorporate functional requirements for the redesign, a connectivity to measure the degree of function modularity respective to the entire design is proposed. Utilizing Eqs. 6-8, the conn_i is derived for each function as shown in Figure 3.

	Transmit Motion	Convert Pressure to Torque	ImportAir	House Components	Regulate Output Torque	Prevent Wear	PreventSlippage	Locate Bolt	PreventLeakage	DisengageMotion	$conn_i$	e-conni
Transmit Motion	ΓX	1	0	0	0	0	0	1	0	2]	4	0.0183
Convert Pressure to Torque	1	X	0	0	0	1	0	0	0	0	2	0.1353
Import Air	0	0	X	1	2	0	1	0	2	0	6	0.0025
House Components	0	0	1	X	0	0	1	0	1	0	3	0.0498
Regulate Output Torque	0	0	2	0	X	0	0	0	0	0	2	0.1353
Prevent Wear	0	1	0	0	0	X	0	0	0	0	1	0.3679
Prevent Slippage	0	0	1	1	0	0	X	0	1	0	3	0.0498
Locate Bolt	1	0	0	0	0	0	0	X	0	0	1	0.3679
Prevent Leakage	0	0	2	1	0	0	1	0	X	0	4	0.0183
Disengage Motion	L 2	0	0	0	0	0	0	0	0	X	2	0.1353

Figure 3: FUNCTION-FUNCTION CORRELATION MATRIX (FF)

Simply surveying Figure 2, it is evident that the function 'House components' carries the heaviest environmental burden, 39.7% of the total environmental impact, which makes it the most suitable candidate for redesign.

However, when analyzing the design structure, it becomes clear that the function 'House components' is highly coupled with other product functions. Therefore, the redesign of this particular function becomes rather complex. Thus, there exist cases where the designer has to associate a preference between the environmental impact that can be saved and the ease of the redesign process itself. The desirability measure proposed is an effort to capture this tradeoff, and also estimate the robustness of this decision under uncertainty as discussed in the results section below.

RESULTS

The data from the FIM along with functioncoupling data derived from the FSM were used to construct an IGDT model for the case study. Figure 4 shows the plot of uncertainties with respect to the desirability measure of the function 'House Components'. It is clear that at certain high values of uncertainty the set critical limit of desirability (0.25) is exceeded. This underscores the importance of assessing the uncertainties present in the FIM. Figure 4 also shows the robustness plot between a value of desirability and the corresponding uncertainties present. As shown the functions, 'Prevent Wear' and 'Locate Bolt' do not have a significant drop in the value of desirability with increasing values of uncertainty. Therefore, they are robust selections from a redesign perspective. On the other hand, 'House Components' has a higher desirability measure at zero uncertainty, but drops off rather rapidly.

The plots in Figure 4 are useful in making a particular decision only if the desirability measure of one function dominates the others for all values of critical desirability. However, as shown in the above case there is a switch in dominance depending on the value of critical desirability. In such cases, unless the interval of critical desirability is negligible, there exists a region where an alternative cannot be chosen without providing additional

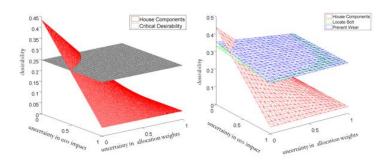


Figure 5: 3D UNCERTAINTY PLOTS OF THE IMPACT WRENCH

information to the decision maker [20]. This required information provides details of trade-offs between competing uncertainties. Scaling weights specified by [21] is one such way of trading-off uncertainties. The mathematical representation of an information-gap model with scaling factors is given by Eq. (15):

(15)
$$U(\alpha, \tilde{u}) = \left\{ u : \left| \frac{u_n - \tilde{u}_n}{\tilde{u}_n} \right| \le s_n \alpha \ n = 1, 2, \dots, N \right\}, \qquad \alpha \ge 0$$

where s_n is a unitless scaling factor that modifies the magnitude of α to be of appropriate scale for each uncertain variable, u_n . Scaling factors are determined on available prior knowledge of the nature of uncertainties in question. In the present case however, the designer has access to no such information. Therefore, equal scaling factors are adopted for modeling uncertainties. If the decision maker in the future can obtain reliable information on the nature of these uncertainties, IGDT can be employed with those specific scaling parameters. By the use of scaling parameters, the problem is condensed into trading of robustness between the critical value of desirability and a baseline value of uncertainty, which contains information on both the eco-impact and function-structure uncertainties.

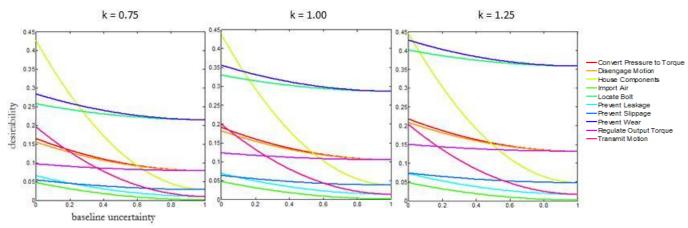


Figure 4: NORMALIZED 2D ROBUSTNESS PLOTS WITH VARYING (k) VALUES

Figure 5 displays the robustness factors with equal scaling factors, for three different values of preference factors (k). When it is assumed that there is no uncertainty, the function 'House Components' achieves the maximum of desirability of 0.44, and is the obvious candidate for redesign. However, as the baseline uncertainty increases, the alternative to be chosen switches. For example, in the plot with k=1, the functions 'House Components' and 'Prevent Wear' intersect at the baseline uncertainty value of 0.13 (13% deviation from the nominal). Thus, beyond this value of uncertainty, 'Prevent Wear' achieves a higher desirability measure and is to be chosen as the function to be redesigned. This indicates that the function 'House Components' is not robust to uncertainty as much as 'Prevent Wear'. If a designer is prepared to accept a critical desirability (maximum achievable value of desirability) of 0.34, then 'Prevent Wear' should be chosen for redesign due to its robustness to uncertainty. Or else, if the designer is certain that the uncertainty in his calculations lies under 0.13, 'House Components'. The other significant feature observed from the above figures is that as the need for product modularity becomes more important during, redesign 'Prevent Wear' tends to approach the maximum desirability value of 'House Components' and beyond k=1.33 emerges as the function which has both the highest value of desirability measure robustness to uncertainty. Thus, it dominates all other functions in entirety, and is the logical choice for redesign, without the need for further deliberations from the designer.

CONCLUSIONS AND FUTURE WORK

The aim of this manuscript was to incorporate a formal uncertainty framework within the FIM. IGDT was successfully incorporated within the FIM to represent uncertainties in environmental impact as well function allocation weights. Using IGDT, it is shown that decisions taken without any regard to uncertainty may lead the designer down the wrong path. The case study conducted in the C.H ½ inch impact wrench highlights the fact that IGDT can determine the range of uncertainty for which a particular product function has the highest expected utility.

The above methodology uses equal fractional scaling within the IGDT, as there are no existing means of obtaining this data. Future work will involve conducting studies among designers, to elicit this data. The desirability measure will be expanded to incorporate other elements such as cost, available manufacturing method

that contribute towards redesign complexity. Finally, an objective methodology to estimate the preference factor (k) will be researched upon.

ACKNOWLEDGMENTS

The authors would like to thank the Division of Environmental and Ecological Engineering at Purdue University for providing financial support in the form of a curriculum development grant. The authors would also like to thank Senthil K. Chandrasegaran from the School of Mechanical Engineering, Purdue University for his contribution. This work was also partially funded by the NSF under Grants EEC 0935074, MANU **1100619** and the Purdue Center for Advanced Manufacturing and Product Lifecycle Management (PLM) center. The contents of this manuscript do not necessarily reflect the views or opinions of either of these agencies.

REFERENCES

- [1] Choi, J., Nies, L., & Ramani, K. (2008). A framework for the integration of environmental and business aspects toward sustainable product development. Journal of Engineering Design, 19(5), 431-446. doi:10.1080/09544820701749116.
- [2] ISO 14040:2006, Environmental management-Life Cycle Assessment-Principles and framework. International Organization for Standardization, 2006a
- [3] Heijungs, R., "Identification of key issues for further investigation in improving the reliability of life-cycle assessments," Journal of Cleaner Production. Vol. 4, Issues 2-4. 159-166. 1996.
- [4] Maurice, B., Frischknecht, R., Coelho-Schwirtz, V., Hungerbuhler, K., "Uncertainty analysis in life cycle inventory. Application to the production of electricity with French coal power plants." *Journal of Cleaner Production*. Vol. 8. Issue 2, 95-108. 2000.
- [5] Hoffmann, M. R., Martin, S. T., Choi, W., "Environmental Application of Semiconductor Photocatalysis," Chemical Reviews. 95 (1) 69-96. 1995.
- [6] Chevalier, J. L., Le Teno, J. F., "Life Cycle Analysis with Ill-Defined Data and its Application to Building Products," International Journal of LCA. 1 (2) 90-96. 1996
- [7] Beccali, G., Beccali, M., Cellular, M., "Fuzzy set application in life cycle inventory of building materials," Building and the environment. International conference, Paris France 1997. 275-282.

- [8] Duncan, S. J., Bras, B., Pardis, C. J. J., "An approach to robust decision making under severe uncertainty in life cycle design," International Journal of Sustainable Design. Volume 1, Number 1/2008 45-59
- [9] Eagan, P. and L. Weinberg. 1999. "An application of analytic hierarchy process techniques to streamlined life-cycle analysis of two anodizing processes." Environmental Science and Technology 33(9): 1495–1500.
- [10] Khan, F.I., R. Sadiq, and B. Veitch, 2004, "Life cycle iNdeX (LlnX): a new indexing procedure for process and product design and decision-making." Journal of Cleaner Production. 12 (1) 59-76. 2004.
- [11] Mohamadabadi, H.S., G. Tichkowsky, and A. Kumar 2007, "Development of a multi-criteria assessment model for ranking of renewable and non-renewable transportation fuel vehicles. Energy 34 (1). 112-125.
- [12] Karthik Ramani, Devarajan Ramanujan, William Z. Bernstein, Fu Zhao, John Sutherland, Carol Handwerker, Jun-Ki Choi, Harrison Kim, and Deborah Thurston, "Integrated Sustainable Life Cycle Design: A Review", J. Mech. Des. 132, 091004 (2010), DOI:10.1115/1.4002308
- [13] Ben-Haim, Y. 2001. Information-gap decision theory. Academic Press, San Diego, California, USA.
- [14] Devanathan, S., Ramanujan, D., Bernstein, W. Z., Zhao F., Ramani, K., 2010. "Integration of Sustainability Into Early Design Through the Function Impact Matrix." Journal of Mechanical Design. 132 (8) 1-8.
- [15] Regan, H. M., Y. Ben-Haim, B. Langford, W. G. Wilson, P. Lundberg, S. J. Andelman, and M. A. Burgman. 2005. Robust decision making under severe uncertainty for conservation management. Ecological Applications 15: 1471–1477
- [16] Zarem, K., M. P. Moghaddam, M. K. S. El Eslami, "Electricity procurement for large consumers based on Information Gap Decision Theory. 38 (1) 234-2
- [17] Duncan, S. J., Bras, B., Pardis, C. J. J., "Applying Info-Gap Theory to Remanufacturing Process Selection Affected by Severe Uncertainty," ASME Conference Proceedings. 2007 Vol. 15.
- [18] Suh, Nam P., The Principles of Design, Oxford University Press, New York, 1990.
- [19] Sosa, M. E., Steven D. Eppinger, Craig M. Rowles, "A Network Approach to Define Modularity of Components in Complex Products," Journal of Mechanical Design Vol. 129 2007 1118-1129

- [20] Duncan, S. J., (2007) Including Severe Uncertainty into Environmentally Benign Life Cycle Design Using Information Gap-Decision Theory. PhD thesis. Georgia Institute of Technology.
- [21] Ben-Haim, Y., and A. Laufer. 1998. Robust reliability of projects with activity-duration uncertainty. ASCE Journal of Construction Engineering and Management 124:125–132.