This paper presents a novel method to achieve absolute three-dimensional shape measurement solely using square binary patterns. This method uses six patterns: three low-frequency phase-shifted patterns and three phase-shifted high-frequency patterns. The phase obtained from the low-frequency phase temporally unwraps the phase obtained from high-frequency patterns. The projector is defocused such that the high-frequency patterns produce a high-quality phase, but the phase retrieved from low-frequency patterns has a large harmonic error that fails the two-frequency temporal phase unwrapping process. In this paper, we develop a computational framework to address the challenge. The proposed computational framework includes four major approaches to alleviate the harmonic error problem: (i) use more than one period of low-frequency patterns enabled by a geometric constraint-based phase unwrapping method; (ii) artificially apply a large Gaussian filter to low-frequency patterns before phase computation; (iii) create an error lookup table to compensate for harmonic error; and (iv) develop a boundary error correction method to alleviate problems associated with filtering. Both simulation and experimental results demonstrated the success of the proposed method.

Temporal phase unwrapping can unwrap the phase by referring to the additionally acquired information. Over the history, the DFP system can employ two-wavelength and multiwavelength phase-shifting algorithms [5–7] or hybrid binary coding with phase-shifting methods [8,9] for temporal phase unwrapping. For high-speed applications, it is always desirable to use the lower number of fringe patterns; as a result, the two-frequency phase-shifting algorithm is preferable. The phase obtained from the low-frequency phase unwraps the phase obtained from high-frequency patterns pixel by pixel without referring to neighboring pixels. However, the capability of the two-frequency method is significantly limited by the noise impact [10]. To reduce the noise impact, Hyun and Zhang recently proposed the enhanced two-frequency phase-shifting algorithm [11] by employing the geometric-constraint-based phase unwrapping method [12].

A typical DFP system requires sinusoidal fringe patterns for high-quality 3D shape measurement. However, owing to the use of 8-bit representation of the sinusoidal fringe patterns, the measurement speed is typically limited to be up to 120 Hz if a digital-light-processing (DLP) projector is used [1], albeit use of other projection techniques such as LED arrays [13], a laser speckle [14], or a mechanical projector.
solving these three equations leads to the texture

By experimental validation, and, finally, Section 5 summarizes

In optical metrology, various phase-shifting algorithms have been extensively employed due to their accuracy and robustness [2]. Typically, it achieves higher measurement accuracy with more fringe images. Despite its sensitivity to noise, a three-step phase-shifting algorithm is still preferable for high-speed applications since it requires the least number of patterns for pixelwise phase computation. Mathematically, three phase-shifted fringe images with equal phase shifts can be described as

\[ I_1(x, y) = I_0(x, y) + I''(x, y) \cos(\phi - 2\pi/3), \]

\[ I_2(x, y) = I_0(x, y) + I''(x, y) \cos(\phi), \]

\[ I_3(x, y) = I_0(x, y) + I''(x, y) \cos(\phi + 2\pi/3), \]

where \( I_0(x, y) \) is the average intensity or texture, \( I''(x, y) \) is the intensity modulation, and \( \phi \) is the phase. Simultaneously solving these three equations leads to the texture

\[ I(x, y) = (I_1 + I_2 + I_3)/3, \]

as well as the phase

\[ \phi(x, y) = \tan^{-1} \left[ \frac{\sqrt{3}(I_1 - I_3)}{2I_2 - I_1 - I_3} \right]. \]

Mathematically, Eq. (5) yields phase values ranging from only \(-\pi\) to \(\pi\) with modulus of \(2\pi\); and this phase is often referred to as a wrapped phase with \(2\pi\) discontinuities. A spatial or temporal phase unwrapping is typically needed to remove \(2\pi\) discontinuities for 3D shape measurement. Phase unwrapping essentially determines the integer number \(k(x, y)\) for each pixel \((x, y)\) to unwrap phase \(\phi(x, y)\). In other words, the relationship between the unwrapped phase \(\Phi(x, y)\) and the wrapped phase \(\phi(x, y)\) is

\[ \Phi(x, y) = k(x, y) \times 2\pi + \phi(x, y). \]

Here \(k(x, y)\) is often regarded as fringe order. If fringe order \(k(x, y)\) can be uniquely determined, the unwrapped phase \(\Phi(x, y)\) is regarded as absolute phase; and if \(k(x, y)\) has an unknown integer number of the overall offset, the unwrapped phase is regarded as relative phase. Spatial phase unwrapping typically gives only relative phase while temporal phase unwrapping tends to recover absolute phase.

Unlike conventional laser interferometry systems, DFP systems can project sufficiently low-frequency fringe patterns that can recover phase uniquely without phase unwrapping (i.e., \(\Phi(x, y) = \phi(x, y)\)), where \(\Phi(x, y)\) refers to the unwrapped phase); and the low-frequency phase can be directly used to determine fringe order for the high-frequency phase \(\phi(x, y)\) as

\[ k(x, y) = \text{Round} \left( \frac{\Phi'(x, y) \times (T^l/T^h) - \phi'/T^h}{2\pi} \right). \]

Here, \(\text{Round}(\cdot)\) is the round operator that gives the closest integer number, \(T^l\) is the low-frequency fringe period in pixels, and \(T^h\) is the high-frequency fringe period in pixels. It should be noted that since the low-frequency phase is scaled up by a factor of \((T^l/T^h)\) its noise could introduce error to fringe order determination: the larger the number is, the more incorrect points it could introduce. Therefore, this two-frequency phase unwrapping algorithm is not extensively employed due to its sensitivity to noise. The next section introduces the enhanced two-frequency phase unwrapping algorithm that can increase the robustness of the two-frequency approach.

### B. Enhanced Two-frequency Phase Unwrapping Algorithm

An et al. [12] developed the absolute fringe order determination method using the inherent geometric constraints of a standard DFP system (i.e., a single camera and a single projector). Briefly, such a phase unwrapping method works if the projection matrices \(P^c\) and \(P^p\) for the camera and the projector are known. These projection matrices represent the mathematical relationship between a point in a 3D world coordinate system \((x^w, y^w, z^w)\) and their projection point on the camera or projector sensor plane \((u, v)\) coordinates. Mathematically, given a virtual plane at \(z = z^w\), each camera pixel \((u', v')\) can be mapped to projector pixel coordinates \((u^p, v^p)\), defining a phase value \(\Phi_0\); and for a \(z = z_{\text{min}}\) plane, the mapped phase map, or \(\Phi_{\text{min}}\), is called minimum phase map. \(\Phi_{\text{min}}\) is a function of depth plane \(z_{\text{min}}\), fringe period \(T\), and the projection matrices \(P^c\) and \(P^p\),

\[ \Phi_{\text{min}}(u', v') = f(z_{\text{min}}, T, P^c, P^p). \]

An et al. [12] proved that for a given depth range, the fringe order \(k(x, y)\) can be determined by
The geometric constraint-based phase unwrapping method permits the use of more than one period of low-frequency fringe patterns for pixel-wise temporal phase unwrapping. Therefore, the enhanced two-frequency phase unwrapping method [11] essentially uses the geometric constraint phase unwrapping method to unwrap low-frequency phase, and the unwrapped low-frequency phase is then used to unwrap the high-frequency phase. Since the period of low-frequency fringe patterns is reduced, the noise impact is alleviated.

C. Proposed Temporal Phase Unwrapping Method

Figure 1 shows the overall computational framework of the proposed temporal phase unwrapping method. A large Gaussian filter is applied to the low-frequency binary fringe patterns that are further used to compute the wrapped phase \( \phi_l \). An LUT is then used to correct the phase error in the wrapped low-frequency phase. The minimum phase \( \Phi_{\min} \) is then applied to unwrap the low-frequency phase to generate the unwrapped phase map \( \Phi_l \). The unwrapped low-frequency phase then temporally unwraps the high-frequency phase pixel by pixel. This subsection will elucidate the details of the proposed computational framework.

As discussed in the previous subsection, the enhanced two-frequency phase-unwrapping algorithm improves the robustness of the two-frequency phase unwrapping approach. As discussed by An et al. [12], the geometric constraint-based temporal phase unwrapping algorithm has limited depth range to be approximately \( T^\alpha / \tan(\alpha) \). Here \( T^\alpha \) is the one period of fringe span on the object space, and \( \alpha \) is the angle between the projector and the camera. To alleviate this depth range limitation, one possible solution is to reduce the angle \( \alpha \) between the projector and the camera. However, it is not desirable since the measurement quality will be compromised. Therefore, due to the limited depth range of the geometric constraint-based phase unwrapping algorithm, the low-frequency fringe period still has to be a rather large number for practical applications. If the high-frequency binary patterns are defocused to be high-quality sinusoidal fringe patterns, the binary structure remains obvious for the low-frequency fringe patterns. Therefore, the corresponding phase obtained from low-frequency fringe patterns still has a large error.

Figure 2 shows an example of binary patterns with two different frequencies (\( T^h = 18 \) pixels and \( T^l = 540 \) pixels). Applying a \( 9 \times 9 \) Gaussian filter (standard deviation \( \sigma = 3 \) pixels) to the high-frequency binary pattern shown in Fig. 2(a) generates a good-quality sinusoidal fringe pattern shown in Fig. 2(c). However, if the same Gaussian filter is applied to the low-frequency binary pattern, as shown in Fig. 2(b), the binary structures remain sharp and clear.

A three-step phase-shifting algorithm is then applied to those blurred binary patterns to compute phase maps. Figure 3(a) shows the wrapped phase \( \Phi^h \) from the high-frequency blurred binary patterns. The phase error is then computed by subtracting unwrapped phase \( \Phi^l \) by the ideal phase \( \Phi^l' \), i.e., \( \Delta \Phi^h = \Phi^h - \Phi^l' \). Figure 3(c) shows the cross-section of the phase error. Clearly, the phase error is very small: root-mean-square (rms) error is approximately 0.005 rad. Similarly, Figs. 3(b) and 3(d), respectively, show the wrapped phase and the phase error for the low-frequency fringe patterns. Obviously, the phase error is very large (rms error: 0.278 rad), as expected.

Figure 4(a) shows the unwrapped high-frequency phase with a lot of incorrectly unwrapped points. This is easy to understand since it is not sufficient for a conventional algorithm to temporally unwrap the high-frequency phase pixel by pixel due to the large binary structural errors shown in Fig. 3(d).
Therefore, it is often assumed that it is not possible to realize two-frequency temporal phase unwrapping using the three-step phase-shifting algorithm and the square binary patterns.

This paper proposes to apply a large Gaussian filter to the low-frequency binary patterns before applying the phase-shifting algorithm. In particular, we suggest using the low-frequency binary patterns before applying the phase-shifting algorithm and the square binary patterns. In this example case, the Gaussian filter we used is $87 \times 87$ pixels with a standard deviation of 29. The phase error is then computed after applying such a large filter. Figure 5(a) shows the phase error. Apparently, the phase error is reasonably small now (approximately rms error of 0.061 rad). We then created an LUT, $\Delta \Phi^h$, as a function of wrapped phase $\Phi^h$. Figure 5(b) shows the created LUT with 256 elements. The low-frequency phase is compensated by the error LUT before being used for temporal phase unwrapping.

The rationale of applying such a large Gaussian filter is that the defocusing effort of the projector will not significantly change the phase error distribution for the low-frequency binary patterns. Figure 6(a) shows the phase rms error with different filter sizes; (b) cross-section of the phase error map for FS = 77 pixels, 87 pixels, and 97 pixels (rms error is 0.086, 0.061, and 0.039 rad, respectively).

Constant. The reason is that when the filter size is large enough, the blurred pattern is very close to ideal sinusoidal patterns. Figure 6(b) illustrates that when the filter size is changed by $\pm 10$ pixels, the maximum error difference is 0.03 rad; and such a small change will not affect the fringe order determination for temporal phase unwrapping. Figure 4(b) shows one cross-section of the unwrapped high-frequency phase from our method; clearly the entire phase is properly unwrapped.

To alleviate the problem, we developed a boundary correction algorithm that is applied to both row and column directions of the unwrapped high-frequency phase map. Taking one row of pixels as an example, we first removed the background data and extracted all the indices of remaind pixels as $\{i_1, \Phi_1, i_2, \Phi_2, \ldots, i_r, \Phi_r\}$, where $i_j$ is the index of the pixel position in a row and $\Phi_i$ denotes the phase value of pixel $i_j$. We defined the first and last $r$ pixels on this raw data as the indexed pixels to be corrected and assume that surface geometry on these pixels is "smooth" (i.e., without abrupt changes causing more than $\pi$ changes for the phase from one pixel to its neighborhood pixels).
Figure 7 illustrates how the boundary correction algorithm works on the first \( r \) pixels. The red solid circle \((i_j, \Phi_j)\) is the pixel to be corrected, and the blue circles are those inner neighbors. Because of the surface “smooth” assumption, the correct phase \( \Phi_j \) is supposed to be close to the phase of its neighbors. The corrected phase was determined by

\[
\Phi_j' = \Phi_j + 2\pi \times \text{Round} \left( \frac{\Phi_{m} - \Phi_j}{2\pi} \right),
\]

where \( \Phi_{m} \) is the median phase value of \( m \) inner neighbors \((i_{j+1}, i_{j+2}, ..., i_{j+m})\). We used the median function to avoid a large error from spike noise on an individual pixel. After correction, we updated corrected pixel phase \((i_j, \Phi_j')\) (red hollow circle shown in Fig. 7) and used the updated value for the following pixel correction. For the first \( r \) pixels on each row, we started the correction process from pixel \((i_r, \Phi_r)\) and repeated the same correction process for the rest of pixels \((i_j, \Phi_j)\) in the order of \( j = r - 1, r - 2, ..., 1 \). For the last \( r \) pixels we implemented the similar boundary correction algorithm. Note that now \( m \) inner neighbors for pixel \((i_j, \Phi_j)\) lie on the interval \([i_{j-m}, i_{j-1}]\), and the order to process data is from right (i.e., outside) to left (i.e., inner). After boundary correction on each row, we also applied the same boundary correction process for each column to further reduce the unwrapping artifacts.

It is important to note that the LUT was generated using the ideal binary patterns from simulation data. Such an LUT was used to compensate phase error for the phase obtained from camera-captured fringe patterns. The size of the Gaussian filter used to generate LUT could be different from that applied to experimental fringe patterns. For example, we suggest using a Gaussian filter with a size of 1/6 fringe period, and a standard deviation of 1/18 fringe period to generate LUT. To maximize the effectiveness of the LUT-compensation algorithm, the filter size should be adapted based on a given hardware system setup. The reason is that to generate the same level of blurring, the Gaussian filter size applied to the projected image should be different from that applied to the capture image if the projected image and capture image has a different pixel size. Practically, such a scaling factor can be determined through calibration. For example, one can capture a uniform flat board and determine the fringe period on the camera space. The Gaussian filter size should be scaled by \( T_l / T_p \), where \( T_l \) is the camera captured fringe period and \( T_p \) is the projected fringe period.

3. SIMULATION

We first performed some simulations to verify the performance of our proposed method. In these simulations, three phase-shifted square binary patterns with low frequency and three phase-shifted square binary patterns with high frequency are generated. The low-frequency binary pattern period is \( T_L = 520 \) pixels, and the high-frequency period is \( T_H = 18 \) pixels. The fringe period ratio, \( T_L / T_H = 540/18 = 30 \), is very large and often fails the conventional two-frequency phase unwrapping algorithm. In addition, Gaussian noise with zero mean and 0.01 standard deviation was added to the fringe pattern to evaluate the robustness of the proposed algorithm.

Figures 8 and 9 show simulation results. Figures 8(a) and 9(a), respectively, show one of the low-frequency patterns with added noise and one cross-section of the pattern; and Figs. 8(b) and 9(b), respectively, show one of the high-frequency patterns and its cross-section. It should be noted that all these fringe patterns were applied to a \( 9 \times 9 \) pixel Gaussian filter with a standard deviation of 3 pixels before adding Gaussian noise. Although the high-frequency pattern appears sinusoidal, the binary structures of the low-frequency pattern is very obvious, as expected.

We applied a large Gaussian filter \((90 \times 90 \) pixels with a standard deviation of 30 pixels\) to the low-frequency patterns. Figures 8(c) and 9(c), respectively, show the filtered pattern and its cross-section. Even after filtering, the structured pattern is still not sinusoidal, and thus the resultant phase, as shown in Figs. 8(e) and 9(e) still cannot be directly used for temporal phase unwrapping without problems. We then applied the LUT compensation algorithm to reduce the low-frequency phase error and the geometric constraint-based phase unwrapping algorithm to unwrap the low-frequency phase. The low-frequency phase was then used to unwrap the high-frequency phase shown in Fig. 8(d) and one cross-section of the phase shown in Fig. 9(d). Figures 8(f) and 9(f) show the results. Clearly, even with such a large fringe period ratio and such a large noise, the proposed algorithm can still work properly.

As a comparison, Fig. 10 shows the results if the large Gaussian filter is applied to the low-frequency patterns, and the LUT error compensation is not implemented. Obviously, phase unwrapping completely fails if the filter is not applied, and some phase unwrapping artifacts occur [e.g., those bumps shown in Fig. 10(d)] if the LUT is not used.
These simulation results confirmed that the proposed method can successfully unwrap phase with large noise and with a large fringe period ratio.

4. EXPERIMENT

To test the performance of our proposed method for practical applications, we developed a 3D shape measurement system, shown in Fig. 11, which includes a complementary metal-oxide-semiconductor (CMOS) camera (Model: Pointgrey GS3-U3-23S6M-C) and a DLP projector (Model: Texas Instruments LightCrafter 4500). The camera was attached with a 25 mm focal length lens (Model: Fujinon CF25HA-1) and the resolution was 1152 × 720 pixels. The projector’s resolution was 912 × 1140 pixels. The entire system was calibrated by the structured light system calibration method developed by Li et al. [18]. For all experiments, we used a fringe period of $T^h = 18$ pixels for three phase-shifted high-frequency binary patterns, and a fringe period of $T^l = 420$ pixels for three phase-shifted low-frequency binary patterns.

We first employed the proposed absolute phase recovery framework to measure a white board. Figure 12 shows the measurement results. Figure 12(a) shows one of the three phase-shifted high-frequency fringe patterns. Figure 12(b) shows one of the three phase-shifted low-frequency binary patterns. After applying a large Gaussian filter (71 × 71 pixels with a standard deviation of 23.67 pixels) to the low-frequency binary patterns, the filtered image is shown in Fig. 12(c). By applying the three-step phase-shifting algorithm to the high-frequency fringe patterns, the wrapped phase map can be obtained, as shown in Fig. 12(d). We also calculated the low-frequency wrapped phase based on filtered phase-shifted fringe images and applied an LUT-based error compensation approach discussed in Subsection 2.C to obtain the compensated phase map, which is shown in Fig. 12(e). We then employed the enhanced two-frequency phase unwrapping algorithm explained in Section 2.B to obtain the absolute unwrapped phase that was further used to reconstruct a 3D shape using the calibrated system parameters. Figure 12(f) shows the 3D results. As a comparison, Fig. 12(g) shows the result when the LUT error compensation was not applied.

To better visualize the difference between the results with and without adopting the LUT-based error compensation, the same cross-section of these two 3D shapes is plotted in Fig. 13(a). Note that we removed the gross slope of the plane depth curves for all plots in Fig. 13. For such a flat board measurement with small depth variance, the reconstructed 3D result without LUT error compensation is similar to the one with LUT error compensation. Although the flat board is successfully reconstructed overall, one can observe that there is some incorrectly reconstructed geometry near the boundaries, and this problem was the unwrapping artifacts discussed in Subsection 2.C.

We then employed the boundary error correction algorithm. In all our experiments, we chose $r = 81$ pixels from each image boundary as unreliable pixels, and $m = 5$ neighboring pixel to
determine median phase value that was used as reference for correction. Figure 12(h) shows the final 3D result after employing the aforementioned boundary correction algorithm. Clearly, the reconstructed result has “smooth” geometry for the entire 3D surface, demonstrating that proposed boundary correction can effectively correct the boundary problems. To further validate the performance of our algorithm, we also applied a conventional temporal phase unwrapping algorithm [8] to measure the same plane. Figure 12(i) shows the 3D measurement results using the temporal phase unwrapping algorithm. Figure 13(b) shows the cross-sections for both the 3D result reconstructed from our algorithm and that from the temporal phase unwrapping algorithm. Clearly, they perfectly overlap with each other, demonstrating the success of our proposed method for measuring an absolute 3D smooth surface like a plane.

We also measured an owl statue with more complex surface geometry to further verify the performance of our proposed algorithm. Figure 14(a) shows one of the high-frequency phase-shifted fringe images, Fig. 14(b) shows one of the low-frequency fringe images, and Fig. 14(c) shows the resultant low-frequency image after applying a large Gaussian filter. Figure 14(d) is the final high-frequency unwrapped phase obtained from the proposed algorithm. The reconstructed 3D geometry without LUT compensation is illustrated in Fig. 14(e), having obvious surface discontinuities caused by incorrect phase unwrapping. Figure 14(f) shows the result after employing the LUT error compensation step. It clearly shows that the 3D central area is significantly improved. However, some areas near the boundary regions are not correctly reconstructed. We then employed the aforementioned boundary correction algorithm, and Fig. 14(g) shows the result. This figure shows that most boundary problems are properly addressed.

Similarly, we employed the temporal phase unwrapping algorithm to measure the same statue, and the result is shown in Fig. 14(h).

Figure 15 shows the close-up views for these 3D reconstructions for better visual comparisons. Figures 15(a) and 15(b), respectively, show the zoom-in view of the 3D result before and after LUT compensation. Again, the LUT-based error compensation algorithm can improve measurement quality by reducing unwrapping artifacts. Figure 15(c) shows the zoom-in view of final measurement result whose boundary artifacts are significantly alleviated compared to the result before employing the boundary correction algorithm. Figure 15(d) shows the same region of the zoom-in view for the result obtained from the conventional temporal phase unwrapping algorithm. Compared to the ground truth shown in Fig. 15(d), the region near the right ear of our final result (highlighted within the red-dashed window) shown in Fig. 15(c) has an overall shift in depth. In this area, the neighbors we used for boundary correction have large depth difference from the pixel...
to its neighboring pixels that does not satisfy our assumption that the local geometry near the corrected boundary must be “smooth.”

Figure 16 shows the same cross-sections for all those 3D reconstructions shown in Fig. 15 to further visualize the differences. Figure 16(b) shows that these two curves overlap well except for about 10 pixels near the most left end where the boundary correction algorithm failed. This, once again, was caused by the violation of surface smoothness assumption. This complex statue experiment demonstrated that, for a complex object measurement, the LUT-based error compensation can effectively reduce unwrapping artifacts within the central area, and the boundary correction algorithm can further reduce the unwrapping artifacts near the boundary region. However, there might still remain unwrapping artifacts near the boundary if the surface smoothness assumption is not satisfied near the boundary regions; and this is the limitation of our proposed method.

Finally, we tested our algorithm on multiple isolated objects: two separated balls were measured at the same time. One of the captured high-frequency fringe patterns is shown in Fig. 17(a). For multiple-object measurement, we first removed the background and then found the largest two connected components to separate these two balls. The separated results based on the texture images are shown in Figs. 17(b) and 17(c). We applied our algorithm to calculate the absolute phase for the separated image of each object whose unwrapped phase is shown in Figs. 17(d) and 17(e), respectively. The unwrapped phase maps were combined into one final absolute phase map, as shown in Fig. 17(f). Figure 17(g) shows the reconstructed geometry using our proposed method. As a comparison, Fig. 17(h) shows the 3D measurement result using the conventional temporal phase unwrapping algorithm.

Once again, we plotted the cross-sections of the measurement result using our proposed method and that using the conventional temporal phase-unwrapping method. Figure 18 shows two overlapped cross-sections. They are perfectly overlapped, as expected, since the sphere surfaces are completely smooth. In summary, all these experimental results clearly demonstrated that our proposed method can satisfactorily recover absolute 3D geometry if the surface geometry is “smooth” near the boundary regardless whether it is a simple object or multiple objects. However, the proposed method could still result in some artifacts near boundary if surface geometry is not “smooth” on the boundary.

Fig. 15. Close-up views of the 3D reconstructions of the owl statue. (a) Zoom-in view of Fig. 14(e); (b) zoom-in view of Fig. 14(f); (c) zoom-in view of Fig. 14(g); (d) zoom-in view of Fig. 14(h). (c) and (d) highlight one region that our proposed method fails to correctly reconstruct the 3D absolute geometry.

Fig. 16. Cross-section of 3D results shown in Fig. 14. (a) One cross-section of Figs. 15(a) and 15(b); (b) one cross-section of Figs. 15(c) and 15(d).

Fig. 17. Experimental results on simultaneously measuring two separated balls. (a) One of three high-frequency phase-shifted fringe patterns; (b) separation of the first ball; (c) separation of the second ball; (d) high-frequency absolute unwrapped phase of the first ball; (e) high-frequency absolute unwrapped phase of the second ball; (f) final high-frequency absolute unwrapped phase; (g) final 3D result of proposed method; (h) 3D result of conventional method.

Fig. 18. Cross-sections of 3D measurement results of multiple objects shown in Figs. 17(g) and 17(h).
5. SUMMARY
This paper has presented a novel method for absolute 3D shape measurement that uses only six square binary patterns for an enhanced two-frequency phase unwrapping algorithm. The projector is only required to be defocused so that high-frequency patterns can generate a high-quality phase. A Gaussian filter is applied to low-frequency fringe images for artificially reducing phase error caused by the binary structures of the low-frequency patterns. Then an LUT-based error compensation was developed to reduce harmonic error impact, and a boundary correction algorithm was also proposed to further alleviate boundary unwrapping artifacts introduced by Gaussian filtering. Experimental results demonstrated the success of our proposed method to measure the absolute shape of complex geometry objects, as well as the multiple isolated objects, despite that there might still be unwrapping artifacts if the surface geometry near boundary is not “smooth.”

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