High-resolution 3D profilometry with binary phase-shifting methods

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This paper presents a novel pixel-level resolution 3D profilometry technique that only needs binary phase-shifted structured patterns. This technique uses four sets of three phase-shifted binary patterns to achieve the phase error of less than 0.2%, and only requires two sets to reach similar quality if the projector is slightly defocused. Theoretical analysis, simulations, and experiments will be presented to verify the performance of the proposed technique. © 2011 Optical Society of America

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1. Introduction

3D profilometry based on digital sinusoidal fringe projection techniques has been playing an increasingly important role in optical metrology due to the rapid advancement of digital video display technology [1]. However, challenges remain to perform high-quality 3D shape measurement with off-the-shelf digital video projectors. One of the major issues is to generate ideal sinusoidal fringe patterns because of the projector's nonlinear gamma effect.

To circumvent this problem, we recently reported a technique that only needs binary structured patterns [2]. The sinusoidal fringe patterns were generated by properly defocusing the binary structured ones. Su *et al.* has also used the defocusing technique to generate ideal sinusoidal fringe patterns by defocusing with a Ronchi grating [3]. However, because it uses a mechanical grating, the phase-shift error is dominant. In contrast, because the digital binary fringe projection technique does not have a phase-shift error, it turned out to have more advantages besides eliminating the nonlinear gamma problem: it allows for an unprecedentedly high-speed 3D profilometry with a phase-shifting technique using the digital-light-processing (DLP) Discovery projection platform [4], and permits the 3D profilometry speed bottleneck elimination of an off-the-shelf DLP projector [5]. However, this technique is not trouble-free. Because the projector must be properly defocused to generate high-quality sinusoidal fringe patterns, there are two major problems: (1) the smaller measurement range, and (2) the challenge of calibrating the defocused projector [2].

This paper proposes a novel method that only requires binary structured patterns to realize pixel-level spatial resolution 3D profilometry. This technique is based on our theoretical analysis and experimental findings. Our theoretical analysis shows that the low frequency (less than eleventh order) harmonics of a square wave only introduce 6X phase error for a three-step phase-shifting technique with a phase-shift of 1/3 period (or $2\pi/3$), and this phase error can be significantly eliminated by averaging two sets of fringe patterns with a phase-shift of 1/12 period (or $\pi/6$). If the projector is slightly defocused, meaning that the frequency components beyond tenth order harmonics are suppressed to a negligible level, the phase error caused by the binary patterns could be reduced to be less than RMS 0.2%, which is less than the quantization error of an 8 bit camera ($1/2^8 \approx 0.4\%$).

Our further analysis shows that when the projector is close to being in focus, the next dominant error frequency doubled, i.e., 12X. This type of phase error can be eliminated by introducing another two sets of fringe patterns with a phase-shift of 1/24 period (or $\pi/12$) from the first two sets. Averaging the phases

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obtained from these four sets will bring down the error to be less than 0.2%. Because under most measurement scenarios, the projector or the camera are not perfectly in focus, using these 12 binary patterns is sufficient to achieve high-quality 3D profilometry.

Of course, if higher accuracy is required, additional sets of binary phase-shifted patterns with similar analysis can be adopted to further improve the measurement quality. By this means, only binary patterns are necessary to achieve the spatial resolution of the conventional sinusoidal fringe patterns based method. Therefore, this technique allows for achieving high measurement accuracy with binary patterns instead of sinusoidal ones, which might introduce better means for 3D profilometry because it is significantly easier to generate binary patterns than to generate ideal sinusoidal ones.

Section 2 introduces the principle sinusoidal and binary phase-shifting algorithms. Section 3 shows some simulations to verify the proposed binary phase-shifting techniques. Section 4 presents some experimental results, and Sec. 5 summarizes this paper.

2. Principle

A. Single Three-Step Phase-Shifting Algorithm

Phase-shifting methods are widely used in optical metrology because of their speed and accuracy [6]. A single three-step phase-shifting (SPS) algorithm with a phase-shift of $2\pi/3$ can be described as

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos(\phi - 2\pi/3), \quad (1)$$

$$I_2(x,y) = I'(x,y) + I''(x,y)\cos(\phi), \quad (2)$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos(\phi + 2\pi/3). \quad (3)$$

Where $I'(x,y)$ is the average intensity, $I''(x,y)$ the intensity modulation, and $\phi(x,y)$ the phase to be solved for. Simultaneously solving these three equations gives the phase

$$\phi(x,y) = \tan^{-1}\left[\sqrt{3}(I_1 - I_3)/(2I_2 - I_1 - I_3)\right]. \quad (4)$$

The phase obtained in Eq. (4) ranges from $-\pi$ to $+\pi$ with 2π discontinuities. A phase unwrapping algorithm can be adopted to obtain the continuous phase [7]. The phase unwrapping is to locate the 2π discontinuity positions and remove them by adding or subtracting multiples of 2π. In other words, the phase unwrapping step is to find an integer number $k(x,y)$ for each point (x,y) so that the continuous phase can be obtained as

$$\Phi(x,y) = 2\pi \times k(x,y). \quad (5)$$

Here, $\Phi(x,y)$ is the unwrapped phase. Once the continuous phase map is obtained, 3D shape can be recovered if the system is calibrated [8].

B. Dual Three-Step Phase-Shifting Algorithm

The SPS works well if the fringe patterns are ideally sinusoidal in profile. However, for binary phase-shifted fringe patterns, if the projector is not properly defocused, some binary structures will appear, and the phase error will be significant. To learn how to compensate for this type of phase error, the binary structured patterns are analyzed.

The cross section of a binary structured pattern is a square wave, thus, understanding the effect of a binary structured pattern can be simplified to study a square wave. A normalized square wave with a period of 2π can be written as

$$y(x) = \begin{cases} 0 & x \in [(2n-1)\pi, 2n\pi) \\ 1 & x \in [2n\pi, (2n+1)\pi) \end{cases}. \quad (6)$$

Here, n is an integer number. The square wave can be expanded as a Fourier series

$$y(x) = 0.5 + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin[(2k+1)x]. \quad (7)$$

To understand how each harmonics affects the measurement quality, we analyzed the phase error by each frequency component. The phase error is obtained by finding the difference between the base phase $\Phi^b(x,y)$ obtained from the fundamental frequency and the phase obtained from the combination of the fundamental frequency and the particular high-frequency harmonics, $\Phi^k(x,y)$. The base phase can be theoretically computed by applying Eq. (4). The fringe patterns with $(2k+1)$-th order harmonics frequencies can be written as

$$I_1^k(x,y) = I'(x,y) + I''(x,y)\{\cos(\phi - 2\pi/3) + \cos[(2k+1)(\phi - 2\pi/3)]/(2k+1)\}, \quad (8)$$

$$I_2^k(x,y) = I'(x,y) + I''(x,y)\{\cos(\phi) + \cos[(2k+1)\phi]/(2k+1)\}, \quad (9)$$

$$I_3^k(x,y) = I'(x,y) + I''(x,y)\{\cos(\phi + 2\pi/3) + \cos[(2k+1)(\phi + 2\pi/3)]/(2k+1)\}. \quad (10)$$

Similarly, the wrapped phase can obtained by using a similar equation as Eq. (4)

$$\phi^k(x,y) = \tan^{-1}\left[\sqrt{3}(I_1^k - I_3^k)/(2I_2^k - I_1^k - I_3^k)\right]. \quad (11)$$

Phase $\phi^k(x,y)$ can be unwrapped to determine the $\Phi^k(x,y)$. The phase error is thus defined as

$$\Delta\Phi^k(x,y) = \Phi^k(x,y) - \Phi(x,y). \quad (12)$$

Figure 1 shows the phase error for each harmonics if a three-step phase-shifting algorithm is utilized. It shows that the third, ninth, and fifteenth harmonics

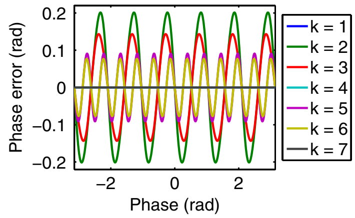

Fig. 1. (Color online) Phase error caused by different harmonics. k is the integer used in Eq. (7).

($k = 1$, 4, 7) will not bring phase error because a phase-shift of $2\pi/3$ is utilized. It also indicates that the low frequency harmonics (less than eleventh) introduces 6X phase error. If another phase map with a phase-shift of 1/12 period (or $\pi/6$) is obtained, it can be used to compensate for this error by averaging it with the other phase map. Because two sets of fringe patterns are used, this technique is called the dual three-step phase-shifting (DPS) method.

C. Quadratic Three-Step Phase-Shifting Algorithm

Figure 1 also shows that the secondary phase error is 12X. This means that if another two sets of fringe patterns have a phase-shift of 1/24 period (or $\pi/12$) between the first two sets, the 12X phase error can be eliminated. Because four sets of fringe patterns are used to eliminate both 6X and 12X phase error, this technique is called the quadratic three-step phase-shifting (QPS) method. After applying QPS, the residual phase errors are induced by harmonics higher than fifteenth order, which can be negligible if the patterns are not perfectly focused (or squared). It is important to notice that the 12X phase error is induced by over tenth-order harmonics. If the patterns are slightly defocused, this type of error can be suppressed to a negligible level, thus, the DPS method might be sufficient.

3. Simulations

Simulations were performed to verify the performance of the proposed methods. In this simulation, we simulated a square wave with a fringe period of 96 pixels. The defocusing is realized by applying a Gaussian smoothing filter, and the different degrees of defocusing are achieved by using different breath of filters. Larger size of Gaussian filters are realized by applying smaller size ones multiple times [9]. In this research, we utilize a 9-pixel Gaussian filter with a standard deviation of 1.5 pixels. If this filter is applied once, the square wave will be deformed as shown in Fig. 2(a). Because the filter size is very small in comparison with the square wave period (96 pixels), the shape of the square wave is well preserved. If an SPS algorithm is applied, the phase error is very large (RMS 0.22 rad or 3.46%) as shown in Fig. 2(b). The DPS method reduces the error to be 1.07% (3.5 times smaller). The phase error obtained from the QPS method makes is negligible (0.10%) in comparison with the quantization error of an 8 bit camera ($1/2^8 \approx 0.4\%$). This simulation confirmed that the QPS algorithm can generate satisfactory result even when the binary patterns are close to ideal.

If the same filter applies four times, the square wave will be further deformed but still has clear binary structures, as shown in Fig. 2(c). Figure 2(d) shows that the phase errors are RMS 1.61%, 0.11%, and 0.02% for the SPS, DPS, and QPS methods, respectively. It should be noticed that the DPS can reduce the phase error to be approximately 0.11%, which is negligible. This simulation confirmed that when the binary patterns are slightly defocused, the DPS is sufficient to provide high-quality 3D shape measurement.

4. Experiments

Experiments were also conducted to test the proposed method. In the experiment, we used a USB CCD camera (The Imaging Source DMK 21BU04) and the LED digital-light-processing projector (Dell M109S). The camera is attached with a 12 mm focal length Megapixel lens (Computar M1214-MP). The resolution of the camera is 640×480. The projector has a resolution of 858×600 with a lens of $F/2.0$ and $f = 16.67$ mm.

We first measured a uniform white surface with the proposed technique. Figure 3 shows the measurement results. In this experiment, we used a very wide fringe pattern, where the fringe pitch (number of pixels per period) is 96 projector's pixels. The first row shows the results when the fringe patterns are close to being in focus. The binary structures are clearly shown in the fringe patterns as illustrated in Fig. 3(b). Figure 3(c) shows that even on the wrapped phase map, the phase error is very obvious. Figures 3(d)–3(f) show the phase error maps for the SPS, DPS, and QPS, respectively. It can be seen from

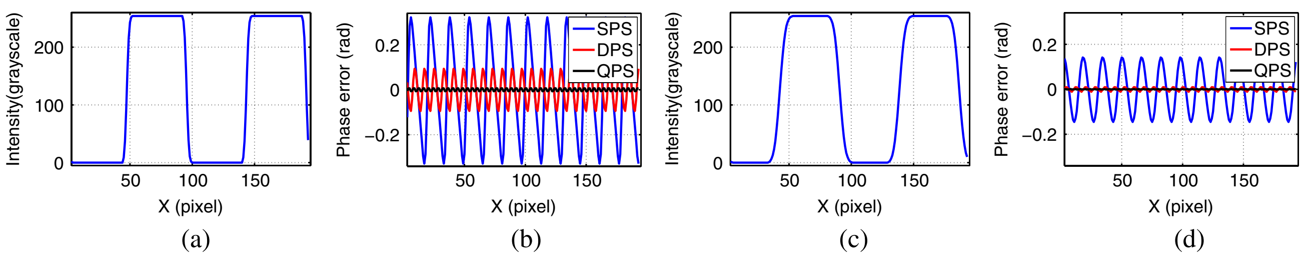

Fig. 2. (Color online) Phase errors with using different phase-shifting methods under different degrees of defocusing. (a) Close to being an ideal square wave; (b) Phase errors for the SPS, DPS, and QPS methods are RMS 3.46%, 1.07%, and 0.10%, respectively; (c) Slightly blurred square wave; (d) Phase errors for the SPS, DPS, and QPS methods are RMS 1.61%, 0.11%, and 0.02%, respectively.

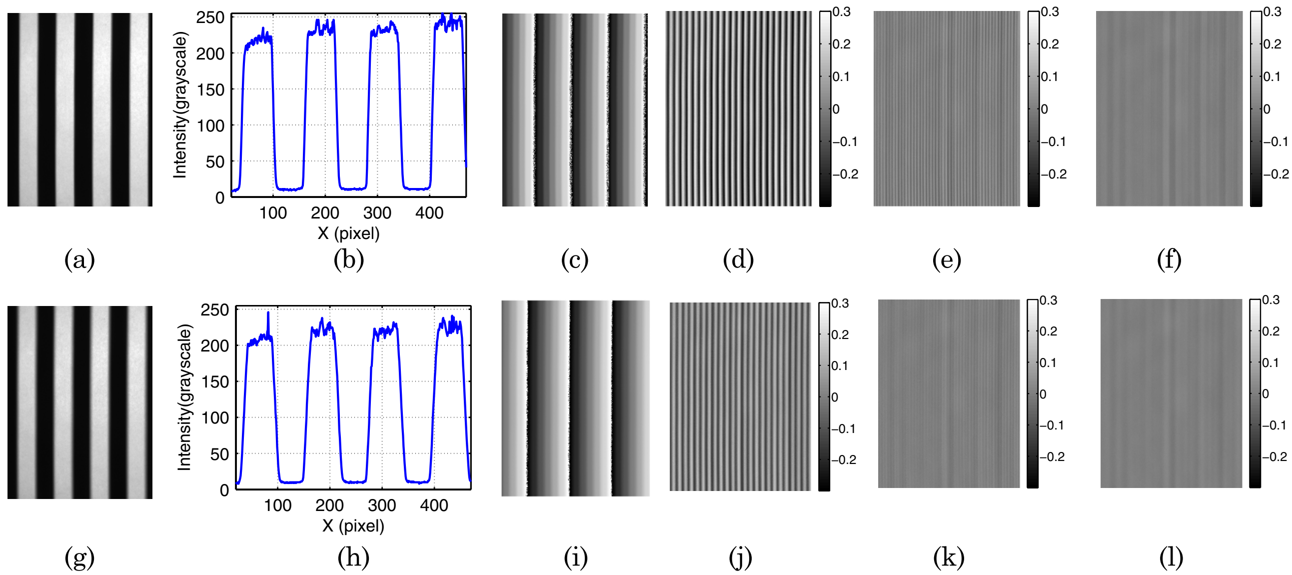

Fig. 3. (Color online) Phase errors with using different phase-shifting methods. (a) Fringe pattern is close to being in focus; (b) 320th row cross section; (c) Wrapped phase map; (d)–(f) Phase error maps for the SPS, DPS, and QPS methods, respectively; (g) Fringe pattern is slightly defocused; (h) 320th row cross section; (i) Wrapped phase map; (j)–(l) Phase error maps for the SPS, DPS, and QPS methods, respectively.

the error map that the SPS method generates significant error, while the QPS method reduces the error to a negligible level. The second row of Fig. 3 shows that when the fringe patterns are slightly defocused, the DPS method is sufficient to perform high-quality measurement.

Figure 4 shows the cross sections of the phase error maps shown in Fig. 3. Figure 4(a) shows the phase errors generated by different methods when the projector is close to being in focus. It can be seen that when the projector is close to being in focus, the SPS method clearly does not generate reasonable quality of measurement, while the DPS method improves its quality dramatically. And the phase error caused by the QPS method is less than 0.21%, which is very low. Figure 4(b) shows the results when the projector is slightly defocused. For this case, the DPS and the QPS does not make much difference, thus, a DPS method is sufficient to perform high-quality measurement. These experiments demonstrated that the real measurements conform to our simulation results.

A more complex 3D sculpture was also measured. Figure 5 shows the results with different SPS, DPS, and QPS algorithms under the same defocusing degree. These experiments indicated that when the projector is close to being in focus, a QPS method can perform high-quality 3D profilometry, while only the DPS method is needed for slightly defocused binary patterns. It should be noted that the binary patterns are very wide: the fringe pitch is 96 projector's pixel.

5. Summary

This paper has presented binary phase-shift methods for high-resolution 3D profilometry. Because this technique allows the binary method to perform pixel-level spatial resolution when the projector is close to being in focus, it solved the two very challenging problems (smaller depth range and difficulty of defocused projector calibration) for the technique

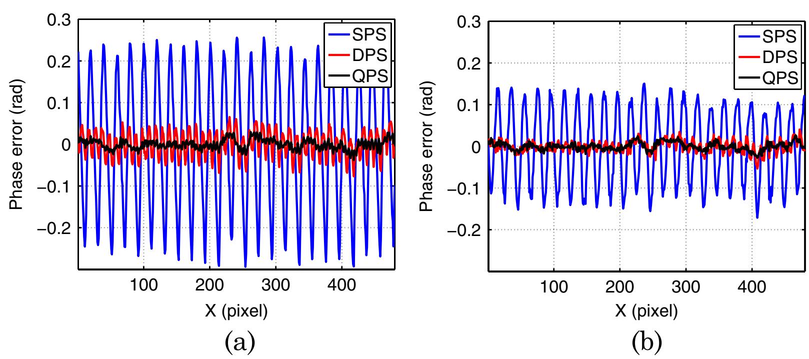

Fig. 4. (Color online) Phase errors with using different phase-shifting methods. (a) Cross sections of the phase error shown in the first row of Fig. 3. The phase errors are 2.67%, 0.46%, and 0.17% for the SPS, DPS, and QPS methods, respectively; (b) Cross sections of the phase error shown in the second row of Fig. 3. The phase errors are 1.48%, 0.21%, and 0.14% for the SPS, DPS, and QPS methods, respectively.

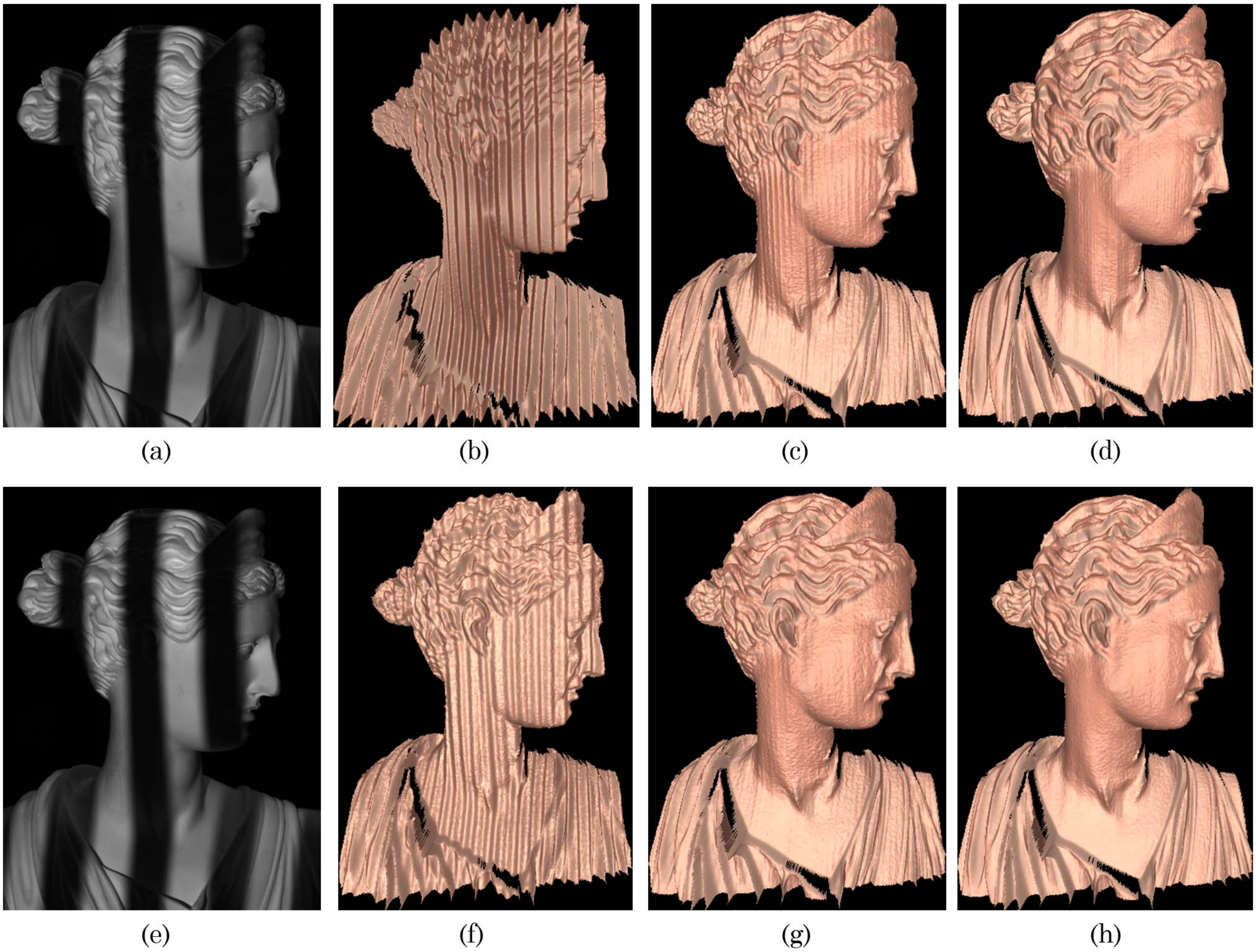

Fig. 5. (Color online) Experimental results with different binary phase-shifting methods. The top row shows the results when the projector is close to being in focus, and the bottom row shows the results when the projector is slightly defocused. (a) One of the binary fringe patterns; (b) 3D result with the SPS method; (c) 3D result with the DPS method; (d) 3D result with the QPS method; (e) One of the binary fringe patterns; (f) 3D result with the SPS method; (g) 3D result with the DPS method; (h) 3D result with the QPS method.

to achieve sinusoidal phase-shifting methods by defocusing binary structured ones. This technique, however, is at the cost of increasing the number of fringe patterns used, which will reduce the measurement speed. Nevertheless, this proposed method has great value in 3D profilometry when only binary patterns can be used (e.g., grating), and it simplifies the digital fringe projection system development without worrying about the nonlinearity of the projector.

References

1. S. Gorthi and P. Rastogi, "Fringe projection techniques: Whither we are?," Opt. Lasers Eng. **48**, 133–140 (2010).
2. S. Lei and S. Zhang, "Flexible 3D shape measurement using projector defocusing," Opt. Lett. **34**, 3080–3082 (2009).
3. X. Y. Su, W. S. Zhou, G. Von Bally, and D. Vukicevic, "Automated phase-measuring profilometry using defocused projection of a Ronchi grating," Opt. Commun. **94**, 561–573 (1992).
4. S. Zhang, D. van der Weide, and J. Olvier, "Superfast phase-shifting method for 3D shape measurement," Opt. Express **18**, 9684–9689 (2010).
5. Y. Gong and S. Zhang, "Ultrafast 3D shape measurement with an off-the-shelf DLP projector," Opt. Express **18**, 19743–19754 (2010).
6. D. Malacara, ed., *Optical Shop Testing*, 3rd ed. (Wiley, 2007).
7. D. C. Ghiglia and M. D. Pritt, *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software* (Wiley,1998).
8. S. Zhang and P. S. Huang, "Novel method for structured light system calibration," Opt. Eng. **45**, 083601 (2006).
9. I. I. Hirschman and D. V. Widder, *The Convolution Transform* (Princeton, 1955).