Novel phase-coding method for absolute phase retrieval

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This Letter presents a novel absolute phase recovery technique with phase coding. Unlike the conventional gray-coding method, the codeword is embedded into the phase and then used to determine the fringe order for absolute phase retrieval. This technique is robust because it uses phase instead of intensity to determine codewords, and it could achieve a faster measurement speed, since three additional images can represent more than $8(2^3)$ unique codewords for phase unwrapping. Experimental results will be presented to verify the performance of the proposed technique. © 2012 Optical Society of America

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Three-dimensional (3D) shape measurement is quite important in many areas ranging from manufacturing to medicine. Numerous techniques have been developed, including Moiré, holography, and fringe projection. Among these methods, the digital fringe projection technique has been exhaustively studied and widely applied in many fields due to its simple setup, automatic data processing, and high-speed and high-resolution measurement capabilities [1]. However, rapidly and simultaneously measuring multiple 3D objects with strong texture variations remains challenging.

There are absolute phase techniques that can solve this problem, such as two-wavelength [2,3], multiwavelength [4,5], temporal phase-unwrapping [6], and gray-coding + phase-shifting [7] methods. Among these methods, the gray-coding + phase-shifting technique is one of the better methods by designing a unique codeword assigned to each 2π phase-change period. Each codeword consists of a sequence of binary structured patterns, and the codewords change from period to period according to the rules of gray code. However, the codeword is limited to 2^M (M is the number of binary patterns used). For example, three binary coded patterns can generate up to eight codewords. Moreover, this technique is less robust for measuring high contrast surfaces, because the gray-coding method determines the code from image intensity rather than its phase.

We propose a novel approach to embedding the codeword into the phase domain. Specifically, instead of encoding the codeword into binary intensity images, the codeword is embedded into the phase range $[-\pi, +\pi)$ of phase-shifted fringe images (e.g., three images for a three-step phase-shifting algorithm). Since the codeword is a finite number that can quantify the phase into discrete levels, this technique has the following merits: (1) less sensitivity to surface contrast, ambient light, and camera noises; (2) fast measuring speed, since it only requires three additional images to determine a fringe order larger than eight (better than a gray-coding method). Therefore, this method provides a novel method for absolute phase retrieval and has the potential to improve the measurement quality, especially when simultaneously measuring multiple high-contrast objects.

Phase-shifting methods are widely used in optical metrology because of their speed and accuracy [8]. A three-step phase-shifting algorithm with a phase shift of $2\pi/3$ can be described as

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos(\phi - 2\pi/3), \quad (1)$$

$$I_2(x,y) = I'(x,y) + I''(x,y)\cos(\phi), \quad (2)$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos(\phi + 2\pi/3), \quad (3)$$

where $I'(x,y)$ is the average intensity, $I''(x,y)$ the intensity modulation, and $\phi(x,y)$ the phase to be solved for. Solving Eqs. (1)–(3) leads to

$$\phi(x,y) = \tan^{-1}\left[\sqrt{3}(I_1 - I_3)/(2I_2 - I_1 - I_3)\right]. \quad (4)$$

The arctangent function will result a value range of $[-\pi, +\pi)$ with 2π discontinuities. A spatial phase-unwrapping algorithm [9] can be used to remove the 2π discontinuities but has problems when there are abrupt surface changes or multiple isolated objects need to be simultaneously measured. The absolute phase techniques, such as the aforementioned gray-coding combined with phase-shifting methods, can circumvent this problem via spatial phase unwrapping, but they have their limitations, as addressed previously.

Unlike conventional codeword generation from binary intensity images, we propose embedding the codeword into phase domain. Figure 1 shows the principle of the proposed phase-coding framework. Figure 1(a) shows that from three phase-shifted fringe images, the wrapped phase $\phi(x,y)$ with 2π discontinuities can be obtained. If we know a stair phase $\phi^s(x,y)$ with the stair changes

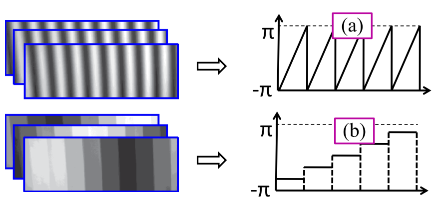

Fig. 1. (Color online) Principle of the proposed technique. (a) Wrapped phase obtained from sinusoidal fringe patterns, (b) codewords extracted from encoded fringe patterns.

perfectly aligned with the 2π discontinuities, the stairs can be used to determine the fringe order for phase unwrapping. If each stair is unique, the stair information can be treated as a codeword to remove 2π jumps point by point, and thus to obtain the absolute phase. In this research, the codeword is represented as the phase ranging from $-\pi$ to $+\pi$, with each stair height as shown in Fig. 1(b). For example, to generate N codewords, ϕ^s is quantized into N levels with a stair height of $2\pi/N$, and the phase ϕ^s is encoded with a set of phase-shifted fringe images. By this means, the coding is realized in the phase domain. It is well known that the phase is more robust to carry on information than intensity and is less sensitive to the influence of sensor noise, ambient light, surface properties, etc. This proposed technique is inherently better than the gray-coding method. Moreover, the maximum unique number it can generate for a gray-coding method is 2^M for M binary images. Therefore, it typically requires more than three patterns to recover absolute phase. In contrast, the proposed method only requires three fringe images to generate more than eight unique numbers. Therefore, it has the potential to increase measurement speed, since fewer images are required to recover one 3D frame.

Here we summarize the procedures required to retrieve absolute phase using the proposed method.

Step 1:

Embed the codeword into the phase with a stair phase function:

$$\phi^s(x,y) = -\pi + [x/P] \times \frac{2\pi}{N}. \tag{5}$$

Here $[x/P] = k$ is the truncated integer representing fringe order; P the fringe pitch, or the number of pixels per period; and N the total number of fringe periods.

Step 2:

Put the stair phase ϕ^s into phase-shifted fringe patterns:

$$I_k(x,y) = I'(x,y) + I''(x,y)\cos(\phi^s + \delta_k). \tag{6}$$

Here, δ_k is the phase shift. For the three-step phase-shifting algorithm we use, $\delta_k = -2\pi/3, 0, +2\pi/3$.

Step 3:

Obtain the stair phase from the coded fringe patterns through phase wrapping. Since the stair phase is encoded into the value ranging from $-\pi$ to $+\pi$, no spatial phase unwrapping is required.

Step 4:

Determine fringe order k from the stair phase:

$$k = \mathrm{Round}[N(\phi^s + \pi)/(2\pi)]. \tag{7}$$

Here, $\mathrm{Round}(x)$ determines the closest integer.

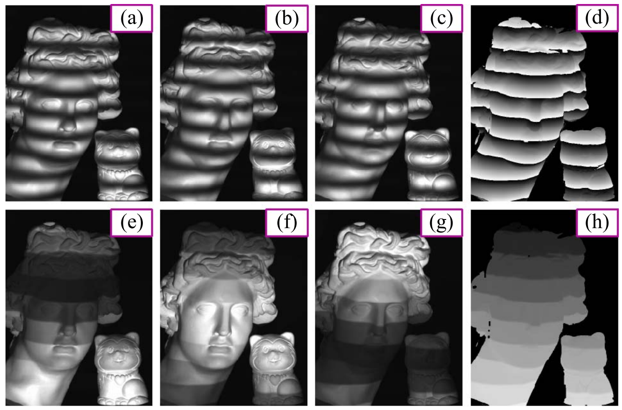

Fig. 2. (Color online) (a)–(c) Three sinusoidal phase-shifted fringe images, (d) wrapped phase map, (e)–(g) three phase encoded fringe patterns, (h) wrapped stair phase map.

Step 5:

Convert wrapped phase $\phi(x,y)$ to absolute phase. Once the fringe order k is determined, the absolute phase can be obtained by

$$\Phi(x,y) = \phi(x,y) + k \times 2\pi. \tag{8}$$

We developed a system to test the proposed method. The system includes a digital-light-processing (DLP) projector (Samsung SP-P310MEMX) and a digital CCD camera (Jai Pulnix TM-6740CL). The camera uses a 16 mm focal length megapixel lens (Computar M1614-MP). The camera resolution is 640×480, with a maximum frame rate of 200 frames/s. The projector has a resolution of 800×600 with a projection distance of 0.49–2.80 m.

To examine the viability of the absolute phase retrieval, two sculptures were simultaneously measured. The geometric dimensions are about 180 mm(H) $\times$ 100 mm(W) $\times$ 100 mm(D) and 60 mm(H) $\times$ 40 mm(W) $\times$ 40 mm(D). Three coded and three sinusoidal phase-shifted fringe patterns were projected and captured sequentially. Figures 2(a)–2(c) show the three sinusoidal phase-shifted patterns, from which the wrapped phase can be calculated, as shown in Fig. 2(d). Figures 2(e)–2(g) show the three coded phase-shifted patterns, from which the wrapped stair phase can be calculated, as illustrated in Fig. 2(h).

The stair phase we obtained varies from $-\pi$ to $+\pi$ with phase noise. Figure 3(a) shows the 140th column cross section of Fig. 2(h), which depicts small noises on the stairs. To obtain the fringe order, Eq. (7) is adopted to normalize the stair phase and to quantize the stair phase into integers $k(x,y)$. Figure 3(b) shows the resultant codeword. Figure 3(c) shows the 140th column cross section of the wrapped phase in Fig. 2(d). We can see the phase jumps and the resultant codeword changes are well aligned. Figure 3(d) shows the absolute phase after applying Eq. (8).

To verify the robustness of the proposed method, stair encoded fringe patterns with different exposure levels were captured. During the experiment, the camera aperture was adjusted to capture brighter or darker fringe

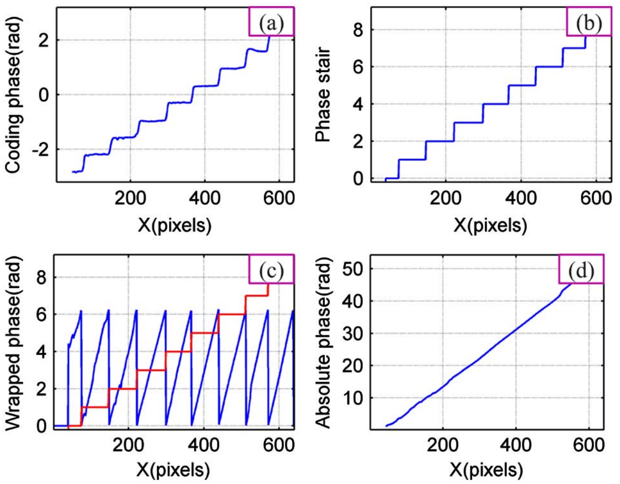

Fig. 3. (Color online) Framework for absolute phase retrieval. (a) One cross section of the original extracted coded phase, (b) codeword after normalization and quantization, (c) the same cross sections of the wrapped phase and the codeword, (d) recovered absolute phase.

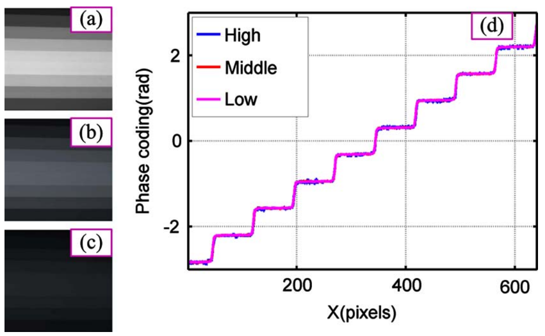

Fig. 4. (Color online) Stair phase recovery for different quality fringe patterns. (a)–(c) One of the phase coded fringe patterns with high exposure, middle exposure, and low exposure, respectively; (d) cross sections of the extracted stair coded phase.

patterns. Figures 4(a)–4(c) respectively show three fringe patterns with high exposure, middle exposure, and low exposure. Figure 4(d) shows the cross sections of the resultant stair phase maps. This experiment shows that the quality of the stair phase ϕ^s does not drastically change as the fringe image brightness drops. In contrast, our prior experience told us that it is quite difficult to correctly recover the codeword from a gray-coding method if the pattern is as dark as the one shown in Fig. 4(c). We also did a similar experiment with a very inexpensive and low-quality camera (The Imaging Source DMK 21BU04). Similar results were obtained, and thus the proposed technique is also quite robust to camera noises.

More complex sculptures were also measured under different exposures. Figures 5(a)–5(c) show the fringe patterns, and Figs. 5(d)–5(f)show the reconstructed 3D results. The same calibration method in [10] was adopted to reconstruct the 3D shape. From these results, we can see that the fringe orders were, again, correctly identified even when the fringe quality was as low as that shown in Fig. 5(c), and the 3D information was also well

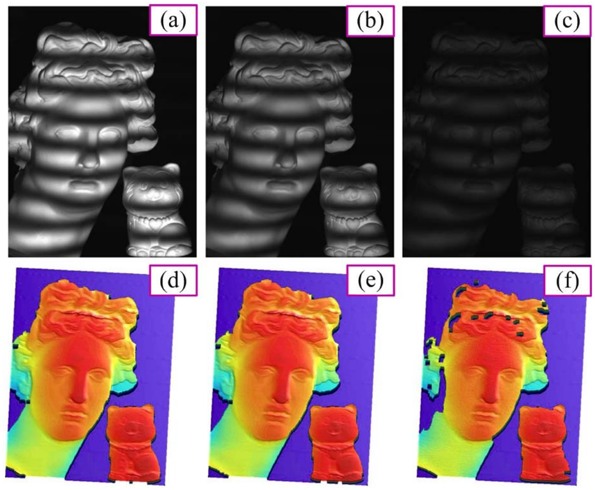

Fig. 5. (Color online) 3D shape measurement results of more complex sculptures. (a)–(c) One of the fringe patterns with different exposures, (d)–(f) the reconstructed 3D results.

recovered. This further verified that the proposed method can perform robustly for low-quality fringe patterns and can also simultaneously measure multiple objects, since the absolute phase is recovered.

We have presented a novel phase-coding method for absolute phase extraction. Unlike conventional gray-coding plus phase-shifting methods, we embed coding information in the phase. Since the coding information is carried on by the phase, it is less sensitive to surface contrast variations, ambient light, and camera noises. Besides, three additional fringe patterns are sufficient to generate more unique codewords than a conventional gray-coding method, and thus this technique has the potential to improve measurement speed. Experimental results have demonstrated the robustness of the proposed method and the capability of this technique to simultaneously measure multiple objects. It is important to note that because of camera and/or projector noise problems, the phase noise will limit the number of unique codewords to be encoded into three fringe patterns. However, because the codeword is encoded into the phase, the noise effect is drastically reduced. If the limitation indeed occurs, either more fringe patterns or a better hardware system could be used to address this problem.

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