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Composite phase-shifting algorithm for absolute phase measurement

Song Zhang *

Mechanical Engineering Department, Iowa State University, Ames, IA 50011, United States

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ABSTRACT

This paper presents a method to recover absolute phase by using only four images: three phase-shifted patterns and one stair pattern. The stair pattern is designed in such a way that the stair changes are perfectly aligned with the phase jumps, and thus absolute phase can be recovered by referring to the stair pattern. Due to system noises and camera and/or projector blurring, a computational framework is also proposed. Because this technique only requires four fringe images for absolute phase recovery, it has the merit of measurement speed. And since the absolute phase is obtained, this technique is suitable for measuring step-height objects. We have developed a digital fringe projection system to verify the performance of the proposed technique.

1. Introduction

3D shape measurement is very important to numerous disciplines, and a number of techniques have been developed including stereo vision, structured light, and digital fringe projection and phase shifting methods [1]. Among which, real-time 3D shape measurement is increasingly needed because of the advancement of computing technology [2].

For a high-speed 3D shape measurement system, reducing the required number of images to reconstruct one 3D frame is vital. This is because, in general, the more images used, the slower the measuring speed, albeit the better measuring quality could be achieved. For a digital fringe projection technique, a three-step phase-shifting algorithm is usually adopt for real-time applications because it uses the minimum number of fringe images to uniquely solve for the phase pixel by pixel. However, the three-step phase-shifting algorithm requires a spatial phase unwrapping algorithm to obtain continuous phase, which cannot be used to measure discontinuous surface or the surfaces with step height more than π [3]. There are quite a number of techniques developed to measure arbitrary 3D surfaces with discontinuities, such as two-wavelength phase-shifting [3,4], multiple-wavelength phase-shifting [5], optimal frequency selection [6], temporal phase unwrapping [7], and gray-coding plus phase-shifting [8] techniques. However, they typically requires more than four images in order to obtain absolute phase. This is not desirable for high-speed applications.

If only one additional image is required to unwrap the phase temporally for a three-step phase-shifting algorithm, the 3D shape measurement speed is affected only to its minimum. A digital fringe projection system makes it feasible because it can generate arbitrary profile fringe patterns. This paper will present a composite algorithm to solve this problem by using only four fringe images. Besides three phase-shifted fringe images, an additional stair image is used. The stair image has the stair changes precisely aligned with the phase discontinuities, and has a unique grayscale value for each stage. Therefore, from the stair image, the unique fringe order number k, used for phase unwrapping, can be determined, and the phase can be unwrapped point by point by referring to the stair image. Due to system noises and camera and/or projector blurring, a computational framework is also proposed to robustly determine fringe order k. We will present some experimental results to verify the performance of the proposed technique.

The paper is organized as follows: Section 2 explains the principle of the proposed technique; Section 3 presents the proposed framework to determine fringe order number; Section 4 shows experimental results; Section 5 discusses the advantages and limitations of the proposed technique; and Section 6 summarizes this paper.

2. Principle

Phase-shifting methods are widely used in optical metrology because of their speed and accuracy [9]. For the real-time 3D shape measurement system we developed, a three-step phase-shifting algorithm with a phase shift of $2\pi/3$ is used [10]. The intensity of three fringe images can be written as:

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos(\phi - 2\pi/3), \quad (1)$$

* Tel.: +1 515 294 0723; fax: +1 515 294 3261.
E-mail address: song@iastate.edu

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$$I_2(x,y) = I'(x,y) + I''(x,y)\cos(\phi), \tag{2}$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos(\phi + 2\pi/3). \tag{3}$$

where $I'(x,y)$ is the average intensity, $I''(x,y)$ is the intensity modulation, and $\phi(x,y)$ is the phase to be solved for. Simultaneously solving Eqs. (1)–(3), we can obtain

$$\phi(x,y) = \tan^{-1}[\sqrt{3}(I_1 - I_3)/(2I_2 - I_1 - I_3)], \tag{4}$$

$$I'(x,y) = (I_1 + I_2 + I_3)/3, \tag{5}$$

$$I''(x,y) = \sqrt{3(I_1 - I_3)^2 + (2I_2 - I_1 - I_3)^2}/3, \tag{6}$$

$$\gamma(x,y) = \frac{I''(x,y)}{I'(x,y)}. \tag{7}$$

where $\gamma(x,y)$ is the data modulation that indicates the quality of fringe for each point, with 1.0 being the best. This information is usually used to determine the background of the image. Eq. (4) gives phase value with the range of $[-\pi, +\pi)$ with a modus of 2π. A traditional approach to remove the 2π discontinuity is to apply a spatial phase unwrapping algorithm [11]. Essentially, by analyzing the phase values of neighborhood pixels and assuming the surface smoothness, a phase unwrapping step can remove 2π jumps by adding or subtracting multiple times of 2π. That is, the relationship between the wrapped phase and the unwrapped one can be written as:

$$\Phi(x,y) = \phi(x,y) + k(x,y) \times 2\pi. \tag{8}$$

Here, $\Phi(x,y)$ denotes the unwrapped phase, and $k(x,y)$ is an integer number.

If a stair image is used, that is perfectly aligned with the 2π phase discontinuities, the $k(x,y)$ can be determined from the stair images. Fig. 1 illustrates the proposed algorithm. Assume that the fringe stripes are vertical, a stair image can be generated as:

$$I_s^p(x,y) = floor[(x + P/2)/P] \times S. \tag{9}$$

Here P is the fringe pitch (number of pixels per fringe period). *floor*() removes the decimals of a floating point data while keeps its integer part. S is the intensity level for each stair.

Because the object surface reflectivity might not be uniform, normalizing the structured images is necessary to accurately determine integer $k(x,y)$. The normalization procedure is actually straightforward because from Eqs. (1)–(3), the maximum and minimum intensity for each pixel can be obtained

$$I_{min}(x,y) = I'(x,y) - I''(x,y), \tag{10}$$

$$I_{max}(x,y) = I'(x,y) + I''(x,y). \tag{11}$$

Assume that the captured stair image is $I_s(x,y)$, this image can be normalized by the following equation:

$$I_s^n(x,y) = \frac{I_s(x,y) - I_{min}(x,y)}{I_{max}(x,y) - I_{min}(x,y)}. \tag{12}$$

Then the integer number $k(x,y)$ can be determined from the normalized stair image as

$$k(x,y) = I_s^n(x,y) \times \frac{R}{S}. \tag{13}$$

Here R is the fringe intensity range generated by the computer (e.g., 250) with a maximum value of 255 for a digital fringe projection system. Once $k(x,y)$ is determined, the phase can be unwrapped point by point using Eq. (8).

One may notice that we have used a similar algorithm for 3D shape compression [12]. In that case, the algorithm was only realized through simulation under perfect conditions: there is no ambient light and the surface reflectivity is uniform. However, we found that the practical problem is very complicated, and it is required to develop a novel computational framework to apply such an algorithm to handle real world case. The computational framework will be discussed next.

3. Framework for absolute phase recovery

Theoretically, the integer $k(x,y)$ can be obtained by applying Eq. (13). However, it is practically difficult due to many issues including noises, surface reflectivity variations, lens defocusing, and camera re-sampling. In this research, we propose the following framework to recover absolute phase.

Step 1: *Normalize stair image*: This step is to apply Eq. (12).

Step 2: *Segment wrapped phase into regions through image processing techniques*: This step needs to identify regions so that there is no 2π changes within each region, the unwrapping integer $k(x,y)$ is the same. To increase the robustness of the algorithm, we use $\pi/4$ instead of π as the metric to find the jumps.

The phase edge was detected by a conventional edge-detection algorithm such as Canny filtering. These edge points can be regarded as walls to separate different regions. However, because of noises, we found that the wall were not continuous, i.e., the edge line segments are broken. To connect those lines, we then applied a Wiener filter to the edge image to connect those broken edge line segments. After the edges are correctly located, the regions can be segmented by finding the connected components separated by the edges.

Step 3: *Determine k(x,y) for each region for phase unwrapping*: Since within each region $k(x,y)$ is a constant, we simply averaged all the intensities of each region on the stair image to reduce the noise effect. We then apply Eq. (13) to determine $k(x,y)$ to unwrap the phase.

Step 4: *Unwrap edge points*: After previous step, the phase should be correctly unwrapped within each region. However, the points on edges have not been processed yet. To process edge points, we approximate the phase value by interpolating the neighborhood unwrapped phase points $\Phi^0(x,y)$. The true absolute

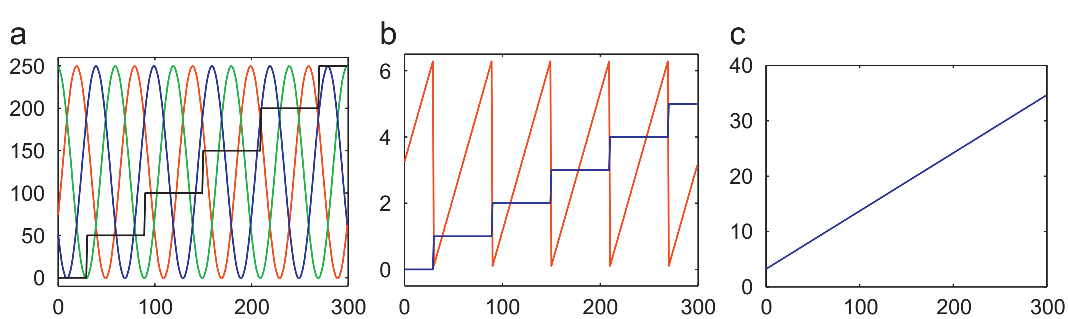

Fig. 1. Schematic diagram of the proposed composite phase-shifting algorithm. (a) Four fringe images used: three phase-shifted sinusoidal fringe images, and one stair image; (b) the 2π discontinuities of the phase is precisely aligned with the normalized stair image intensity changes; (c) the unwrapped phase.

phase value can be determined by

$$\Phi(x,y) = \text{Round}\left[\frac{\Phi^0(x,y)-\phi(x,y)}{2\pi}\right] \times 2\pi + \phi(x,y). \quad (14)$$

4. Experiments

We developed a 3D shape measurement system to verify the performance of the proposed technique. The hardware system includes a CCD camera (Jai Pulnix TM6740-CL), and a digital-light-processing (DLP) projector (Samsung SP-P310MEMX). The camera is attached with a 16 mm focal length lens (Computar M1614-MP) at F/1.4 to 16C. The camera has a resolution of 640×480 with a maximum frame rate of 200 frames per second. In this research, the camera exposure time was set to be 1/30 s, and captures at 30 frames per second. The projector has a resolution of 800×600 with a projection distance of 0.49–2.80 m.

We firstly measured a complex 3D statue. Figs. 2–4 show the measurement results. In this experiment, four images are captured for 3D shape recovery: three phase-shifted fringe patterns (Fig. 2(a)–(c)), and one stair pattern (Fig. 2(d)). From three phase-shifted fringe images, the wrapped phase can be obtained, as shown in Fig. 2(e). To segment the phase regions, the Canny edge detection technique was adopted to find the edges on the wrapped phase, and Fig. 2(f) shows the result. It is important to note that a threshold of $\pi/4$ instead of π was used to find phase jumps caused by abrupt surface changes. As can be seen from the resultant picture, different phase regions are still connected through background areas since there is abrupt phase changes on the background. To separate those regions, we calculated the mask image, as shown in Fig. 2(g), from the data modulation using Eq. (7). Another Canny edge detection was applied to determine the edges on the mask image. Combining the edges from this image and that from the phase images will result in the edge image shown in Fig. 2(h). This figure shows that phase regions were separated pretty well. However, we found that the edges were not always continuous due to noises. Fortunately, those gaps are usually one or two pixel wide. We then applied a Wiener filter to connect those broken edge segments. Fig. 2(i) shows the resultant image after applying the Wiener filter to the edge image shown in Fig. 2(h). The last step of region identification is to label each region with a different number, as shown in Fig. 2(j).

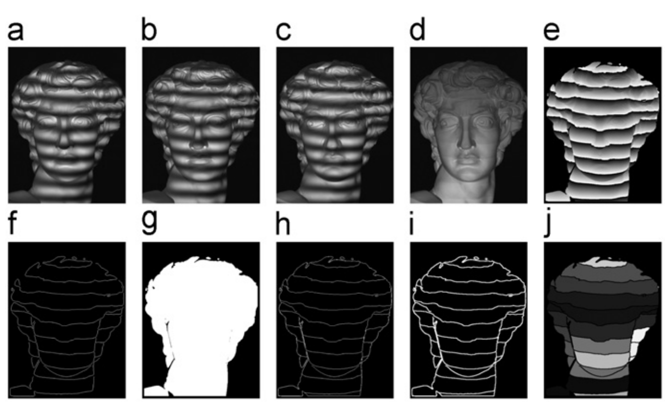

Fig. 2. Measurement example of a 3D statue. (a)–(c) Three phase-shifted fringe images; (d) stair image; (e) wrapped phase; (f) canny edge detection on wrapped phase image; (g) mask; (h) combined edge from the mask image with the edge from the wrapped phase; (i) connect small broken edge line segments after applying the Wiener filter; (j) labeled region.

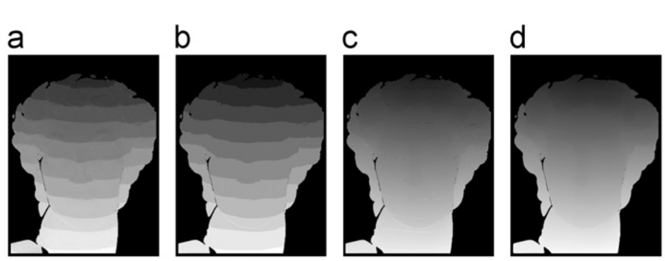

Fig. 3. Measurement example of a 3D statue (continued from Fig. 2). (a) Normalized stair image; (b) stair image averaged region by region; (c) unwrapped phase using the fringe order determined from (b); (d) unwrapped phase after correcting edge phase information.

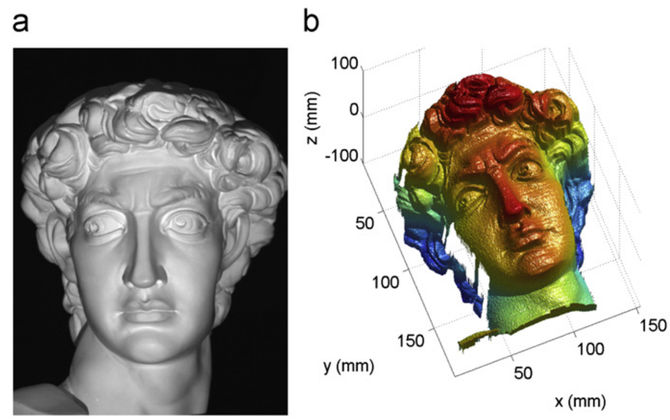

Fig. 4. Measurement example of a 3D statue (continued from Fig. 3). (a) Photograph of the statue; (b) recovered 3D shape.

Once all phase regions are labeled, the next step is to find the fringe order from the stair image. Obviously, the originally captured stair image (shown in Fig. 2(d)) cannot be used because of the surface reflectivity variations from one point to the other. To accommodate this issue, the stair image has to be normalized. Fig. 3(a) shows the normalized stair image. This image shows pretty good uniformity within each stair region but still contains significantly noise. We found that it is very difficult to determine the fringe order directly from this normalized image because of noises. Since we have labeled phase regions with each region being the same fringe order, we simply averaging the stair image to reduce the noise effect. Fig. 3(b) shows the clean result after averaging, and thus the fringe order can be better determined. Fig. 3(c) shows the unwrapped phase with the fringe order determined from Fig. 3(b). The result was quite good except those edge points shown in Fig. 2(h) where the noises were not treated. Fortunately, those problematic areas are usually one to two pixels wide, the associated problems can be eliminated by applying a median filter. As explained in Section 3, the median filtering will result in some artifacts on the final measurement results, which can be further corrected by applying Eq. (14). Fig. 3(d) shows the final unwrapped phase. It should be noted that due to the noise of the system, using Eqs. (10) and (11) to normalize the stair image is not accurate. In this research, we projected and captured additional uniform images for this purpose.

From the unwrapped phase the 3D shape can be recovered. In this research, we utilized a very simple reference-plane-based phase-to-height conversion algorithm introduced in Reference [13]. Fig. 4(a) shows a photograph of the object, and Fig. 4(b) shows the measurement results plotted in 3D. This experiment clearly shows that the 3D surface profile is well captured, and the reconstructed 3D shape is of high quality.

To further demonstrate the capability of the proposed absolute phase measurement technique, a step-height object with a known height of 50 mm was also measured. Fig. 5 shows the measurement

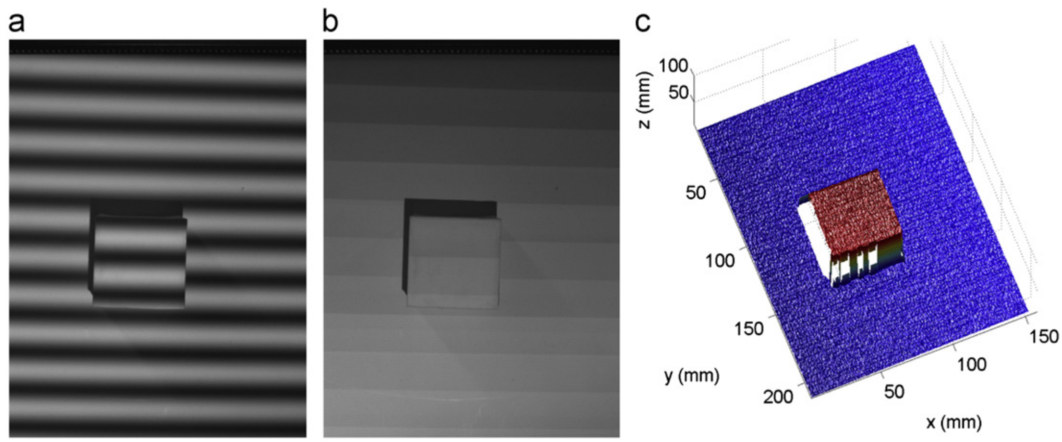

Fig. 5. Measurement example of a step-height object. (a) One of the three phase-shifted fringe patterns; (b) stair image; (c) recovered 3D shape.

results. It clearly shows that the proposed technique can be used to measure abrupt surface changes, such as the step-height object.

5. Discussions

The proposed technique has the following advantages:

- *High measurement speed*. Comparing with other absolute phase recovery technique with a phase-shifting method which usually requires five or more fringe patterns, the proposed technique only requires four fringe patterns. Because fewer number of fringe patterns are needed, the achievable measurement speed is potentially higher, assuming that the computer power is sufficient to keep up with the capturing speed.
- *High measurement quality*. Comparing with the Fourier method [14] which might require less fringe patterns for absolute phase recovery, the proposed technique utilizes a phase-shifting technique which is inherently more robust, can handle more complex surfaces, and can achieve better quality measurement.
- *Robust*. The proposed computational framework treats the phase regions as a whole to obtain fringe order, which is significantly less sensitive to random noises.

Yet, this proposed technique is not trouble free, and it has the following limitations:

- *Region identification*. The proposed technique heavily relies on the proper region identification. We used a $\pi/4$ threshold to identify the phase discontinuities for edge detection. It works pretty well for most cases, however, if the object surface changes result in a phase difference between neighboring pixels within this range, the proposed technique will not be able to correctly identify the location of jumps. However, unlike a conventional spatial phase-unwrapping algorithm where the incorrectly unwrapped phase may affect the other areas, the proposed technique will not propagate to any other regions.
- *Stair image normalization*. We found that using Eq. (12) is not very robust mainly because the quantization error, the nonlinearity and the noise of the system. Since only 8 bits or 256 grayscale values are available for the projector, the resolution error is fairly significant. Secondly, in a real system, we usually only use about 200 grayscale values due to the nonlinearity issue of the projector. Combining with the projector noise, the calculated maximum and minimum images are not sufficiently accurate for dark areas. In this research, we thus projected those images for normalization. With the hardware technology advancements, we believe that those additional images are not necessary to adopt the proposed technique.
- *Maximum achievable absolute phase*. Because a stair image is used to resolve absolute phase, and usually only 255 grayscale values available for a digital video projector, the maximum stairs is usually less than 25 since the stair height is larger than 10. In order words, it can only resolve 10 periods of fringe patterns or maximum of 20π absolute phase range.

6. Summary

This paper has presented a technique to recover absolute phase by utilizing four fringe images. Experimental results have been presented to demonstrate the viability of the techniques. Because this proposed technique only uses four fringe images to recover one 3D frame, it is desirable for high-speed applications.

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