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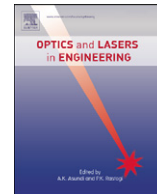
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## Optics and Lasers in Engineering

journal homepage: [www.elsevier.com/locate/optlaseng](http://www.elsevier.com/locate/optlaseng)

## 3D shape measurement with 2D area modulated binary patterns

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## ARTICLE INFO

## Article history:

Received 17 January 2012

Received in revised form

3 March 2012

Accepted 5 March 2012

Available online 23 March 2012

## Keywords:

3D shape measurement

Binary defocusing

Area modulation

Fringe analysis

## ABSTRACT

This paper presents a novel area-modulation technique for three-dimensional (3D) shape measurement with binary defocusing. Specifically, this technique modulates local  $2 \times 2$  pixels to create five grayscale values to enhance fringe quality when the projector is not perfectly in focus. With this novel technique, we will show that the phase error is approximately  $1/3$  of the square binary method when fringe pattern is dense and the projector is nearly focused.

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## 1. Introduction

High-speed, high-resolution three-dimensional (3D) shape measurement based on fringe analysis has been increasingly used in the past decades [1]. However, it is usually difficult to achieve both high speed and high measurement accuracy.

Our recent study has demonstrated numerous merits of utilizing the binary defocusing technique for high-speed 3D shape measurement. Utilizing the digital light processing (DLP) Discovery projector and the binary defocusing technique, we have successfully achieved unprecedentedly high-speed 3D shape measurement with a phase-shifting algorithm [2]. However, we found that it is difficult for such a technique to achieve high-quality 3D shape measurement for a long depth range especially when the projector is nearly focused. Xu et al. has proposed a technique to significantly reduce the phase error when the projector is not properly defocused [3]. However, the residual error is not negligible for high accuracy 3D shape measurement.

Ayubi et al. has proposed a sinusoidal pulse width modulation (SPWM) technique to increase measurement depth range [4]. The SPWM technique improves the sinusoidality of the less-defocused fringe pattern by shifting the high-order harmonics to higher frequencies so that they can be easier to be suppressed by defocusing. We have proposed an optimum pulse width modulation (OPWM) technique to further improve the SPWM method [5]. The OPWM technique selectively eliminates unwanted frequencies component by solving for a nonlinear optimization problem. Both techniques showed advantages over the square binary

method (SBM) when the fringe stripe is wide. However, due to the discrete fringe generation nature, when the fringe is dense, neither of them performs better than the SBM in a digital fringe projection system [6]. This is because there are not sufficient pixels to manipulate along one direction ( $x$  or  $y$ , but not both).

This paper presents a technique that modulates the binary patterns in both  $x$  and  $y$  directions to improve fringe quality even for dense fringes. Because it has another dimension to control, it could outperform the SBM. This technique comes from our two observations: (1) it is easier to generate sinusoidal patterns by defocusing a triangular wave than a square wave and (2)  $2 \times 2$  pixel cells of the projector can be easily blended together if the projector is not perfectly in focus. Therefore, we can manipulate the  $2 \times 2$  pixels to generate more grayscale values than two (0 s and 1 s), which makes it viable to emulate a triangular wave with a small fringe period. With this novel technique, we will show that the error is approximately  $1/3$  that of the SBM when fringe is dense and the projector is nearly focused. Both simulations and experiments have shown that the phase error induced by this technique is approximately  $1/3$  of the squared binary when the fringe stripe is 16 pixels width.

Section 2 explains the principle of each technique. Sections 3 and 4 respectively show simulation and experimental results, and finally Section 5 summarizes this paper.

## 2. Principle

## 2.1. Four-step phase-shifting algorithm

Phase-shifting algorithms are widely used in optical metrology because of their measurement speed and accuracy [7]. Numerous

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phase-shifting algorithms have been developed including three step, four step, and five step. In this paper, we use a four-step phase-shifting algorithm with a phase shift of  $\pi/2$ . Four fringe images can be described as

$$I_k(x,y) = I'(x,y) + I''(x,y) \cos[\phi + (k-1)\pi/2], \quad (1)$$

where  $k=1, 2, 3, 4$ ,  $I'(x,y)$  is the average intensity,  $I''(x,y)$  the intensity modulation, and  $\phi(x,y)$  the phase to be solved for

$$\phi(x,y) = \tan^{-1}[(I_4 - I_2)/(I_1 - I_3)]. \quad (2)$$

This equation provides the phase ranging  $[-\pi, +\pi]$  with  $2\pi$  discontinuities. A continuous phase map can be obtained by adopting a spatial or temporal phase unwrapping algorithm. In this research, we used the temporal phase unwrapping framework introduced in [8].

## 2.2. Sinusoidal fringe pattern generation by defocusing

Sinusoidal fringe patterns can be generated by defocusing square binary patterns. Mathematically, the square binary wave can be described as

$$f(x) = \begin{cases} 1, & x \in [nT, nT + T/2), \\ 0, & x \in [nT + T/2, nT + T). \end{cases} \quad (3)$$

Here  $n$  is an integer and  $T$  is the period of the square wave. Such a square wave can also be mathematically represented as

Fourier series:

$$f(x) = 0.5 + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{2k+1} \sin\left[\frac{2\pi(2k+1)}{T}x\right]. \quad (4)$$

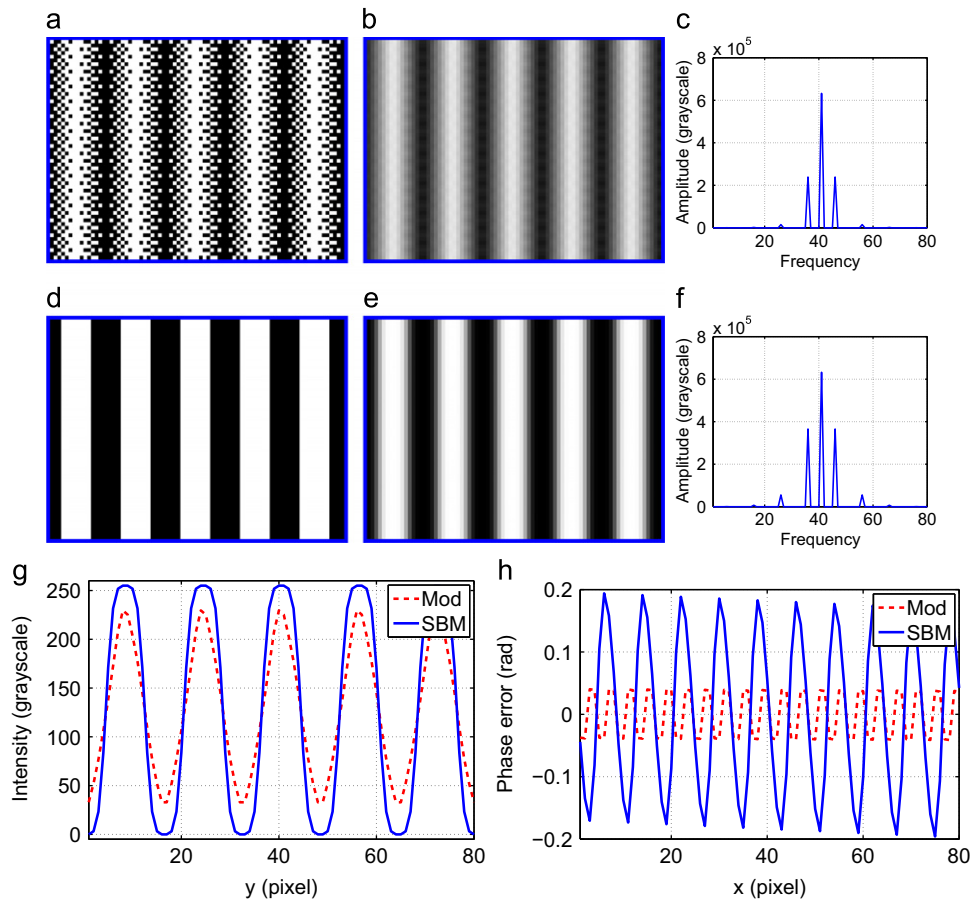
Sinusoidal fringe patterns can also be generated by defocusing triangular patterns. The mathematical description of a triangular wave is

$$f(x) = \begin{cases} 2x/T - n, & x \in [nT, nT + T/4), \\ -2x/T + n + 1.5, & x \in [nT + T/4, nT + 3T/4), \\ 2x/T - n - 1.5, & x \in [nT + 3T/4, nT + T). \end{cases} \quad (5)$$

A triangular wave with the period of  $T$  and an amplitude of 0.5 can be described as Fourier series:

$$f(x) = 0.5 + \frac{4}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)^2} \sin\left[\frac{2\pi(2k+1)}{T}x\right]. \quad (6)$$

The defocusing technique essentially suppresses all high-order harmonics but preserves the fundamental frequency. Eqs. (4) and (6) indicate that the magnitudes of high-order harmonics of a triangular wave drop much more rapidly than those of the square wave. For example, the influence of the 3rd harmonics for a triangular wave is approximately equivalent to that of the 9th harmonics for the square wave. Therefore, it is easier for a triangular pattern to become a sinusoidal pattern by defocusing than a square binary pattern. Yet, theoretically, comparing with the squared wave, the triangular wave has lower contrast should only the fundamental frequency be considered. However, if the ideal sinusoidal fringe pattern is generated by defocusing, the fringe contrast of the triangular wave could be much higher



**Fig. 1.** Simulation results. (a) modulated pattern; (b) filtered pattern of (a); (c) frequency spectrum of (b); (d) square binary pattern; (e) filtered pattern of (d); (f) frequency spectrum of (e); (g) horizontal cross sections of the simulated patterns; (h) phase errors: root-mean-square (rms) 0.130 rad for SBM, and rms 0.039 rad for the proposed method. It is important to notice that the fringe contrast is also reduced for the area-modulated pattern.

because the greater amount of defocusing is required to generate sinusoidal pattern for the square wave. The defocusing not only suppresses the high-order harmonics, but also reduces the fringe contrast accordingly.

### 2.3. Area modulation technique

Unlike the SPWM or OPWM technique where the fringe patterns are modulated in one dimension ( $x$  or  $y$  but not both), better sinusoidal fringe patterns could be generated by modulating the pattern in both  $x$  and  $y$  dimensions. For instance, Xian and Su have demonstrated that by manufacturing grating with high accuracy, high-quality sinusoidal fringe patterns can be generated by slight defocusing [9]. However, this technique requires precisely controlling numerous tiny cells ( $\mu\text{m}$ ), which is not practical for the digital fringe projection technique where the pixel size is usually large, and the number of pixels are small.

In this research, we propose to modulate local  $2 \times 2$  pixels to enhance fringe quality. On average, if only one pixel is 1 whilst the rest are 0s, the equivalent grayscale is  $1/4$ . Similarly, if only 2 pixels are 1s, the equivalent grayscale is  $0.5$ . Comparing with the whole patterns, the  $2 \times 2$  pixels could be easily blended together if the projector is not perfectly in focus, and thus by modulating these four pixels, we can generate five equivalent grayscale values instead of two. If the binary patterns are generated by a sequence of  $0, 1/4, 2/4, 3/4, 1, 3/4, 2/4, 1/4, 0, \dots$ . A triangular wave with five intensity levels can be generated. By this means, it is easy to prove that 16 pixels are sufficient to generate one triangular period with five grayscale values, thus this technique could improve measurement quality when the fringe is fairly dense (period of 16 pixels) and the projector is nearly focused.

In general, for a digital fringe projection technique, it is possible to avoid phase-shift error if a proper phase-shifting algorithm is adopted due to the discrete fringe generation nature. For a conventional sinusoidal fringe generation technique, an arbitrary fringe pitch (number of pixels per fringe period) could be utilized, which allows the use of any phase-shifting algorithms. However, the area-modulated binary pattern provides a restriction on the choice of phase-shifting algorithms. For example, the  $2 \times 2$  area modulated pattern could result in a fringe pitch of 16, which will induce phase-shift errors if a three-step phase-shifting algorithm is used, and thus we adopt a four-step phase-shifting algorithm.

### 3. Simulations

Simulations were carried out to verify the performance of the proposed technique. Fig. 1(a) shows the modulated binary patterns with a period of 16 pixels; and the corresponding square binary pattern is shown in Fig. 1(d). Interestingly, even before

defocusing, the modulated pattern appears better sinusoidal than the square binary one. These fringe patterns were then smoothed by a small Gaussian filter:  $7 \times 7$  pixels in size with a standard deviation of 1.17 pixels. The smoothed patterns are shown in Fig. 1(b) and (e) respectively. The cross sections of the filtered patterns are plotted in Fig. 1(g) which showed that the modulated pattern indeed appears to be triangular. To check the fringe quality, the frequency spectra are plotted in Fig. 1(c) and (f). It can be seen that the high-order harmonics amplitudes of the modulated pattern are much smaller than those of the square binary pattern. This indicates indeed that the proposed pattern is better sinusoidal. One might notice that the fringe pattern shown in Fig. 1(b) is not uniform vertically. These were caused by the fact that the area-modulated patterns are not vertically uniform (unlike squared binary pattern), and the amount of defocusing is small. However, this local features does not introduce significant phase errors, as demonstrated in this simulation as well as the experimental results.

If the fringe patterns are shifted  $1/4$  period (or 4 pixels for fringe period of 16 pixels) a time, the aforementioned four-step phase-shifting algorithm can be employed to calculate the phase. The phase errors can be determined by taking the differences from the phase obtained from ideal sinusoidal patterns. The cross sections of the phase error maps are shown in Fig. 1(h). This figure shows that the phase error introduced by the proposed method is much smaller than that by the SBM: approximately  $1/3$  times. These simulations demonstrated that the proposed technique outperforms the SBM when the fringe is dense, and thus the measurement quality could be significantly improved.

### 4. Experiments

Experiments were also carried out to verify the performance of the proposed technique. The hardware system includes a CCD camera (Pulnix TM6740-CL) and a digital-light-processing (DLP)

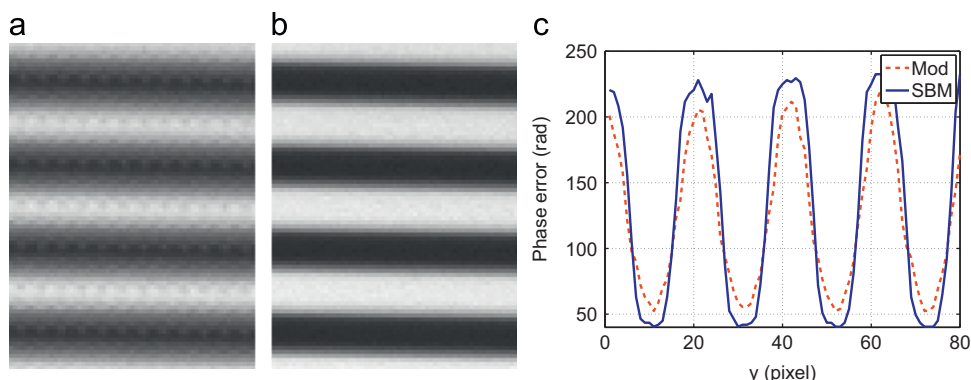


Fig. 2. Measurement result of a flat board. (a) One of the modulated fringe patterns; (b) one of the SBM patterns; (c) vertical cross sections of these patterns.

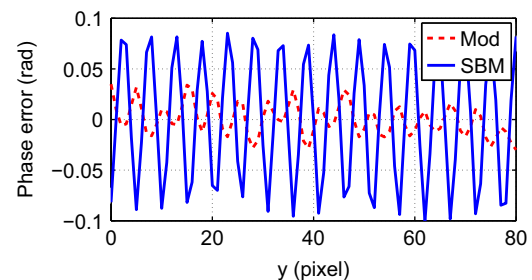


Fig. 3. Phase error comparison of the flat board measurement. The phase error for the SBM is rms 0.063 rad, and that for the area modulated patterns is rms 0.016 rad.

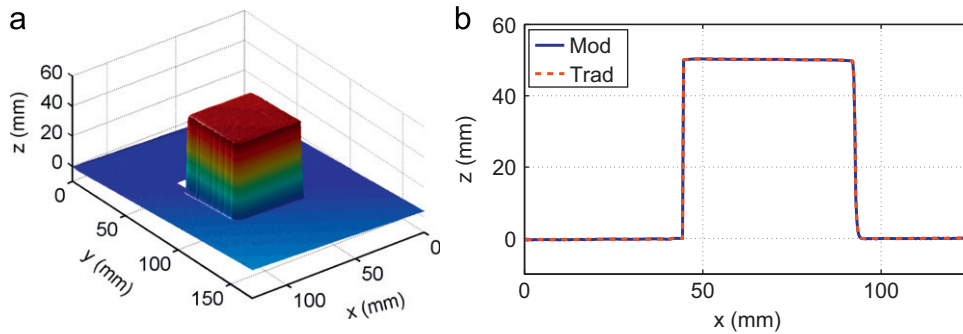


projector (Samsung SP-P310MEMX). The camera uses a 16 mm focal length lens (Computar M1614-MP), with an image resolution of  $640 \times 480$ . The projector and the camera remained untouched between experiments.

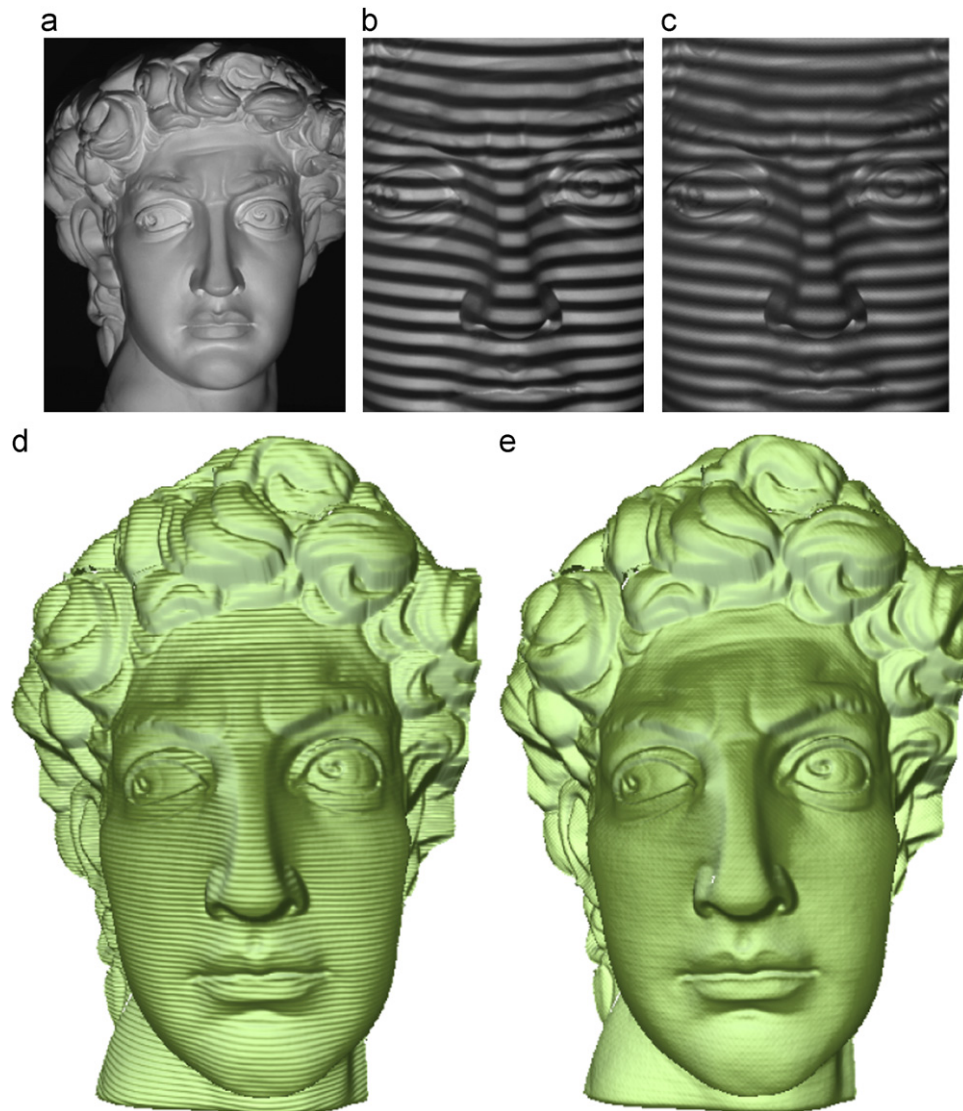
We first measured a uniform white board with fringe period of 16 pixels. Fig. 2 shows the results. Fig. 2(a) and (b) respectively

shows the modulated pattern and the SBM pattern. Their cross sections are shown in Fig. 2(c), which depicts that the area-modulated pattern appears triangular while the SBM shows clear binary structures.

Fig. 3 plots the phase errors that were computed as the method introduced in [3]. Again, it shows that the phase error



**Fig. 4.** Step-height object measurement result. (a) 3D plot; (b) cross sections of 3D result from modulated pattern and from the conventional sinusoidal fringe projection technique.



**Fig. 5.** Measurement result of a complex statue. (a) Photograph; (b) zoom-in view of the square binary pattern; (c) zoom-in view of the modulated pattern; (d) 3D result using SBM; (e) 3D result using the proposed method.

from the proposed method is less than 1/3 of that from the SBM. This experiment also verified that the proposed area-modulation technique only results in very small phase error, thus enabling relatively higher accurate 3D shape measurement.

We also evaluated the system accuracy by measuring a known step-height (50 mm) object, and compared it against the conventional fringe projection technique with projector's nonlinear gamma correction. Fig. 4 shows the result. In this research, the phase was unwrapped using the temporal phase unwrapping framework introduced in [8]. Fig. 4(b) shows that the difference between these two was very small: approximately rms 0.2%. This further verified the success of the proposed technique.

Finally, we measured a more complex 3D statue whose size is approximately  $190\text{ W} \times 419\text{ H} \times 165\text{ D mm}^3$ . Fig. 5 shows the measurement results. Again the measurement quality using the proposed technique is drastically better than that of the SBM when the fringe stripe is narrow (period of 16 pixels), and the projector is nearly focused.

## 5. Conclusions

This paper has presented an area-modulation technique to dramatically improve 3D shape measurement quality using the binary defocusing method when the fringe is very dense and the

projector is nearly focused. Our simulations and experiments showed that the error is approximately 1/3 of that with the square binary defocusing method. Because the proposed method can use nearly focused binary patterns, it can significantly increase the measurement depth range for the binary defocusing technique.

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