

# Superfast multifrequency phase-shifting technique with optimal pulse width modulation

Yajun Wang and Song Zhang\*

Department of Mechanical Engineering, Iowa State University, Ames, Iowa 50011, USA

\*Corresponding author: song@iastate.edu

**Abstract:** The technique of generating sinusoidal fringe patterns by defocusing squared binary structured ones has numerous merits for high-speed three-dimensional (3D) shape measurement. However, it is challenging for this method to realize a multifrequency phase-shifting (MFPS) algorithm because it is difficult to simultaneously generate high-quality sinusoidal fringe patterns with different periods. This paper proposes to realize an MFPS algorithm utilizing an optimal pulse width modulation (OPWM) technique that can selectively eliminate high-order harmonics of squared binary patterns. We successfully develop a 556 Hz system utilizing a three-frequency algorithm for simultaneously measuring multiple objects.

© 2011 Optical Society of America

**OCIS codes:** (120.0120) Instrumentation, measurement and metrology; (320.7100) Ultrafast measurements; (120.2650) Fringe analysis.

---

## References and links

1. S. Gorthi and P. Rastogi, "Fringe projection techniques: whither we are?" *Opt. Lasers Eng.* **48**, 133–140 (2010).
2. X.-Y. Su, W.-S. Zhou, G. V. Bally, and D. Vukicevic, "Automated phase-measuring profilometry using defocused projection of a Ronchi grating," *Opt. Commun.* **94**(13), 561–573 (1992).
3. S. Lei and S. Zhang, "Flexible 3-D shape measurement using projector defocusing," *Opt. Lett.* **34**(20), 3080–3082 (2009).
4. S. Lei and S. Zhang, "Digital sinusoidal fringe generation: defocusing binary patterns VS focusing sinusoidal patterns," *Opt. Lasers Eng.* **48**(5), 561–569 (2010).
5. S. Zhang, "Flexible 3-D shape measurement using projector defocusing: extended measurement range," *Opt. Lett.* **35**(7), 931–933 (2010).
6. Y. Wang and S. Zhang, "Optimal pulse width modulation for sinusoidal fringe generation with projector defocusing," *Opt. Lett.* **35**(24), 4121–4123 (2010).
7. D. Malacara, ed., *Optical Shop Testing*, 3rd ed. (John Wiley and Sons, 2007).
8. D. C. Ghiglia and M. D. Pritt, *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software* (John Wiley and Sons, 1998).
9. K. Creath, "Step height measurement using two-wavelength phase-shifting interferometry," *Appl. Opt.* **26**(14), 2810–2816 (1987).
10. C. E. Towers, D. P. Towers, and J. D. Jones, "Optimum frequency selection in multifrequency interferometry," *Opt. Lett.* **28**(11), 887–889 (2003).
11. V. G. Agelidis, A. Balouktsis, and I. Balouktsis, "On applying a minimization technique to the harmonic elimination PWM control: the bipolar waveform," *IEEE Power Electron. Lett.* **2**, 41–44 (2004).
12. S. Zhang, D. van der Weide, and J. Olivier, "Superfast phase-shifting method for 3-D shape measurement," *Opt. Express* **18**(9), 9684–9689 (2010).
13. M. Schaffer, M. Grosse, and R. Kowarschik, "High-speed pattern projection for three-dimensional shape measurement using laser speckles," *Appl. Opt.* **49**(18), 3622–3629 (2010).

---

## 1. Introduction

Three-dimensional (3D) shape measurement is quite important in many areas ranging from manufacturing to medicine. Numerous techniques have been developed including Moiré, holog-

raphy, and fringe projection [1]. Among these methods, the digital fringe projection technique has been exhaustively studied and widely used in many fields due to its simple setup, automatic data processing, high-speed and high-resolution measurement capabilities.

For a digital fringe projection system, ideal sinusoidal fringe patterns are used to retrieve 3D information. Su et al. [2] and Lei and Zhang [3] has proposed a technique that is to generate sinusoidal fringe patterns by properly defocusing squared binary structured ones using a mechanical grating (former) or a digital video projector (latter), which we call a squared binary method (SBM). The SBM has numerous advantages for 3D shape measurement especially for high-speed applications [4]: (1) It is less sensitive to the synchronization between the camera and the projector; (2) The nonlinear gamma of the projector does not affect the measurement quality; (3) The exposure time can be much shorter than the projector's single channel projection time for an off-the-shelf digital light processing (DLP) projector, thus it could enable a superfast 3D shape measurement technique.

For the measurement of a single object without step heights, SBM can perform well if the projector is properly defocused [4]. To extend the measurement range of the defocusing technique, Zhang proposed a method to obtain absolute phase by introducing binary coded patterns [5]. Because it permits point-by-point absolute phase extraction, the defocusing technique is further enhanced and the high-speed multi-object measurement becomes possible. However, this method is not trouble free. Because sinusoidal fringe patterns are generated by defocusing, the binary coded ones are blurred, which makes it difficult to correctly determine the designed code. As a result, the perfectly designed code might not work. This problem is deteriorated for high-speed applications since motion could introduce additional misalignment.

To further improve the capability of high-speed multi-object measurement, this paper proposes a multifrequency phase-shifting (MFPS) method with an optimal pulse width modulation (OPWM) technique. OPWM is a recently developed technique to improve the fringe quality for the SBM [6]. The SBM performs well only when the fringe stripes are narrow but shows significant problems if the fringe stripes are wide. Moreover, an MFPS cannot be realized because it is impossible to obtain high quality sinusoidal fringe patterns with different breadth under the same defocusing degree. By selectively eliminate undesired harmonics, OPWM technique permits the generation of high-quality fringes with different breadths under a slightly defocused projector. Therefore, the MFPS method, an effective and widely-adopted approach for multi-object measurement, becomes feasible using the defocusing technique. Moreover, because OPWM still uses binary patterns, it maintains the aforementioned advantages of the defocusing technique. In this research, we have developed a 5,000 Hz fringe projection system, that could realize 556 Hz 3D shape measurement using a three-frequency phase-shifting method for simultaneous multiple objects measurement.

## 2. Principle

### 2.1. Three-step phase-shifting algorithm

Phase-shifting algorithms are widely used in optical metrology because of their measurement speed and accuracy [7]. Numerous phase-shifting algorithms have been developed including three step, four step, double three step, and five step. In this paper, we use a three-step phase-shifting algorithm with a phase shift of  $2\pi/3$ . Three fringe images can be described as,

$$I_1(x, y) = I'(x, y) + I''(x, y) \cos(\phi - 2\pi/3), \quad (1)$$

$$I_2(x, y) = I'(x, y) + I''(x, y) \cos(\phi), \quad (2)$$

$$I_3(x, y) = I'(x, y) + I''(x, y) \cos(\phi + 2\pi/3). \quad (3)$$

Where  $I'(x, y)$  is the average intensity,  $I''(x, y)$  the intensity modulation, and  $\phi(x, y)$  the phase to be solved for. The phase,  $\phi(x, y)$ , and the texture,  $I'(x, y)$ , can be solved for from these equations

$$\phi(x, y) = \tan^{-1} \left[ \sqrt{3}(I_1 - I_3)/(2I_2 - I_1 - I_3) \right], \quad (4)$$

$$I'(x, y) = (I_1 + I_2 + I_3)/3. \quad (5)$$

Equation (4) provides the phase ranging  $[-\pi, \pi)$  with  $2\pi$  discontinuities. This  $2\pi$  phase jumps can be removed to obtain the continuous phase map by adopting a phase unwrapping algorithm [8]. However, such a spatial phase unwrapping algorithm has limitations: it could not be used when large step height exists which may cause the phase change more than  $\pi$ , or multiple objects need to be measured simultaneously.

## 2.2. Multifrequency phase-shifting (MFPS) algorithm

As aforementioned, the phase we obtain from a single-frequency method is within the range of  $[0, 2\pi)$ . When a fringe pattern contains more than one stripes, the phase need to be unwrapped to obtain the continuous phase map. This means that if another set of wider fringe patterns (a single fringe stripe can cover the whole image) is used to obtain a phase map without  $2\pi$  discontinuities. The second phase map can be used to unwrap the other one point by point without spatial phase unwrapping. To obtain the phase of the wider fringe patterns, there are two approaches: (1) use very long wavelength directly; and (2) use two short wavelengths to generate an equivalent long one. The former is not very commonly used because it is difficult to generate high-quality wide fringe patterns due to noises or hardware limitations. Thus the latter is more frequently adopted. This subsection will briefly explain the principle of this technique.

Theoretically, the relationship between the absolute phase  $\Phi$  and the wavelength  $\lambda$ , and the height  $h(x, y)$  can be written as

$$\Phi = [C \cdot h(x, y)/\lambda] \cdot 2\pi. \quad (6)$$

Here  $C$  is a system constant. Therefore, for  $\lambda_1 < \lambda_2$  with absolute phase being  $\Phi_1$  and  $\Phi_2$ , respectively, their difference is

$$\Delta\Phi_{12} = \Phi_1 - \Phi_2 = [C \cdot h(x, y)/\lambda_{12}^{eq}] \cdot 2\pi. \quad (7)$$

Here,  $\lambda_{12}^{eq} = \lambda_1\lambda_2/|\lambda_2 - \lambda_1|$  is the equivalent wavelength between  $\lambda_1$  and  $\lambda_2$ . If  $\lambda_2 \in (\lambda_1, 2\lambda_1)$ , we have  $\lambda_{12}^{eq} > \lambda_2$ . In reality, we only have the wrapped phase,  $\phi_1$  and  $\phi_2$ . We know that the relationship between the absolute phase is  $\Phi$  and the wrapped phase  $\phi = \Phi \pmod{2\pi}$  with  $2\pi$  discontinuities. Here the modulus operator is to convert the phase to a range of  $[0, 2\pi)$ . Taking the modulus operation on Eq.(7) will lead to

$$\Delta\phi_{12} = [\Phi_1 - \Phi_2] \pmod{2\pi} = [\phi_1 - \phi_2] \pmod{2\pi}. \quad (8)$$

$\Delta\phi_{12} = \Delta\Phi_{12} \pmod{2\pi}$ . If the wavelengths are properly chosen, so that the resultant equivalent wavelength  $\lambda_{12}^{eq}$  is large enough to cover the whole range of image (i.e.,  $|C \cdot h(x, y)/\lambda_{12}^{eq}| < 1$ ). In this case, the modulus operator does not affect anything, thus no phase unwrapping is required.

However, because the existence of noises, two-frequency technique is usually not sufficient [9]. Practically, at least three frequency fringe patterns are required for point by point absolute phase measurement. The multifrequency technique is designed so that the equivalent widest fringe stripe can cover the whole image [10].

Assume another set of fringe patterns with wavelength ( $\lambda_3$ ) are used, the equivalent wavelength between  $\lambda_1$  and  $\lambda_3$  will be  $\lambda_{13}^{eq} = \lambda_1\lambda_3/|\lambda_3 - \lambda_1|$ . We will have

$$\Delta\phi_{13} = [\phi_1 - \phi_3] \pmod{2\pi} = \{[C \cdot h(x, y)/\lambda_{13}^{eq}] \cdot 2\pi\} \pmod{2\pi}, \quad (9)$$

$$\Delta\phi_{123} = (\Delta\phi_{13} - \Delta\phi_{12}) \pmod{2\pi} = \{[C \cdot h(x, y)/\lambda_{123}^{eq}] \cdot 2\pi\} \pmod{2\pi}. \quad (10)$$



### 3. Experimental results

We have developed a superfast system that is composed of a DLP Discovery projection system, a high-speed CMOS camera, and a self-developed synchronization circuit. The DLP Discovery projection system includes a DLP Discovery board (D4000), an ALP High Speed and an optical module (S3X). The camera used in this system is Phantom V9.1. The synchronization circuit takes the projection timing signal and generate timing signal to trigger the camera.

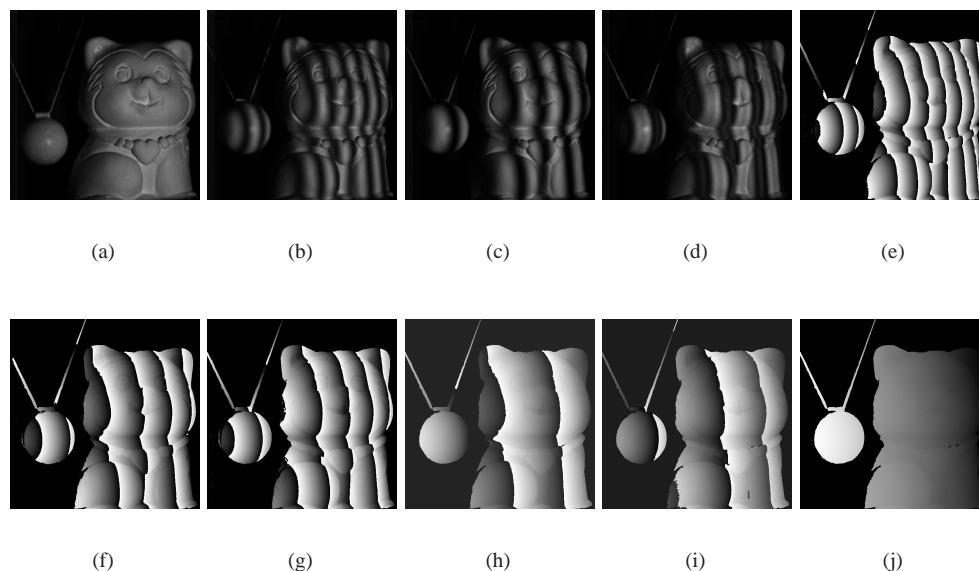


Fig. 2. (a) Photograph of the captured scene; (b) One fringe pattern ( $\lambda_1 = 60$  pixels); (c) One fringe pattern ( $\lambda_2 = 90$  pixels); (d) One fringe pattern ( $\lambda_3 = 102$  pixels); (e) Wrapped phase  $\phi_1$ ; (f) Wrapped phase  $\phi_2$ ; (g) Wrapped phase  $\phi_3$ ; (h) Equivalent phase difference  $\Delta\phi_{12}$ ; (i) Equivalent phase difference  $\Delta\phi_{13}$ ; (j) Resultant phase  $\Delta\phi_{123}$ .

To test the proposed algorithm, we chose three frequency fringe patterns with  $\lambda_1 = 60$ ,  $\lambda_2 = 90$ , and  $\lambda_3 = 102$  pixels. It can be found that the resultant equivalent fringe wavelength is 765 pixels. In other words, if we use the projector to generate 765 pixel wide fringe patterns, no spatial phase unwrapping is needed to recover absolute phase.

To demonstrate that this technique can be used to perform superfast 3D shape measurement for multiple objects, we chose our system to switch and capture 2D fringe patterns at 5,000 Hz, and measured two separate objects. The camera image resolution is set as  $576 \times 576$  pixels. In this experiment, there are two separate objects, a static sculpture on the right, and a swinging pendulum ball on the left, as shown in Fig. 2(a). The diameter of the spherical ball is about 17 mm, the stature is approximately  $60$  (H)  $\times$   $45$  (W) mm in the viewing volume of the camera. Figure 2(b)-2(d) show the distorted OPWM fringe patterns with different frequencies. Figures 2(e)-2(g) respectively show the wrapped phase maps extracted from those frequencies. The corresponding equivalent wrapped phase for  $\lambda_1$  and  $\lambda_2$  is shown in Fig. 2(h), and that for  $\lambda_1$  and  $\lambda_3$  is shown in Fig. 2(i). Finally, the phase map of the longest equivalent wavelength can be obtained from these two equivalent phase maps, and the result is shown in Fig. 2(j). It can be seen from this figure that this phase map has no  $2\pi$  discontinuities, thus no spatial phase unwrapping is needed.

From this longest equivalent wavelength phase map, the shortest one with  $\lambda_1 = 60$  pixels can be unwrapped point by point that can then be used to recover 3D information. In this research, we use the calibration technique introduced in Ref. [12] to calibrate our system. Because this is an approximation method, the accuracy is not very high. We found that with the defocusing technique, the measurement error is approximately root-mean-squared (rms) 0.19 mm over a measurement depth range of 6.35 mm. We cannot implement any existing high-accuracy structured light system calibration technique because they all require the projector to be in focus, which is not the case for our system. We are exploring a new method to accurately calibrate a defocused projector. If successful, it will significantly improve the measurement accuracy of our system. At current stage, we believe our measurement accuracy is not as high as other high-speed 3D shape measurement techniques such as the one described in Ref. [13].

Figure 3(a) shows the image after averaging three fringe patterns with the shortest wavelength. Fig. 3(b) shows the reconstructed 3D result. It can be seen that even though OPWM technique can recover 3D shape very well, it shows some artifacts on the averaged texture image. This is because the operation of OPWM is in phase domain without considering the texture information. It can be seen that the proposed method can be used to obtain absolute phase point by point, and can be used to simultaneously measure multiple objects.

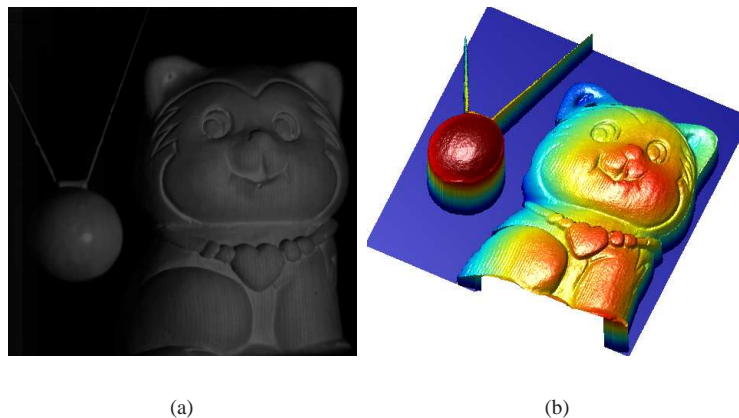


Fig. 3. (a) Averaged image of the object (Media 1); (b) 3-D result (Media 2).

Since the fringe images are captured at 5,000 Hz, and a three-frequency phase-shifting algorithm is used, the 3D shape measurement speed is actually  $5,000/9 = 556$  Hz, which is sufficient to capture the rapid motion, such as a pendulum system. From 900 2D images, 100 3D frames can be recovered. Media 1 shows the video of the averaged texture images, the video is played at 25 fps, which is approximately 1/22 of the capturing speed. The textured images,  $I'(x,y)$  in Eq. (5), are generated by averaging three shortest wavelength phase-shifted fringe patterns. Media 2 shows the 3D reconstructed results at 25 fps. This experiment clearly demonstrated that by combining the OPWM technique with the defocusing technique, a superfast MFPS algorithm can be realized. This proposed technique can be used to measure multiple rapidly moving objects simultaneously.

#### 4. Conclusion

This paper has presented a multifrequency phase-shifting method with an optimal pulse width modulation technique to conduct superfast simultaneous multiple objects measurement. We

have developed a 556 Hz 3D shape measurement system using a three-frequency phase-shifting algorithm. Experiments have been conducted and proved the success of the proposed technique.